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Ciências

Inflaton candidates: from string theory to particle physics

Sravan Kumar Korumilli

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Orientador: Prof. Doutor Paulo Vargas Moniz

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To my loving parents

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I don't want to believe. I want to know
- Carl Sagan



The Starry Night by Vincent Van Gogh, 1889.

Resumo

A inflação constitui um paradigma essencial na cosmologia moderna. Em particular, e de acordo com os comunicados da missão *Planck* em 2015, acerca da medição da radiação cósmica de fundo, há um interesse crescente na procura de candidatos a inflação extraídos de teorias fundamentais e em testar estas propostas. Esta tese apresenta modelos inflacionários que podem ser classificados numa abordagem descendente ou ascendente nas escalas de energia. Na abordagem descendente, apresentamos estudos de cenários inflacionários ligados à teoria de cordas e à supergravidade (SUGRA), seja com campos (múltiplos) 3-formas, com o modelo Dirac-Born-Infeld Galileon, no contexto de uma teoria de campos para cordas ou ainda no modelo α -atrator SUGRA $\mathcal{N} = 1$. Na abordagem ascendente, propomos a construção de um modelo inflacionário baseado numa teoria de grande unificação, complementada com uma simetria conforme, em que estudamos, não só a inflação, mas também implicações no campo da física de partículas. O nosso trabalho de investigação inclui diferentes classes de inflação governadas por campos escalares canónicos, não canónicos ou ainda em contexto de gravidade induzida. A totalidade destes modelos é consistente com os dados obtidos na missão *Planck* e suportados por parâmetros cosmológicos cruciais como o índice espectral escalar n_s , a razão tensor para escalar r ou ainda a não-Gaussianidade primordial. O estudo abordado nesta tese reforça a expectativa que futuras missões observacionais, cujo objectivo seja detetar ondas gravitacionais primordiais e a não-Gaussianidade da radiação cósmica de fundo, possam ajudar a melhor distinguir os modelos inflacionários considerados.

Palavras-chave

Inflação, Teoria de cordas, Supergravidade, Teoria de grande unificação, Ondas gravitacionais primordiais, Não-Gaussianidades.

Abstract

Cosmic inflation is the cornerstone of modern cosmology. In particular, following the *Planck* mission reports presented in 2015 regarding cosmic microwave background (CMB), there is an increasing interest in searching for inflaton candidates within fundamental theories and to ultimately test them with future CMB data. This thesis presents inflationary models using a methodology that can be described as venturing top-down or bottom-up along energy scales. In the top-down motivation, we study inflationary scenarios in string theory and supergravity (SUGRA), namely with (multiple) 3-forms, Dirac-Born-Infeld Galileon model, a string field theory setup and $\mathcal{N} = 1$ SUGRA α -attractor models. In the bottom-up motivation, we construct a grand unified theory based inflationary model with an additional conformal symmetry and study not only inflation but also provide predictions related to particle physics. Our research work includes various classes of inflation driven by scalar fields under a canonical, non-canonical and induced gravity frameworks. All these models are consistent with Planck data, supported by key primordial cosmological parameters such as the scalar spectral index n_s , the tensor to scalar ratio r , together with the primordial non-Gaussianities. Future probes aiming to detect primordial gravitational waves and CMB non-Gaussianities can further help to distinguish between them.

Keywords

Inflation, String theory, Supergravity, Grand unified theories, Primordial gravitational waves, Non-gaussianities.

List of publications

This dissertation is based on the following publications:

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- J. Morais, M. Bouhmadi-López, K. Sravan Kumar, J. Marto, and Y. Tavakoli, “Interacting 3-form dark energy models: distinguishing interactions and avoiding the Little Sibling of the Big Rip,” *Phys. Dark Univ.* **15** (2017) 7-30, arXiv:1608.01679 [gr-qc].
- Shreya Banerjee, Suratna Das, K. Sravan Kumar and T.P. Singh, “On signatures of spontaneous collapse dynamics modified single field inflation,” *Phys. Rev. D* **95**, 103518 (2017) arXiv:1612.09131 [astro-ph.CO].
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List of Abbreviations

Λ CDM	Λ cold dark matter
AdS/dS	(Anti-) de Sitter
CAM	Cosmological attractor models
CMB	Cosmic Microwave Background
CW	Coleman-Weinberg
DBIG	Dirac-Born-Infeld Galileon
EFT	Effective field theory
GL	Goncharov-Linde
GS	Gong and Sasaki
GUTs	Grand unified theories
FLRW	Friedmann-Lemaître-Robertson-Walker
LSS	Large Scale Structure
RHNs	Right handed neutrinos
SBCS	Spontaneous breaking of conformal symmetry
SM	Standard model
SFT	String field theory
SUSY	Supersymmetry
SUGRA	Supergravity
SV	Shafi-Vilenkin
TC	Tachyon condensation

1

Introduction

It took less than an hour to make the atoms, a few hundred million years to make the stars and planets, but five billion years to make man

- George Gamow, *The creation of the Universe*

The classical Big Bang cosmology scenario proposed by G. Gamow in 1946 [1], was supported by the first detection of Cosmic Microwave Background (CMB) reported by A. A. Penzias and R. W. Wilson in 1965 [2]. However, such setting suffered from serious difficulties that became known as horizon and flatness problems [3]. Moreover, the development of Grand Unified Theories (GUTs) in the late 70's [4] predicting the unification of strong, electromagnetic and weak interactions at the energy scales $\sim 10^{16}$ GeV, revealed the possible over production of magnetic monopoles, in the early Universe, which was known as monopole problem [5]. These problems could be solved by means of an accelerated (near de Sitter) expansion of the Universe, as proposed by A. A. Starobinsky and A. H. Guth [6, 7], which is designated as *the theory of cosmological inflation*. This theory was subsequently improved by the proposals of, A. D. Linde [8, 9], A. Albrecht and P. J. Steinhardt [10], which were known as *chaotic inflationary scenario* and *new inflationary scenario*, respectively. Afterwards, V. F. Mukhanov, G. V. Chibisov and S. W. Hawking [11, 12] provided an explanation for the Large Scale Structure (LSS) formation seeded by primordial quantum fluctuations, which made the theory of inflation observationally attractive. In order to have an adequate particle production at the end of inflation, a reheating process [13, 14] is expected, constituting an intermediate stage in the evolution of the Universe, subsequently leading into *a radiation dominated era and then a matter dominated era*. Currently, the *inflationary paradigm* has been widely accepted and stands as an essential mechanism abridging the epoch of quantum gravity, the theory of all fundamental interactions and elementary particles (i.e, physics near the Planck energy scale), to our present day understanding of particle physics. In summary, the theory of inflation combines features imported from particle physics, astrophysics and cosmology to border and connect to a theory of everything.

Once the exponential expansion begins, the Universe rapidly becomes homogeneous, isotropic and spatially flat, which can be described by a Friedmann-Lemaître-Robertson-Walker (FLRW) metric. During the inflationary regime, the Universe scale factor $a(t)$ (which is a function of cosmic time t) increases exponentially, leaving the Hubble parameter $H = \frac{1}{a} \frac{da}{dt}$ almost constant [15]. This implies the comoving Hubble radius $(aH)^{-1}$ to decrease during this period i.e., $\frac{d}{dt} \left(\frac{1}{aH} \right) < 0$ for the time $t_* < t_e$, where t_* , t_e mark the beginning and the end of inflation, respectively. To solve the horizon and flatness problems it is essential that the scale factor during inflation should increase at least $N = 50 - 60$ number of e -foldings where $N = \ln \left(\frac{a(t_e)}{a(t_*)} \right)$ [15].

The required inflationary dynamics can be retrieved either by modifying General Relativity or by the addition of hypothetical matter fields which means modifying either left hand or right hand side of the Einstein equations, given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{m_{\text{P}}^2}T_{\mu\nu}, \quad (1.1)$$

where we fix the units $\hbar = 1$, $c = 1$, $m_{\text{P}}^2 = \frac{1}{8\pi G}$ with the value of reduced Planck mass $m_{\text{P}} = 2.43 \times 10^{18}$ GeV. Here $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, $T_{\mu\nu}$ is the energy-momentum tensor and $g_{\mu\nu}$ is the spacetime metric tensor¹. The scalar field responsible for inflation is usually named as inflaton, when it is a hypothetical matter field or scalaron when it emerges from modified gravity. In this thesis, we are mainly interested in finding inflaton candidates. An adequate period of exponential expansion ending in a reheating epoch can be met when the so called slow-roll parameters ε , η satisfy the following conditions during inflation [16]

$$\varepsilon = -\frac{\dot{H}}{H^2} \ll 1, \quad \eta = \frac{\dot{\varepsilon}}{H\varepsilon} \ll 1, \quad (1.2)$$

where over dot indicates the differentiation with respect to t .

The temperature fluctuations in the CMB are caused by the primordial quantum fluctuations of the scalar degrees of freedom during inflation [15]. In other words, the source of inflationary expansion and LSS of the Universe can be traced back to the dynamics and nature of one or more scalars. Inflationary background fluctuations (primordial modes) are created quantum mechanically at subhorizon scales $k \gg aH$, where k is the comoving wavenumber. The CMB temperature anisotropy and LSS can be explained by the evolution of those fluctuations on superhorizon scales $k \ll aH$.

The quantum fluctuations during inflation can be depicted by the curvature perturbation ζ in the comoving gauge². In the case of single field inflation, ζ gets conserved on superhorizon scales. Therefore, the fluctuations are adiabatic and the power spectrum measured at the time of horizon exit $k \sim aH$, is related to the temperature anisotropies in the CMB [17, 18]. Whereas in the case of multifield inflation, ζ evolves on the superhorizon scales as it is additionally sourced by isocurvature modes. In this case ζ is computed either by using transfer functions or the so called δN formalism [19-23].

The key observables of inflationary scenarios are related to the two-point and higher order correlation functions of curvature perturbation (see Appendix. A.1 for a brief review). The two-point correlation function of ζ defines the scalar power spectrum which predicts the Gaussian distribution of density fluctuations. Inflationary expansion obeying conditions (1.2) predicts that the scalar power spectrum would depart from exact scale invariance. This is quantified by a parameter named scalar spectral index, or scalar tilt, n_s that should differ from unity.

The other prediction of inflationary theory is the primordial gravitational wave power spectra, that can be defined in a similar way to the scalar power spectrum as a two point correlation function of tensor modes. The ratio of tensor to scalar power spectrum r and the tensor tilt n_t , defined in a similar way as n_s , are crucial to test any model of inflation against observations. In the context of the recent results from *Planck* satellite in 2015 [24, 25] and the joint analysis of BICEP2/Keck Array and *Planck* (BKP) [26], the single field inflationary paradigm, emerges as

¹Throughout the thesis, we set the metric signature $(-, +, +, +)$, small Greek letters are the fully covariant indexes.

²The details of other gauge choices can read from [15]. Throughout this thesis we use the notation for curvature perturbation either ζ or \mathcal{R} [15], bearing the fact that the curvature perturbation defined on uniform density hypersurfaces ζ and the comoving curvature perturbation \mathcal{R} are nearly equal in the slow-roll inflation (see [15] for details).

adequate to generate the observed adiabatic, nearly scale invariant and the highly Gaussian density fluctuations imprinted as the CMB temperature anisotropies. Moreover, the data is very much consistent with Λ CDM model³ of the current Universe and the results also indicate that we live in a spatially flat Universe [27].

The CMB observations from *Planck* 2015 [24], constrains the scalar spectral index and the tensor to scalar ratio as

$$n_s = 0.968 \pm 0.006, \quad r < 0.09, \quad (1.3)$$

with respect to *Planck* TT+lowP+WP at 95% confidence level⁴ (CL) which rules out scale invariance at more than 5σ [24]. Furthermore, the data suggests a small running of the spectral index $dn_s/d\ln k = -0.003 \pm 0.007$, which is consistent with the prediction from single field models of inflation [18]. So far, there is no significant detection of primordial tensor modes (from the value of r), which is crucial to fully confirm the inflationary paradigm. A concrete measurement of r relates to an observation of the so called B-mode polarization amplitude, which can only be caused by primordial tensor modes in the CMB radiation. Moreover, the latest results suggest so far no evidence for a blue tilt of the gravitational wave power spectra i.e., $n_t > 0$ from a very preliminary statistical analysis [24, 26]. The proposed post-*Planck* satellites CMBPol, CORe, Prism, LiteBIRD and many other ground based experiments such as Keck/BICEP3 [29-32] are expected to reach enough sensitivity to detect B-modes and establish if $r \sim \mathcal{O}(10^{-3})$.

There is a considerable variety of different models of inflation that can be motivated theoretically, but the degeneracy of the predictions from various models of inflation is an ongoing problem for cosmologists [33, 34]. One way to probe further the nature of the inflaton field is to study the statistics of the perturbations it produces beyond the two-point correlation function [35-37], starting with the three-point function. The latter is parametrized in Fourier space by the bispectrum (defined in Appendix. A.1), a function of the amplitude of three wave vectors that sum to zero as a consequence of momentum conservation. The bispectrum of “local shape”, is a function of three wave numbers that peaks in the squeezed limit where two wave numbers are much larger than the third. The bispectrum of “equilateral shape” tends to zero in the squeezed limit, but peaks when all three wave numbers are similar in size. A third shape is often considered that peaks on folded triangles, where two wave numbers are approximately half of the third. Introducing three parameters $f_{\text{NL}}^{\text{loc}}$, $f_{\text{NL}}^{\text{equi}}$ and $f_{\text{NL}}^{\text{ortho}}$, which parametrize the overall amplitude of a local, equilateral and orthonormal shapes for the bispectrum, *Planck* 2015 data [25] informs us that

$$f_{\text{NL}}^{\text{loc}} = 0.8 \pm 5.0, \quad f_{\text{NL}}^{\text{equi}} = -4 \pm 43, \quad f_{\text{NL}}^{\text{ortho}} = -26 \pm 21, \quad (1.4)$$

at 68% CL.

Any confirmation of non-Gaussianities in the CMB would be a significant information about the nature of the inflaton field [37]. For example, establishing through unequivocal observations and data analysis a local non-Gaussianity would rule out all the single field models of inflation [38].

³ Λ stands for the cosmological constant and the CDM means the Cold Dark Matter.

⁴Here TT+lowP+WP indicates the combined results of *Planck*’s angular power spectrum of temperature fluctuations with low- l polarization (of CMB radiation) likelihood analysis and polarization data from Wilkinson microwave anisotropy Probe (WP) [28].

1.1 Standard scalar field inflation and Planck data

The simplest standard mechanism to set up inflationary expansion is conveyed by a (canonical) scalar field minimally coupled to Einstein gravity dictated by the following action

$$S = \int d^4x \sqrt{-g} \left[\frac{m_{\text{P}}^2}{2} R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right], \quad (1.5)$$

where g is the determinant of the metric $g_{\mu\nu}$.

To sustain inflationary expansion long enough, the general ingredient has been that the potential $V(\varphi)$ needs to dominate over the kinetic term $-\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi$, for which the inflaton is required to be almost constant during inflation. This is achieved by the slow-roll approximation, which can be expressed in terms of potential slow-roll parameters⁵ as

$$\varepsilon_V \equiv \frac{m_{\text{P}}^2}{2} \frac{V'(\varphi)}{V(\varphi)} \ll 1, \quad \eta_V = m_{\text{P}}^2 \frac{V''(\varphi)}{V(\varphi)} \ll 1, \quad (1.6)$$

where ‘a prime’ denotes differentiation with respect to the argument φ . The scalar spectral index n_s and the tensor to scalar ratio r read as [15]

$$n_s = 1 - 6\varepsilon_V^* + 2\eta_V^*, \quad r = 16\varepsilon_V^*, \quad (1.7)$$

where “*” denotes the quantities evaluated at the horizon exit. The energy scale of inflation can be estimated as $M_{\text{inf}} \equiv V_*^{(1/4)} \simeq M_{\text{GUT}} r^{(1/4)}$ and the range of values that the field can take during inflation can be determined by the Lyth bound [39–41]. In Appendix. A.2, we summarize the observational tests of standard single field inflation.

The constraints from *Planck* and BICEP2/Keck array data [24] rule out several potentials for a standard scalar field (see Fig. 1.1), nevertheless the flat ones of the following form

$$V \sim \left(1 - e^{-\sqrt{2/3} B \varphi} \right)^{2n}, \quad (1.8)$$

became successful candidates for the description of inflation and appeared in various scenarios [33, 34, 42]. The parameter B , in the above potential, can lead to any value of $r < 0.09$ with a fixed value for n_s , namely

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{12B}{N^2}. \quad (1.9)$$

We note that the potentials of the form in (1.8) cannot be easily justified field theoretically in the standard scalar description.

In the case⁶ of $B = 1$ the potential is the same as the one in the Einstein frame description of Starobinsky’s $R + R^2$ inflation [6, 43] and also in the Higgs inflation with a non-minimal coupling [44]. Although, these two models occupy a privileged position in the $n_s - r$ plane of *Planck* 2015, it is still not possible to distinguish these two models observationally. The difference between these models is greatly expected to be found at the reheating phase [45], whose observational reach is uncertain in the near future [46].

⁵These are related to the general slow-roll parameters in (1.2) as $\varepsilon \approx \varepsilon_V$, $\eta \approx 4\varepsilon_V - 2\eta_V$ [15].

⁶The potentials with $B \neq 1$ requires more complicated realization of inflation in a fundamental theory which we will discuss later in Sec. 1.3.

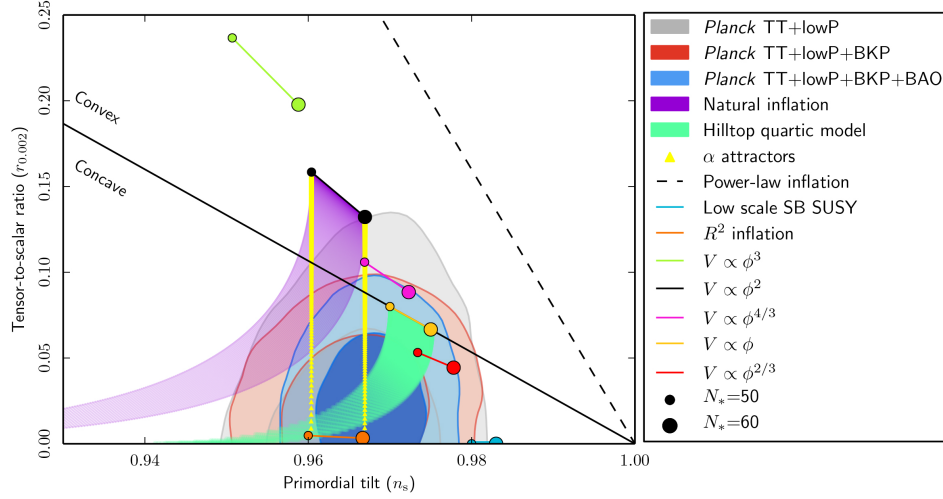


Figure 1.1: Marginalized joint 68% and 95% CL regions for n_s and r at the pivot scale $k_* = 0.002\text{Mpc}^{-1}$ from Planck in combination with other data sets, compared to the theoretical predictions of selected inflationary models.

1.2 Beyond standard scalar inflation?

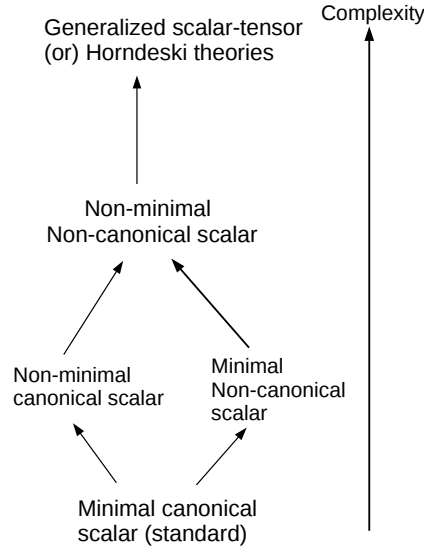


Figure 1.2: In this tree diagram we present the ways towards more elaborated inflationary model building.

The standard scalar field action can be extended (cf. Fig. 1.2) either with a non-minimal coupling to gravity (e.g., Higgs inflation [44]) or with a non-canonical kinetic term⁷. Furthermore, a general scalar-tensor theory was written and is known as Horndeski theory [48], which was shown to be equivalent to generalized Galileon model (G-inflation) [49]. The details about the G-inflationary action and calculations of perturbation spectra are presented in Appendix. A.3.

⁷In general, we can also add additional matter fields to play crucial role along with the inflaton e.g., in the case of Warm inflation, radiation plays crucial role [47]

The standard single field models predicts very much a Gaussian landscape, where any small non-Gaussianities are suppressed by the slow-roll parameters [35], whereas the non-canonical and multifield models predict detectable levels of non-Gaussianities [50-53]. In Ref. [54] shapes of non-Gaussianities in the general scalar-tensor theories were worked out, however the specific predictions are model dependent.

1.3 Top-down vs bottom-up motivations

Inflationary models can be phenomenologically realized in top-down or bottom-up (cf. Fig. 1.3) motivations as described below.

1.3.1 Top-down: Inflation in string theory/supergravity

According to the present observations, the Hubble parameter during inflation can be as large as 10^{13-14} GeV, suggesting the scale of inflation to be of the order of $M_{\text{inf}} \gtrsim 10^{15}$ GeV. These energy scales are acceptable in theories of gravity promising ultraviolet (UV) completion, such as string theory/M-theory and supergravity (SUGRA), hence argued to play a crucial role in inflation [55]. Therefore, during the last years there have been many attempts to understand the inflationary picture from the low energy effective field theories (EFTs) motivated from such fundamental approaches [42, 56-58]. The interest of studying such inflationary scenarios is that it gives the best framework to get some observational indication of these fundamental theories. There has been a plethora of inflationary models in the literature, based on several modifications of the matter or gravity sector inspired from string theory/SUGRA [33]. Given our ignorance on the relation between a UV complete theory and its low energy effective limit, there is a plenty of room to construct models [34, 57] and aim to falsify them against current and future CMB observations.

M-theory, believed to explain all fundamental interactions including gravity, that describes the physics near Planck energy scale, is defined in a 11 Dimensional (11D) spacetime and claims to unify all five versions of superstring theories⁸ [60, 61], as presented in Fig. 1.4 (taken from Ref. [62]). The idea of building inflationary models within string theory allows to test possible 4D low energy EFTs of these five superstring theories. In principle, to obtain a low energy limit of any version of superstring theory into 4D, we need to compactify six extra dimensions on small internal manifold such as Calabi-Yau⁹ and we thus are generically left with many possibilities to construct 4D EFTs [66, 67]. Studying inflation in these theories is therefore most pertinent [61, 68, 69].

Broadly, inflationary scenarios in string theory can be divided into two categories:

1. Open string inflation (e.g., Brane/Anti-brane inflation);
2. Closed string inflation (e.g., Moduli inflation).

A detailed review and recent observational status (with respect to *Planck* 2015 data) of several of these inflationary scenarios, driven by closed and open string fields, can be found in [33, 57,

⁸Which are related by T-, S- dualities [59-61]

⁹Moreover, these compactifications have to be well stabilized to accommodate sufficient conditions for inflation to happen in the resultant EFT. For example, this was successfully prescribed in type IIB string theory through KKLT and KKLMNT scenarios [63-65].

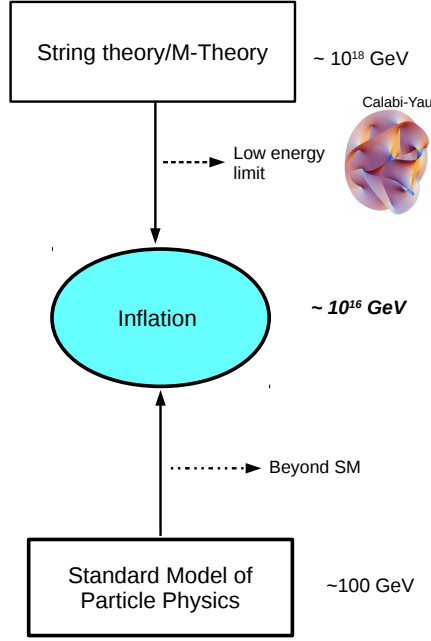


Figure 1.3: In the top-down motivation we build models in the low EFTs of string theory/M-theory which can be realized via compactifications on Calabi-Yau manifolds. In the bottom-up motivation we build models based on the physics beyond the SM of particle physics e.g., in GUTs and MSSM.

58, 70]. Inflation in string theory contains several types of scalar field terms (e.g., Dirac-Born-Infeld (DBI) inflation where the scalar field is non-canonical), with fundamentally motivated choices of potentials, which can be tested by inflationary observables. With more precise CMB data, in the future we may aim to establish the role of string theory in inflationary dynamics, and help to fulfill our understanding of a fundamental theory [71].

Supergravity (SUGRA) is a gauge theory that is an extension of General relativity where we impose a local (gauged) supersymmetry (SUSY) and most SUGRA settings constitute a low energy limit of superstring theory. There are several versions of SUGRA, characterized by the number of massless gravitinos $\mathcal{N} = 1, \dots, 8$. In particular, $D = 4, \mathcal{N} = 1$ SUGRA could be an intermediate step between superstring theory and the supersymmetric standard model of particle physics that we hope to observe at low energies [72-75]. Therefore, it is realistic to construct EFT of inflation in $D = 4, \mathcal{N} = 1$ SUGRA from high scale SUSY breaking¹⁰. Several inflationary models in the past have been constructed in $\mathcal{N} = 1$ SUGRA and they stand out to be an interesting possibility in regard of *Planck* data [42, 77]. Moreover, inflation in SUGRA has the interesting feature of predicting particle DM candidates (e.g., massive gravitino¹¹) [78]. Inflation in SUGRA is usually described by the Kähler potential as well as superpotentials, which depend on the chiral superfields [72, 79]. A brief discussion of SUSY breaking mechanisms for different SUGRA inflationary scenarios can be found in [80].

As mentioned previously, the observational data provided a special stimulus to study inflation with flat potentials of the form (1.8), which became successful candidates [33, 34, 42]. Such potentials are so far shown to occur in the low energy effective models of string theory/SUGRA

¹⁰If SUSY is not found in the current collider experiments, then high scale SUSY breaking is a natural expectation in a UV complete theory; inflation can thus be a testing ground for high scale SUSY breaking [76].

¹¹Gravitino is the supersymmetric partner of graviton with $\text{spin} = \frac{3}{2}$ which can gain mass due to SUSY breaking.

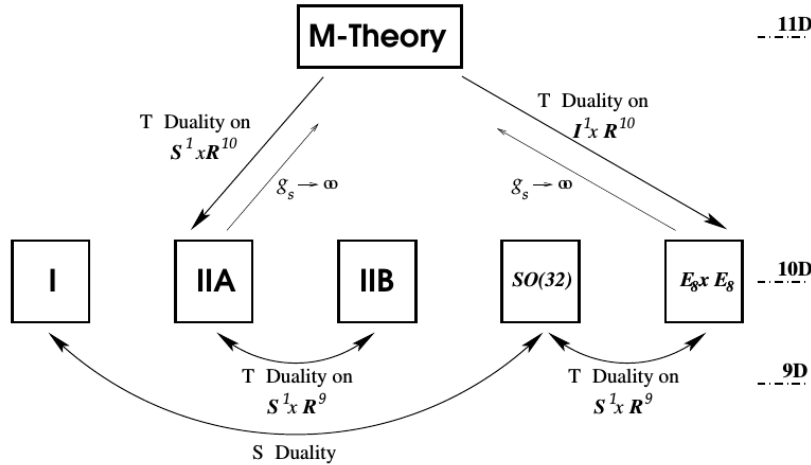


Figure 1.4: The various duality transformations that relate the superstring theories in nine and ten dimensions. T-Duality inverts the radius R of the circle S^1 or the length of the finite interval I^1 , along which a single direction of the spacetime is compactified, i.e. $R \rightarrow l_p^2/R$. S-duality inverts the (dimensionless) string coupling constant g_s , $g_s \rightarrow 1/g_s$, and is the analog of electric-magnetic duality (or strong-weak coupling duality) in four-dimensional gauge theories. M-Theory originates as the strong coupling limit of either the Type IIA or $E_8 \times E_8$ heterotic string theories.

and modified gravity [81-87]. A generic structure of Kähler potentials in SUGRA suitable for inflation and a possible connection to the open/closed string theory were studied in [88].

1.3.2 Bottom-up: Inflation and particle physics

Inflation has convincingly abridge cosmology with our present knowledge of particle physics, through the process of reheating: the scalar mode that drives expansion settles to a (true) vacuum, leading to particle production through a mechanism that depends on how the inflaton oscillates when it reaches to the minimum of the potential [13, 14]. This has strongly motivated the construction of inflationary models within the standard model (SM) of particle physics and beyond. Therefore, in a bottom-up motivation, several particle physics models were proposed including Higgs inflation, within grand unified theories (GUT) [44, 89-91] and Minimally Supersymmetric extensions of SM (MSSM) [33]. The interesting feature of a bottom-up motivation is that these models can be tested outside the scope of CMB e.g., at collider experiments.

In the particle physics context, SM Higgs inflation [44] is particularly interesting due to the fact that Higgs was the only scalar so far found at LHC [92]. Nevertheless, for Higgs to be a candidate for inflaton, it requires a large non-minimal coupling¹². On the other hand, SM is known to be incomplete due to the mass hierarchy problems e.g., the Higgs mass being very low (125 GeV) compared to GUT scale, plus nearly but not quite negligible neutrino masses ($\sim 0.1\text{eV}$). Furthermore, observed matter anti-matter asymmetry and dark matter find no explanation within SM. In this regard, inflationary models beyond SM physics i.e., GUTs and MSSM, are quite natural to explore [95, 96] (see Fig. 1.5 which is taken from [97]). The main advantage of studying inflation in SM extension theories is that in these constructions it is more natural to accommodate the reheating process after inflation and moreover, we can expand the observational tests

¹²It was known that a scalar field with large non-minimal coupling gives rise to a R^2 term considering 1-loop quantum corrections. Consequently, renormalization group (RG) analysis shows that Higgs inflation is less preferable compared to Starobinsky model [93, 94].

beyond CMB, something that is more difficult to achieve when we consider models in the string theory/SUGRA. However, on the other hand there is a hope that the GUTs and MSSM can be UV completed in heterotic superstring theories [98-100].

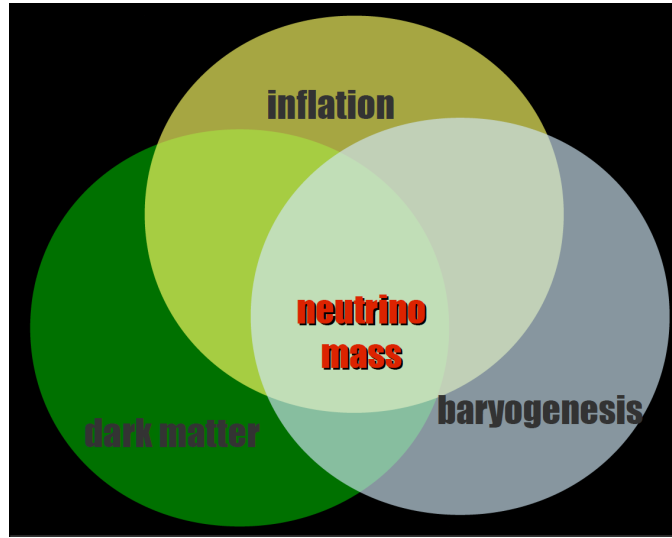


Figure 1.5: Inflation in particle physics motivated models such as GUTs and MSSM are particularly interesting, when considering neutrino masses, DM and baryogenesis. Neutrinos are worthy elements beyond SM particle physics.

1.4 Overview of the thesis

In this thesis, we study the following inflationary scenarios based on string/SUGRA and GUTs, which we show to be compatible with constraints from Planck data [24].

- Chapter 2: Multiple 3-form field inflation and non-Gaussianity

p -form fields¹³ are part of type IIA string theory [60, 61] where they generically appear as the gauge fields of SUSY multiplets. p -form fields are massless in the standard Calabi-Yau compactifications while in the low energy effective theories, p -form fields can gain mass via the Stückelberg mechanism [101, 102]. In the broader class of p -form inflation [103, 104], 3-forms are realized to be viable alternative to scalar field inflation [105, 106]. Moreover, in $D = 4$, 3-form fields are relevant and they have been studied especially in $\mathcal{N} = 1$ SUSY theories with quadratic potentials¹⁴ [107-109]. In this chapter, we generalize the single 3-form inflation with multiple 3-form fields and find suitable (phenomenological) choice of potentials compatible with observations. We also compute the corresponding generation of non-Gaussianities in this model.

- Chapter 3: DBI Galileon inflation

D-branes are fundamental objects in string theory to which open strings are attached, satisfying Dirichlet boundary condition [60]. Inflationary scenarios involving D-branes are associated with the motion of the branes in internal dimensions. These models are promising ones in string cosmology [61]. In particular, the Dirac-Born-Infeld (DBI) inflation has

¹³A covariant tensor of rank p , which is anti-symmetric under exchange of any pair of indices is called p -form.

¹⁴The 3-form potential can be generated by SUSY breaking which also breaks the gauge symmetry.

gained substantial attention in recent years [110-118] via low energy effective versions of type IIB string theory and $\mathcal{N} = 1$ SUGRA [58, 67, 119, 120]. In this chapter we study a well motivated extension of this model, known as DBI Galileon inflation, and show that it enables a wider compatibility with *Planck* data.

- Chapter 4: Effective models of inflation from an SFT inspired framework

Assuming stringy energy scales are relevant at inflation, the field theory of interacting strings i.e., string field theory (SFT) would perhaps be crucial to be accounted [121, 122]. There were early attempts of considering inflation in SFT studied with p -adic strings [123, 124]. In this chapter, admitting non-locality being the distinct feature of SFT which is associated with how the string fields interact (see Appendix. D for details), we introduce a framework motivated from open-closed string field theory coupling; the open string tachyon condensation ends up in an inflationary (in general a constant curvature) background with a stabilized dilaton field. We demonstrate that this configuration leads to interesting effective and viable models of inflation.

- Chapter 5: Non-slow-roll dynamics in α -attractors

The so-called α -attractor models are very successful with Planck data, predicting any value of $r < 0.09$ with $n_s = 0.968$ for $N = 60$. The predictions of this model are strongly connected to the mathematical features of the inflaton's kinetic term [125]. These models were first proposed in $\mathcal{N} = 1$ SUGRA in the context of superconformal symmetries [83]. In this chapter, we study the model in non-slow-roll (or) Hamilton-Jacobi formalism [126, 127], which is different from standard slow-roll approximation discussed in Sec. 1.1.

- Chapter 6: Conformal GUT inflation

Coleman-Weinberg (CW) inflation proposed by Q. Shafi and A. Vilenkin [89, 90] was the first model of inflation proposed in the context of GUTs such as $SU(5)$ and $SO(10)$, where inflation is the result of GUT symmetry breaking. In this chapter, we generalize this model with conformal symmetry whose spontaneous symmetry breaking, in addition to the GUT symmetry, flattens the CW potential. As a result we obtain $n_s \sim 0.96 - 0.967$ and $r \sim 0.003 - 0.005$ for 50 – 60 number of e -foldings. We compute the predictions for proton life time and get values above the current experimental bound [128]. We implement type I seesaw mechanism by coupling the inflaton field to the right handed neutrinos. We further study the reheating and baryogenesis in this model through non-thermal leptogenesis.

2

Multiple 3-form field inflation and non-Gaussianity

One of the basic things about a string theory is that it can vibrate in different shapes or forms, which gives music its beauty

- Edward Witten

Considered as a suitable alternative to the conventional scalar field, single 3-form inflation has been introduced and studied in Ref. [105, 106, 129, 130]. In [129] a suitable choice of the potential for the 3-form has been proposed in order to avoid ghosts and Laplacian instabilities; the authors have shown that potentials showing a quadratic dominance, in the small field limit, would introduce sufficient oscillations for reheating [129] and would be free of ghost instabilities. In [130], it was shown that single 3-form field is dual to a non-canonical scalar field whose kinetic term can be determined from the form of 3-form potential. Therefore, similar to the non-canonical scalar field 3-form field perturbations propagate with a sound speed $0 < c_s \lesssim 1$ which produces effects into inflationary observables. In [130], single 3-form inflation was shown to be consistent with $n_s = 0.97$ for power law and exponential potentials and the corresponding generation of large non-Gaussianities were studied for small values of sound speed.

In this chapter, we extend the single 3-form framework to \mathbb{N} 3-forms and explore their subsequent inflationary dynamics. We particularly focus on two 3-forms scenario for which we compute the power spectra relevant for the observations at CMB. More concretely, we obtain the inflationary observables for suitable choice of potentials and aim to falsify the two 3-forms inflationary scenario. In this regard, this chapter is divided into two main sections. The first section is dedicated to study of the different type of inflationary scenarios driven by two 3-forms. We study the evolution of curvature perturbation on superhorizon scales ($c_s k \ll aH$) effected by the dynamics of isocurvature perturbations. For this we compute the transfer functions that measure the sourcing of isocurvature modes to the curvature modes on superhorizon scales. We obtain the observables such as scalar spectral tilt and its running, tensor to scalar ratio.

In the second section, we compute the non-Gaussianities generated by two 3-forms dynamics. We compute the bispectrum using the fact that 3-form fields are dual to a non-canonical scalar fields. We compute reduced bispectrum f_{NL} on superhorizon scales using our prescription of δN formalism applied to the 3-forms. We predict the values of f_{NL} parameter in different limits of 3-momenta for the same choice of potentials studied in the first section. Finally, in Sec. 2.2.3 we confirm, in particular, that two 3-forms inflationary scenario is compatible with current observational constraints.

In this chapter we follow the units $m_{\text{P}} = 1$.

2.1 Inflation with multiple 3-forms and primordial power spectrum

This section is organized as follows. In Sec. 2.1.1 we identify basic features of \mathbb{N} 3-forms slow-roll solutions, which can be classified into two types. We also discuss how the inflaton mass can be brought to lower energy scales, for large values of \mathbb{N} . In Sec. 2.1.2 we examine the possible inflationary solutions, when two 3-forms are present. There are two classes; solutions not able to generate isocurvature perturbations (type I); and solutions with inducing isocurvature effects (type II). We show that, using a dynamical system analysis¹, the type I solutions does not bring any new interesting features than single 3-form inflation [106, 129]. Type II case, however, characterizes a new behaviour, through curved trajectories in field space. Moreover, type II inflation is clearly dominated by the gravity mediated coupling term which appears in the equations of motion. We present and discuss type II solutions for several classes of potentials, which are free from ghost instabilities [129] and show evidence of a consistent oscillatory behavior at the end of the two 3-forms driven inflation period. In addition, we calculate the speed of sound, c_s^2 , of adiabatic perturbations for two 3-forms and show it has significant variations during inflation for type II solutions. Therefore, our major objective in this chapter is to understand and explore the cosmological consequences of type II solutions. In Sec. 2.1.3 we discussed adiabatic and entropy perturbations for two 3-form fields, using a dualized action [130, 131]. We distinguish, type I and type II solutions with respect to isocurvature perturbations and calculate the power spectrum expression [18]. In Sec. 2.1.4 we present how our inflationary setting can fit the tensor to scalar ratio, spectral index and its running provided by the *Planck* data [132]. In this chapter, we follow the units $m_{\text{P}} = 1$.

2.1.1 \mathbb{N} 3-form fields model

In this section, we generalize the background equations associated to a single 3-form field, which has been studied in [105, 106, 129], to \mathbb{N} 3-form fields. We take a flat FLRW cosmology, described with the metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad (2.1)$$

The general action for Einstein gravity and \mathbb{N} 3-form fields is written as

$$S = - \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \sum_{I=1}^{\mathbb{N}} \left(\frac{1}{48} F_I^2 + V(A_I^2) \right) \right], \quad (2.2)$$

where $A_{\beta\gamma\delta}^{(I)}$ is the I th 3-form field and we have squared the quantities by contracting all the indices. The strength tensor of the 3-form is given by²

$$F_{\alpha\beta\gamma\delta}^{(I)} \equiv 4\nabla_{[\alpha} A_{\beta\gamma\delta]}^{(I)}, \quad (2.3)$$

¹Details presented in Appendix B.

²Throughout this chapter, the Latin index I will be used to refer the number of the quantity (or the 3-form field) or the I th quantity/field. The other Latin indices, which take the values $i, j = 1, 2, 3$, will indicate the three dimensional quantities; whereas the Greek indices will be used to denote four-dimensional quantities and they stand for $\mu, \nu = 0, 1, 2, 3$.

where anti-symmetrization is denoted by square brackets. As we have assumed a homogeneous and isotropic universe, the 3-form fields depend only on time and hence only the space like components will be dynamical, thus their non-zero components (for FLRW background) are given by

$$A_{ijk}^{(I)} = a^3(t) \varepsilon_{ijk} \chi_I(t) \Rightarrow A_I^2 = 6\chi_I^2, \quad (2.4)$$

where $\chi_I(t)$ is a comoving field associated to the I th 3-form field and ε_{ijk} is the standard three dimensional Levi-Civita symbol. Also note that by introducing the more convenient field $\chi_I(t)$, which is related to the corresponding 3-form field by the above relation, we have, subsequently, the following system of equations of motion for \mathbb{N} 3-form fields

$$\ddot{\chi}_I + 3H\dot{\chi}_I + 3\dot{H}\chi_I + V_{,\chi_I} = 0, \quad (2.5)$$

where $V_{,\chi_I} \equiv \frac{dV}{d\chi_I}$. For each value of I , each of the (2.5) are not independent: it is straightforward to see that a peculiar coupling is present through the Hubble parameter derivative, \dot{H} . This fact will play a crucial role, establishing different classes of inflationary behavior when more than one 3-form field is employed. In this setting, the gravitational sector equations are given by

$$\begin{aligned} H^2 &= \frac{1}{3} \left\{ \frac{1}{2} \sum_{I=1}^{\mathbb{N}} [(\dot{\chi}_I + 3H\chi_I)^2 + 2V(\chi_I)] \right\}, \\ \dot{H} &= -\frac{1}{2} \left[\sum_{I=1}^{\mathbb{N}} \chi_I V_{,\chi_I} \right]. \end{aligned} \quad (2.6)$$

Therefore, the mentioned (gravity mediated) coupling between the several \mathbb{N} 3-form fields will act through the gravitational sector of the equations of motion. The total energy density and pressure of the \mathbb{N} 3-form fields read

$$\begin{aligned} \rho_{\mathbb{N}} &= \frac{1}{2} \sum_{I=1}^{\mathbb{N}} [(\dot{\chi}_I + 3H\chi_I)^2 + 2V(\chi_I)], \\ p_{\mathbb{N}} &= -\frac{1}{2} \sum_{I=1}^{\mathbb{N}} [(\dot{\chi}_I + 3H\chi_I)^2 + 2V(\chi_I) - 2\chi_I V_{,\chi_I}]. \end{aligned} \quad (2.7)$$

We rewrite (2.5) as

$$\ddot{\chi}_I + 3H\dot{\chi}_I + V_{,\chi_I}^{\text{eff}} = 0, \quad (2.8)$$

where

$$V_{,\chi_I}^{\text{eff}} \equiv 3\dot{H}\chi_I + V_{,\chi_I} = V_{,\chi_I} \left[1 - \frac{3}{2}\chi_I^2 \right] - \frac{3}{2}\chi_I \left[\sum_{\substack{J=1 \\ J \neq I}}^{\mathbb{N}} \chi_J V_{,\chi_J} \right]. \quad (2.9)$$

In order to describe the dynamics of the 3-form fields, we express the equations of motion in

terms of the variable

$$w_I \equiv \frac{\chi'_I + 3\chi_I}{\sqrt{6}}, \quad (2.10)$$

where $\chi'_I \equiv d\chi_I/dN$ in which the number of e-folds of inflationary expansion is $N = \ln a(t)$. Thus, we get

$$H^2 \chi''_I + (3H^2 + \dot{H}) \chi'_I + V_{,\chi_I}^{\text{eff}} = 0. \quad (2.11)$$

The Friedmann constraint is written as

$$H^2 = \frac{1}{3} \frac{V(\chi_I)}{(1-w^2)}, \quad (2.12)$$

where

$$w^2 \equiv \sum_{I=1}^{\mathbb{N}} w_I^2.$$

Employing the dimensionless variables (2.10), the equations of motion (2.11) can be rewritten in the autonomous form as

$$\begin{aligned} \chi'_I &= 3 \left(\sqrt{\frac{2}{3}} w_I - \chi_I \right) \\ w'_I &= \frac{3}{2} \frac{V_{,\chi_I}}{V} (1-w^2) \left(\chi_I w_I - \sqrt{\frac{2}{3}} \right) + \frac{3}{2} (1-w^2) \frac{1}{V} w_I \sum_{\substack{J=1 \\ I \neq J}}^{\mathbb{N}} \chi_J V_{,\chi_J}, \end{aligned} \quad (2.13)$$

In the whole chapter, we study the sum separable potentials of the form

$$V = \sum_I V_I(\chi_I). \quad (2.14)$$

2.1.1.1 Dual action for \mathbb{N} 3-forms

In general, any p -form in D dimensions has a dual of $(D-p)$ -form [103, 130]. In our case 3-form field (A) and its field tensor four-form (F) are dual to a vector and a scalar field respectively which can be expressed as [130]

$$A_{\mu\nu\rho} = \varepsilon_{\alpha\mu\nu\rho} B^\alpha, \quad F_{\mu\nu\rho\sigma} = -\varepsilon_{\mu\nu\rho\sigma} \phi, \quad (2.15)$$

where $\varepsilon_{\mu\nu\rho\sigma}$ is an antisymmetric tensor.

The corresponding action for the scalar field dual representation of the \mathbb{N} 3-forms is [130, 133]

$$S = - \int d^4x \sqrt{-g} \left[\frac{1}{2} R + P(X, \phi_I) \right], \quad (2.16)$$

where

$$P(X, \phi_I) = \sum_{I=1}^{\mathbb{N}} \left(\chi_I V_{I,\chi_I} - V(\chi_I) - \frac{\phi_I^2}{2} \right), \quad (2.17)$$

with $X = -\frac{1}{2} G^{IJ}(\phi) \partial_\mu \phi_I \partial^\mu \phi_J$.

The dual fields are related to the 3-forms through the following relation [130, 133]

$$X_I = -\frac{1}{2}\partial_\mu\phi_I\partial^\mu\phi_I = \frac{1}{2}V_{,\chi_I}^2. \quad (2.18)$$

For a background unperturbed FLRW cosmology, we can use the dualities defined in (2.18) to write the following relation between a 3-form field and its dual scalar field

$$\phi_I = \dot{\chi}_I + 3H\chi_I. \quad (2.19)$$

Using the above relations, in the Lagrangian (2.17) we can identify the kinetic term to be

$$K(X_I) = \sum_{I=1}^N (\chi_I V_{,\chi_I} - V(\chi_I)). \quad (2.20)$$

Since this kinetic term is only a function of χ_I and not of ϕ_I , this means that the field metric is $G_{IJ}(\phi_J) = 1$. Therefore, we have $X = \sum X_I$. The 3-form fields present on the right-hand side of (2.17) should be viewed as functions of the kinetic terms X_I though the inverse of the relation (2.18).

Following the above relations we compute here the following quantities which we use later in our study

$$P_{,X} \equiv \sum_I P_{,\chi_I} = \sum_I P_{,\chi_I} \left(\frac{\partial\chi_I}{\partial X_I} \right) = \sum_I \frac{\chi_I}{V_{,\chi_I}}. \quad (2.21)$$

And similarly

$$P_{,X_I X_I} = \frac{1}{V_{,\chi_I X_I} V_{,\chi_I}^2} - \frac{\chi_I}{V_{,\chi_I}^3}. \quad (2.22)$$

$$P_{,X_I X_I X_I} = -\frac{V_{,\chi_I X_I X_I}}{V_{,\chi_I X_I}^3 V_{,\chi_I}^2} + \frac{3\chi_I}{V_{,\chi_I}^5} - \frac{3}{V_{,\chi_I}^4 V_{,\chi_I X_I}}. \quad (2.23)$$

$$P_{,I} = -\phi_I = -\sqrt{6}Hw_I. \quad (2.24)$$

Considering the large amount of non-canonical scalar fields studies in cosmology, it might be tempting to think that given a 3-form theory the best way to proceed would be to simply pass to the dual scalar field theory and work solely with scalar field quantities. However, starting from a set of massive 3-form fields makes the task of analytically writing the dual scalar field theory very difficult, except for very particular potentials [130]. This can be seen by noting but the technical difficulty found when one tries to invert (2.18). Yet, in a similar manner to that advocated in Ref. [130] for the single field case, we will see that we can still make use of the dual theory indirectly.

2.1.1.2 Initial conditions and slow-roll inflation

Analogous to the scalar field [134] as well as single 3-form [105, 129] inflationary models, the so-called slow-roll parameters are taken as $\varepsilon \equiv -\dot{H}/H^2 = -d \ln H / dN$ and $\eta \equiv \varepsilon' / \varepsilon - 2\varepsilon$, which, for our model, are given by³

³Equivalently solely in terms of χ_I and w_I , $\eta = \frac{\sum_{I=1}^N 3\left(\sqrt{\frac{2}{3}}w_I - \chi_I\right)(V_{,\chi_I} + V_{,\chi_I X_I} \chi_I)}{\sum_{I=1}^N V_{,\chi_I X_I}}$

$$\varepsilon = \frac{3}{2} \frac{\sum_{I=1}^{\mathbb{N}} \chi_I V_{,\chi_I}}{V} (1 - w^2) , \quad (2.25)$$

$$\eta = \frac{\sum_{I=1}^{\mathbb{N}} \chi'_I (V_{,\chi_I} + \chi_I V_{,\chi_I \chi_I})}{\sum_{I=1}^{\mathbb{N}} \chi_I V_{,\chi_I}} . \quad (2.26)$$

We can see from (2.25) and (2.26) that, for \mathbb{N} 3-form fields, one manner to establish a sufficient condition for inflation (with the slow-roll parameters $\varepsilon \ll 1$ and $\eta \ll 1$) is by means of

$$\begin{cases} 1 - \sum_{I=1}^{\mathbb{N}} w_I^2 & \approx 0 , \\ \chi'_I & \approx 0 . \end{cases} \quad (2.27)$$

It is important, however, to also consider another (albeit less obvious) possibility, which is to have instead

$$\begin{cases} 1 - \sum_{I=1}^{\mathbb{N}} w_I^2 \approx 0 , \\ \sum_{I=1}^{\mathbb{N}} \chi'_I (V_{,\chi_I} + \chi_I V_{,\chi_I \chi_I}) \approx 0 . \end{cases} \quad (2.28)$$

The condition expressed in (2.28) means that the inclusion of more than one 3-form field allows the emergence of an inflationary scenario without even requiring that $\chi'_I \approx 0$. Therefore, we can expect to have different behaviors, in contrast to the ones usually found in models with just one 3-form. The different \mathbb{N} 3-form fields will evolve in an intricate correlated way in order to satisfy (2.28). This possibility will deserve a more detailed analysis in the next sections. We should note that all the derived equations in this section reduce to the single one 3-form case when $\mathbb{N} = 1$, as expected.

2.1.1.3 Inflaton mass

Returning to the condition (2.27), we have

$$\sum_{I=1}^{\mathbb{N}} \chi_I^2 \approx \frac{2}{3} .$$

Note that the Friedmann constraint (2.12) does not hold precisely at $\sum_{I=0}^{\mathbb{N}} \chi_I^2 = 2/3$, and $\chi'_I = 0$. If we assume a symmetric situation, where all w_I are equal during inflation, i.e, if all fields come to the same value during inflation, then $\chi_I(N)$ will take a constant value

$$\chi_p = \sqrt{\frac{2}{3\mathbb{N}}} . \quad (2.29)$$

In this symmetric situation, all the 3-form fields will behave identically during inflation. If \mathbb{N} is very large, the plateau of $\chi_I(N)$ converges towards zero ($\chi_p = \sqrt{\frac{2}{3\mathbb{N}}} \rightarrow 0$ as $\mathbb{N} \rightarrow \infty$, for the symmetric case where all w_n are equal). The initial conditions for the single 3-form inflation case were discussed in [129]. The reduction of the plateau energy scale for \mathbb{N} 3-forms can have a nontrivial consequence, which is to bring the inflaton mass well below Planck mass. This

is illustrated by the following analysis. let us assume that all the 3-form fields behave in the same way, reaching a constant value χ_p during inflation and starting to oscillate by the end of inflation. Subsequently, we rewrite the Friedmann constraint (2.12) for this case as,

$$H^2 = \frac{1}{3} \frac{V}{(1-w^2)}. \quad (2.30)$$

Taking $V_I = V_{0I} f_I(\chi_I)$, and where $f_I(\chi_I)$ are dimensionless functions. Comparing (2.30) with the Friedmann constraint of a single 3-form field case, we get

$$V = \tilde{V}_1, \quad (2.31)$$

where $\tilde{V}_1 = \tilde{V}_{01} \tilde{f}_1(\chi_1)$ is the potential for the single 3-form field case. If we choose $V_{01} = V_{02} = \dots = V_{0n} = V_{0N}$, which means that the energy scales of the potentials are the same, and also assume that $\chi_I = \chi_p$ for all I in (2.31), we get

$$\frac{V_{0N}}{\tilde{V}_{01}} = \frac{\tilde{f}_1(\chi_1)}{N f_N(\chi_N)}. \quad (2.32)$$

Let us consider the power law potential $f = \chi^l$, for a 3-form. If we substitute the corresponding value of the plateau for N 3-form $(\chi_p = \sqrt{\frac{2}{3N}})$ and of the single 3-form case $(\chi_{1p} = \sqrt{\frac{2}{3}})$ in (2.32), then we can have the following ratio of energy scales for the potentials, of N 3-forms and single 3-form

$$\frac{V_{0N}}{\tilde{V}_{01}} = N^{-1+\frac{l}{2}}. \quad (2.33)$$

We can translate this argument in terms of the inflaton mass, which is defined to be the square root of the second derivative of the potential. Therefore, the ratio between the inflaton masses corresponding to the N 3-forms potential (m_N), and the single 3-form (\tilde{m}_1), for a power law potential (χ^l) , is given by

$$\frac{m_N}{\tilde{m}_1} \equiv \sqrt{\frac{V_{0N} \chi_p^{l-2}}{\tilde{V}_{01} \chi_{1p}^{l-2}}} = N^{-\frac{l}{2}}. \quad (2.34)$$

Therefore, it is possible to bring down the mass of the inflaton to lower energy scales by increasing the number of 3-form fields.

2.1.2 Two 3-form fields model

In this subsection, we would like to concentrate on the case where only two 3-form fields are present. Accordingly, we will rewrite some of the equations as follows. Thus, the non-zero

components of (2.4) are

$$A_{ijk}^{(1)} = a^3(t) \varepsilon_{ijk} \chi_1(t), \quad A_{ijk}^{(2)} = a^3(t) \varepsilon_{ijk} \chi_2(t), \quad (2.35)$$

$$(2.36)$$

which implies $A_1^2 = 6\chi_1^2$, $A_2^2 = 6\chi_2^2$. Also, we rewrite equations of motion (2.11) in terms of our dimensionless variables as

$$H^2 \chi_1'' + (3H^2 + \dot{H}) \chi_1' + V_{,\chi_1}^{\text{eff}} = 0, \quad (2.37)$$

$$H^2 \chi_2'' + (3H^2 + \dot{H}) \chi_2' + V_{,\chi_2}^{\text{eff}} = 0, \quad (2.38)$$

where the Friedmann and acceleration equations are given by

$$H^2 = \frac{1}{3} \frac{V_1(\chi_1) + V_2(\chi_2)}{(1 - w_1^2 - w_2^2)}, \quad (2.39)$$

$$\dot{H} = -\frac{1}{2} (\chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2}). \quad (2.40)$$

In order to further discuss suitable initial conditions, the slow roll conditions $\varepsilon, |\eta| \ll 1$ suggests the equation of a circle (of unit radius), as

$$w_1^2 + w_2^2 \approx 1, \quad (2.41)$$

which we rewrite in terms of trivial parametric relations as

$$\begin{aligned} w_1 &\approx \cos \theta, \\ w_2 &\approx \sin \theta. \end{aligned} \quad (2.42)$$

Subsequently, from (2.13), we can establish the initial conditions for the field derivatives

$$\begin{cases} \chi_1' \approx 3 \left(\sqrt{\frac{2}{3}} \cos \theta - \chi_1 \right), \\ \chi_2' \approx 3 \left(\sqrt{\frac{2}{3}} \sin \theta - \chi_2 \right). \end{cases} \quad (2.43)$$

Since (2.42) can be satisfied by assigning many different continuous values of the new parameter θ , we, therefore, anticipate to investigate diverse solutions. More precisely, a particular choice of this parameter will affect the way (2.26), (i.e, the value of η) will depend on the two 3-form fields. Before proceeding, let us mention that for two 3-forms inflation, we choose herein initial conditions for the fields above or below the value given by (2.29), which are expected to influence the number of e -foldings. In particular, we will investigate the asymmetric situation, when each w_I is different⁴, which will provide a new behavior with respect to inflation.

⁴For example, if we take $w_1 = 1$ and $w_{I>1} = 0$, we then find a scenario similar to single 3-form field driving the inflation and where all the other fields approach zero.

2.1.2.1 Type I inflation ($\chi'_I \approx 0$)

As we have established in Sec. 2.1.1.2, the slow-roll conditions enable us to find two types of inflationary solutions, according to relation (2.27)-(2.28). In the following, we investigate them in more detail. In type I solution, the 3-form fields which are responsible for driving the inflationary period, will be displaying $\chi'_I \approx 0$. The following is a stability analysis for this type, presented in a dynamical system context. Whenever necessary, we will complement this study by a numerical discussion.

Let us remind the autonomous system of equations for the field χ_1 ,

$$\chi'_1 = 3 \left(\sqrt{\frac{2}{3}} w_1 - \chi_1 \right), \quad (2.44)$$

$$w'_1 = \frac{3}{2} (1 - (w_1^2 + w_2^2)) \left(\lambda_1 \left(\chi_1 w_1 - \sqrt{\frac{2}{3}} \right) + \lambda_2 \chi_2 w_1 \right), \quad (2.45)$$

and also for χ_2 ,

$$\chi'_2 = 3 \left(\sqrt{\frac{2}{3}} w_2 - \chi_2 \right), \quad (2.46)$$

$$w'_2 = \frac{3}{2} (1 - (w_1^2 + w_2^2)) \left(\lambda_2 \left(\chi_2 w_2 - \sqrt{\frac{2}{3}} \right) + \lambda_1 \chi_1 w_2 \right), \quad (2.47)$$

where $\lambda_I = V_{,\chi_I}/V$. Notice that, (2.44)-(2.45) are coupled with (2.46)-(2.47). With the variables (χ_I, w_n) , let $f_1 := d\chi_1/dN$, $f_2 := dw_1/dN$, $f_3 := d\chi_2/dN$ and $f_4 := dw_2/dN$. The critical points are located at the field space coordinates (x_c) and are obtained by setting the condition $(f_1, f_2, f_3, f_4)|_{x_c} = 0$.

To determine the stability of the critical points, we need to perform linear perturbations around each of them by using $x(t) = x_c + \delta x(t)$; this results in the equations of motion $\delta x' = \mathcal{M} \delta x$, where \mathcal{M} is the Jacobi matrix of each critical point whose components are $\mathcal{M}_{ij} = (\partial f_i / \partial x_j)|_{x_c}$. A critical point is called stable (unstable) whenever the eigenvalues ζ_i of \mathcal{M} are such that $\text{Re}(\zeta_i) < 0$ ($\text{Re}(\zeta_i) > 0$) [135]. If $\text{Re}(\zeta_i) = 0$, then other methods should be employed to further assess the stability of the critical point. Among different approaches, we have the center manifold theorem [135-138] or, alternatively, we can consider a perturbative expansion to nonlinear order as in Refs. [105, 129]. In this work we will follow the last mentioned method, whenever necessary.

The autonomous dynamical (2.44)-(2.47) fixed points are given by

$$\begin{aligned} \chi_{1c} &= \sqrt{\frac{2}{3}} w_1, & w_{1c} &= \sqrt{\frac{2}{3}} \frac{\lambda_1}{\lambda_1 \chi_1 + \lambda_2 \chi_2}, \\ \chi_{2c} &= \sqrt{\frac{2}{3}} w_2, & w_{2c} &= \sqrt{\frac{2}{3}} \frac{\lambda_2}{\lambda_1 \chi_1 + \lambda_2 \chi_2}. \end{aligned} \quad (2.48)$$

If $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$ (otherwise, $V_{1,\chi_1} = 0$ and $V_{2,\chi_2} = 0$), (2.48) can be rewritten as

$$\begin{aligned} \chi_{1c} &= \sqrt{\frac{2}{3}} w_1, & w_{1c} &= \frac{\lambda_1}{\sqrt{\lambda_1^2 + \lambda_2^2}}, \\ \chi_{2c} &= \sqrt{\frac{2}{3}} w_2, & w_{2c} &= \frac{\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}. \end{aligned} \quad (2.49)$$

Generically, with a inflationary stage as a target, the fixed points coordinates must satisfy $w_{1c}^2 + w_{2c}^2 \simeq 1$. This last condition is required to satisfy the slow-roll condition (2.27). Therefore, we can define as well

$$\begin{aligned} w_{1c} &= \cos \theta, \\ w_{2c} &= \sin \theta. \end{aligned} \quad (2.50)$$

Note that, when we consider the field coordinates in (2.48), also χ_{1c} and χ_{2c} are constrained by $\chi_{1c}^2 + \chi_{2c}^2 = 2/3$. Consequently, the dynamical system (2.44)-(2.47) has fixed points with only two independent degrees of freedom, which can be chosen to be the pair (χ_{1c}, w_{1c}) . Therefore, the critical points or inflationary attractors are found by solving the following expression $w_{1c}^2 + w_{2c}^2 = 1$, which upon substitution gives,

$$\left(\sqrt{\frac{2}{3}} \frac{\lambda_1}{\lambda_1 \chi_{1c} + \lambda_2 \chi_{2c}} \right)^2 + \left(\sqrt{\frac{2}{3}} \frac{\lambda_2}{\lambda_1 \chi_{1c} + \lambda_2 \chi_{2c}} \right)^2 = 1. \quad (2.51)$$

It is clear from (2.51) that the location of critical points depends on the choice of 3-form potentials. For example let us take $V_1 = \chi_1^n$ and $V_2 = \chi_2^m$. It follows that

$$\left(\frac{n \left(\sqrt{\frac{2}{3}} \right)^n (\cos \theta)^{n-1}}{n \left(\sqrt{\frac{2}{3}} \cos \theta \right)^n + m \left(\sqrt{\frac{2}{3}} \sin \theta \right)^m} \right)^2 + \left(\frac{m \left(\sqrt{\frac{2}{3}} \right)^m (\sin \theta)^{m-1}}{n \left(\sqrt{\frac{2}{3}} \cos \theta \right)^n + m \left(\sqrt{\frac{2}{3}} \sin \theta \right)^m} \right)^2 = 1. \quad (2.52)$$

From (2.52), $\theta = 0$ and $\theta = \pi/2$ can be called as trivial fixed points independent of the choice of n, m . Satisfying the condition (2.52), for our particular choice of potentials, allows us to also identify non trivial fixed points in the range $0 < \theta < \pi/2$. To easily identify these, we can extract a simple constraint from (2.49), given by

$$\chi_{1c}/\chi_{2c} = \lambda_1/\lambda_2. \quad (2.53)$$

Condition (2.53) is fully consistent with (2.52), except for the trivial fixed points $\theta = 0$ and $\theta = \pi/2$. Let us apply the example where $V_1 = \chi_1^n$ and $V_2 = \chi_2^m$, and substituting in (2.53), We have

$$\frac{n \left(\sqrt{\frac{2}{3}} \cos \theta \right)^{n-2}}{m \left(\sqrt{\frac{2}{3}} \sin \theta \right)^{m-2}} = 1. \quad (2.54)$$

We can read from (2.54) that for identical quadratic potentials, i.e., for $n = m = 2$, (2.53) is satisfied for all values of $0 < \theta < \pi/2$. Identical quadratic potentials is the only case where we can have an infinite number of non trivial fixed points. For any other choice of potentials, i.e., for $n \neq m$, there will only be a finite number of non trivial fixed points.

In Fig. 3.1 we illustrate the evolution of the fields χ_1 and χ_2 for quadratic potentials with $\theta = \pi/2$ and $\theta = \pi/4$. The asymmetry in choosing $\theta \neq \pi/4$ manifests through one of the 3-form fields having a plateau slightly higher than the other.

Type I solutions, as far as the stability analysis, are very similar to the scenario where just single 3-form field is present. The novelty here is that we can have solutions as shown in Fig. 3.1.

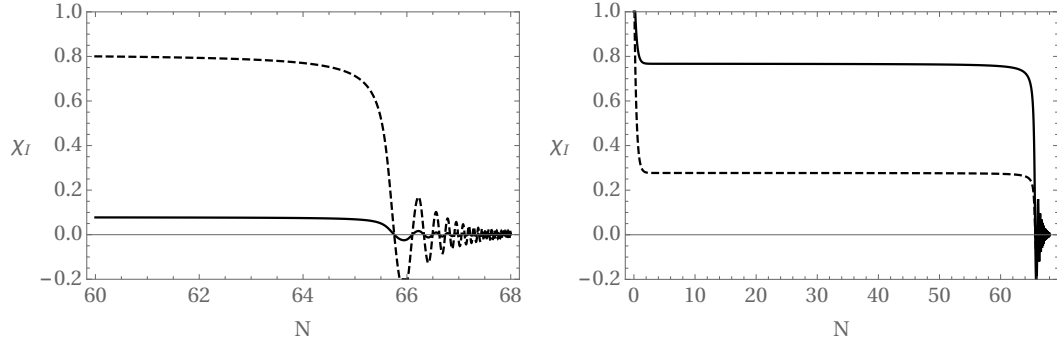


Figure 2.1: Left panel is the graphical representation of the numerical solutions of (2.37) and (2.38) for $\chi_1(N)$ (full line) and $\chi_2(N)$ (dashed line) with $\theta \approx \frac{\pi}{2}$ for the potentials $V_1 = \chi_1^2$ and $V_2 = \chi_2^2$. In the right panel, we depict the graphical representation of the numerical solutions of (2.37) and (2.38) for $\chi_1(N)$ (full line) and $\chi_2(N)$ (dashed line) with $\theta = \frac{\pi}{9}$. We have taken the initial conditions as

$$\chi_1(0) = 2.1 \times \sqrt{\frac{1}{3}} \text{ and } \chi_2(0) = 2.1 \times \sqrt{\frac{1}{3}}.$$

Therein, we have a case where we consider that the 3-form fields χ_1 and χ_2 are under the influence of the same kind of quadratic potential, i.e., $V_n = \chi_n^2$.

We discuss the stability of these fixed points and their stability in the Appendix B for simple potentials. Other combinations of potentials can be tested for stability along the same lines presented there. We summarize the results in Table 2.1.

$V(\chi_1)$	$V(\chi_2)$	existence	stability	Oscillatory regime
χ_1^2	χ_2^2	$0 < \theta < \pi/2$	unstable saddle	yes
$\chi_1^4 + \chi_1^2$	$\chi_2^4 + \chi_2^2$	$\theta = \{0, \pi/4, \pi/2\}$	unstable	yes
$\chi_1^3 + \chi_1^2$	$\chi_2^3 + \chi_2^2$	$\theta = \{0, \pi/4, \pi/2\}$	unstable	yes
$\exp(\chi_1^2) - 1$	$\exp(\chi_2^2) - 1$	$\theta = \{0, \pi/4, \pi/2\}$	unstable	yes
χ_1^2	$\chi_2^4 + \chi_2^2$	$\theta = \{0, \pi/2\}$	unstable	yes
$\exp(-\chi_1^2)$	$\exp(-\chi_2^2)$	$\theta = \{0, \pi/4, \pi/2\}$	unstable	no
χ_1^2	χ_2^4	$\theta = \{0, \pi/3, \pi/2\}$	unstable	yes
$\chi_1^n \quad (n > 2)$	$\chi_2^n \quad (n > 2)$	$\theta = \{0, \pi/4, \pi/2\}$	unstable	no

Table 2.1: Summary of some type I solutions critical points and their properties.

2.1.2.2 Type II inflation ($\chi'_I \neq 0$)

Let us now present the other class of inflationary solution, which was mentioned in the Introduction. This type is associated to the manner asymmetry is present. Let us be more specific. One way to attain this solution consists of choosing an initial value of θ away from the fixed points previously discussed. This corresponds to the curved trajectories in the right panel of Fig. 2.3. Another manner is by choosing different scales of the potentials i.e., $V_{01} \neq V_{02}$. In any case, the inflationary behavior (type II) is similarly affected concerning either way of introducing asymmetry. We should note here that there is no analog for a type II solution within single 3-form driven inflation. To understand this new type of inflationary scenario, let us take $V_1 = \chi_1^2$ and $V_2 = 2\chi_2^2$ (just different slopes), whose numerical solutions are plotted in Fig. 2.2. In Fig. 2.2, the two fields continuously evolve, and at the same time assist each other in order to sustain a slow-roll regime. As we can see from the left panel of Fig. 2.2, one field continues

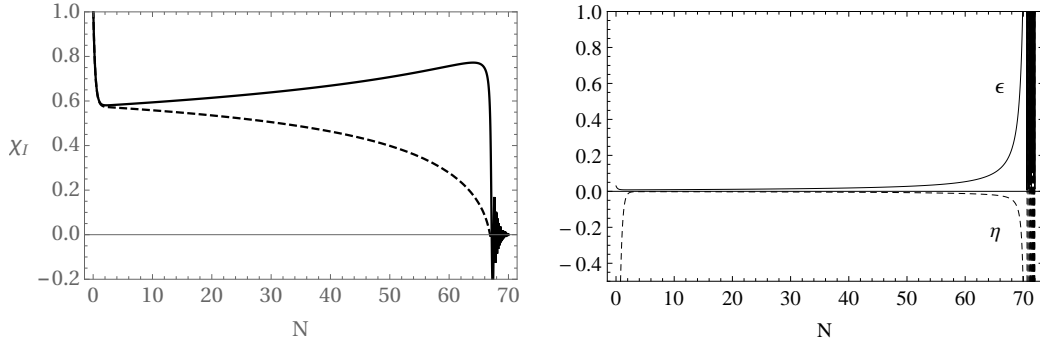


Figure 2.2: In the left panel we have the graphical representation of the numerical solutions of (2.37) and (2.38) for $\chi_1(N)$ (full line) and $\chi_2(N)$ (dashed line) with $\theta = \frac{\pi}{4}$ for the potentials $V_1 = \chi_1^2$ and $V_2 = 2\chi_2^2$. We have taken the initial conditions as $\chi_1(0) = 1.8 \times \sqrt{\frac{1}{3}}$ and $\chi_2(0) = 2.0 \times \sqrt{\frac{1}{3}}$. In the right panel, and for the same initial conditions, we have the graphical representation of the numerical solutions for $\epsilon(N)$ (full line) and $\eta(N)$ (dashed line).

to slowly decrease (dashed line) and the other (full line) starts to increase until it enters in an oscillatory regime. However, in the right panel of Fig. 2.2, we see that the slow-roll parameters evolve (before oscillating) near to zero during the period of inflation. Moreover, from (2.26), the behavior of the two fields are such that even with $\chi'_I \neq 0$, the slow-roll conditions are consistent with inflation. The fact is that the slow roll parameter $\eta \rightarrow 0$ is now due to the constraint (2.28). As previously mentioned, a rather unusual cooperation between the two 3-form fields, emphasized by the mentioned coupling (gravity mediated, through \dot{H}) provides a different inflationary dynamics.

This new type of solution presents a period of inflation with an interesting new feature. More precisely, when one 3-form field decreases, say χ_1 , then the other field, χ_2 , is constrained to increase. However, the increase of the second 3-form field is limited by the fact that, as the first one inevitably approaches zero, then (2.47) becomes

$$w'_2 \sim \frac{3}{2} (1 - w_2^2) \lambda_2 \left(\chi_2 w_2 - \sqrt{\frac{2}{3}} \right), \quad (2.55)$$

with the coupling term $\lambda_1 \chi_1 w_2$ being negligibly small. We see that (2.55) will become zero when w_2 (which is increasing, as is χ_2) will approach 1. At this stage, and inspecting (2.46), it is clear that χ_2 will stop increasing and start to decrease, making $\chi'_2 < 0$. This situation is depicted in the left panel Fig. 2.2, where the decreasing field is reaching zero at the same period where the other stops to increase and also converges to zero. The two 3-form fields behave strongly correlated and assisting each other through the inflationary period. Therefore, this more complex and correlated evolution of the fields can provide a different observational signature when compared to other multifield inflationary models.

The different nature of type I and type II solutions is represented in Fig. 2.3. Therein, we have a parametric plot⁵ of $\chi_1(N)$ and $\chi_2(N)$ in the field space, where the fixed points (cf. in particular the analysis in B.1 and B.2) are located at a pair of coordinates (χ_{1c}, χ_{2c}) , of course associated to a situation where $(\chi'_1, \chi'_2) = 0$.

The two fields rapidly evolve towards this pair of coordinates, (cf. the behavior illustrated in Figs. 3.1 and 2.2) settling there for the inflationary period. Afterwards, and because these

⁵Please note that Fig. 2.3 is *not* a phase space representation.

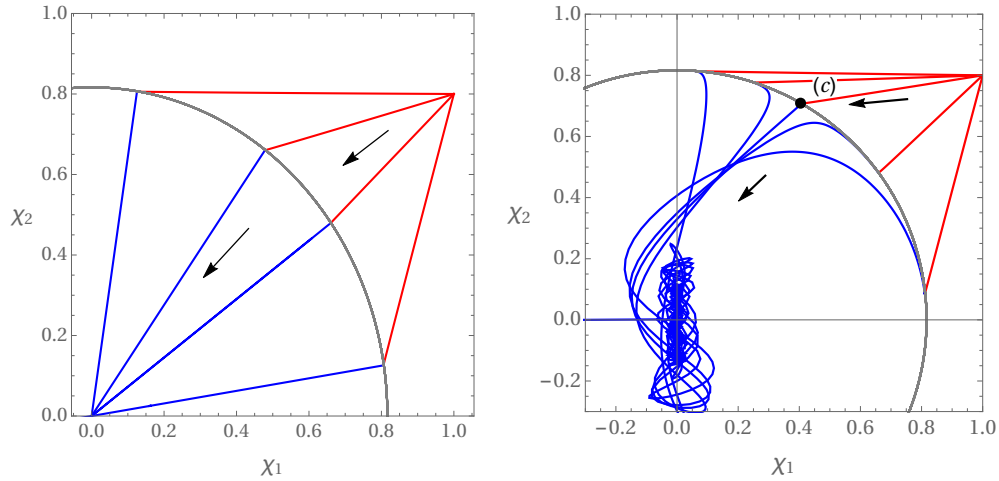


Figure 2.3: This figure represents a set of trajectories evolving in the (χ_1, χ_2) space. These trajectories are numerical solutions of (2.37) and (2.38) and correspond to a situation where we choose $V_1 = \chi_1^2$ and $V_2 = \chi_2^2$ (left panel), as an illustrative example only showing type I solution. All the fixed points are part of the arc of radius $\sqrt{2/3}$ in the (χ_1, χ_2) plane. In the right panel, we have an example, where we have taken $V_1 = \chi_1^2$ and $V_2 = \chi_2^4$, showing type II solutions, except for the trajectory going close to a fixed point with $\theta = \pi/3$ (point C). In addition, in the right panel, we have an illustration of two 3-form fields damped oscillations by the end of inflation. The arrows, in the plots, indicate the direction of time in the trajectories.

fixed points are not stable, the two fields will eventually diverge from it. More precisely, in the left panel of Fig. 2.3 we have the particular case where the two 3-form fields are under the influence of identical quadratic potentials. In this case, only type I solutions are present and the inflationary epochs, occur near the depicted circle. Those fixed points in this figure are all located in the arc of radius $\sqrt{2/3}$ in the (χ_1, χ_2) plane. The right panel, of the same figure, constitutes an example where only one fixed point is present (using (2.52)) between $\theta = 0$ and $\theta = \pi/2$. This fixed point, located at (C) in the right panel, corresponds to a type I solution when $\theta = \pi/3$, for a case where the potentials are $V(\chi_1) = \chi_1^2$ and $V(\chi_2) = \chi_2^4$. All the other depicted trajectories are type II solutions, where the \dot{H} -term coupling mediation plays a crucial role(cf. Fig 2.2). The peculiar oscillatory regime, present the right panel of 2.3, is also characteristic of the coupling term in the effective potential (2.9). We shall discuss the oscillatory behavior in the following.

Oscillatory regime after inflation

The main purpose of here is to present an analytical description of the oscillatory behavior, emerging by the end of inflation for the choice of potentials presented in Table 2.1. This analysis can also be useful for subsequent studies on reheating and particle production, as modeled by the two 3-forms scenario which we postpone for a future work. The interesting aspect that happens with two 3-forms is due to the presence of the \dot{H} coupling term in the effective potential (2.8), which becomes particularly dominant and produces a nontrivial interaction between the 3-form fields in the type II case. At this point, we must note that this property is more general, in the sense that the conclusion drawn for two fields can be easily extended when more 3-form fields are included. The choice of potential plays an important role regarding the presence of a consistent oscillatory behavior, which successfully avoid ghost instabilities by the end of inflation. This is illustrated for single 3-form inflation in the Ref. [129, 139]. Based on the studies

of single 3-form inflation, we chose potentials containing quadratic behavior. Moreover, we must emphasize that the oscillatory regime for two 3-forms case is different from single 3-form inflation, due to the presence of the coupling term in the equations of motion. An exception is the case of identical quadratic potentials, i.e., taking $V_I = \chi_I^2$, where we can reasonably ignore the effect of coupling. This is the special case where two 3-form fields oscillate almost independently.

To illustrate this, let us first consider that the two fields are subjected to quadratic potentials $V_I = \frac{1}{2}m_I^2\chi_I^2$. For simplicity we work with the equations of motion in t time (2.5). The equation of motion (2.5) for the 3-form field χ_I can be approximated in the small field limit ($\chi_I \rightarrow 0$) by neglecting the effect of coupling term in the effective potential (2.9) as,

$$\ddot{\chi}_I + 3H\dot{\chi}_I + m_I^2\chi_I \approx 0. \quad (2.56)$$

From the Friedmann constraint (2.6) we have that during inflation H slowly decreases, since $\dot{H} < 0$. When inflation ends, $m_I^2 \sim H^2$, and subsequently the 3-form fields begin to coherently oscillate at scales $m_I^2 \gg H^2$. The evolution of χ_I at the oscillatory phase can be studied by changing the variable $\chi_I = a^{-3/2}\bar{\chi}_I$, so that (2.56) becomes

$$\ddot{\bar{\chi}}_I + \left(m_I^2 - \frac{9}{4}H^2 - \frac{3}{2}\dot{H}\right)\bar{\chi}_I \approx 0. \quad (2.57)$$

Using the approximations $m_I^2 \gg H^2$ and $m_I^2 \gg \dot{H}$, the solution to (2.57) can be written as

$$\bar{\chi}_I = C \sin(m_I t). \quad (2.58)$$

where C is a the maximum amplitude of the oscillations. Thus the solution for χ_I can be written as

$$\chi_I = C a^{-3/2} \sin(m_I t). \quad (2.59)$$

An interesting aspect arises in the small field limit when one of the two 3-form fields potentials is not quadratic. Let us suppose the situation described in B.2, with one field subjected to a quartic potential, $V_2 = \lambda\chi_2^4$. This discussion is related to the oscillatory phase we see in the right panel of Fig. 2.3, regarding the type II case. This combination of potentials has the peculiar feature to induce an oscillatory regime, more precisely, that for a single 3-form field it would be absent under the quartic potential due to the presence of a ghost term [129]. In the limit $\chi_1, \chi_2 \rightarrow 0$, towards the oscillatory phase, the field χ_1 will be approximately described by (2.59). Therefore the 3-form field χ_1 undergoes a damped oscillatory regime due to the dominance of quadratic behavior. However, the second field χ_2 , also undergoes an oscillatory regime, not caused by the quartic potential but due to the coupling term, $V_{2,\chi_2}^{\text{eff}}$, dominance in (2.38). The equation of motion (2.37) for the 3-form field becomes (in the small field limit, $\chi_1, \chi_2 \rightarrow 0$, near the oscillatory phase),

$$\ddot{\chi}_2 + 3H\dot{\chi}_2 + \left(4\lambda\chi_2^3 - \frac{3}{2}m_1^2\chi_1^2\chi_2\right) \approx 0. \quad (2.60)$$

The nonlinear differential equation (2.60) is explicitly affected by the oscillatory behavior of χ_1 , which could cause something similar to a parametric resonance effect in particle production

[139]. The effective potential also carries a cubic term, which turns the equation difficult to solve. However, we can conjecture that for two 3-forms inflation, at least one of the potentials must contain a quadratic behavior, which forces all the other fields to undergo a consistent oscillatory phase due to the influence of the coupling term. In the case of the single 3-form inflation, there is no oscillatory behavior for quartic potential, a fact that the authors in [129] explain by means of ghost instabilities. Therefore, we present a new choice of potential i.e., $V_1 = \chi_1^2$ and $V_2 = \chi_2^4$, which can avoid ghost instabilities due to the presence of consistent oscillatory phase. A similar oscillatory regime is present when assisted inflation with two scalar fields is studied by means of an explicit quartic coupling in the action [140].

Varying speed of sound for two 3-form fields

In the following we examine how the type II solutions establish pressure perturbations with varying speed of sound.

Adiabatic perturbations are defined by

$$\frac{\delta P}{\dot{P}} = \frac{\delta \rho}{\dot{\rho}},$$

where P and ρ are the pressure and energy density of the system. Pressure perturbations can in general be expanded as a sum of an adiabatic and a non adiabatic perturbations (δP_{nad}), which is given by [141]

$$\delta P = \delta P_{nad} + c_s^2 \delta \rho,$$

where $c_s^2 = \dot{P}/\dot{\rho}$ is the adiabatic sound speed for scalar perturbations in a thermodynamic system⁶. When an adiabatic system is composed with multiple scalar fields ϕ_n , we have that

$$\frac{\delta \phi_i}{\dot{\phi}_i} = \frac{\delta \phi_j}{\dot{\phi}_j}. \quad (2.61)$$

The condition (2.61) is consequently valid for any two scalar field systems. The above condition can also be applicable for a system of \mathbb{N} 3-forms because its action can (at least formally) always be dualized and reduced to an action with \mathbb{N} non canonical scalar fields [130].

The general expression for the adiabatic sound speed for \mathbb{N} 3-form fields is defined as

$$c_s^2 = \frac{\dot{P}_{\mathbb{N}}}{\dot{\rho}_{\mathbb{N}}}. \quad (2.62)$$

If we take (2.7) within the slow roll approximation $\chi_I'' \ll V_I(\chi_I)$, we get, generally

$$c_s^2 = \frac{\sum_{n=1}^{\mathbb{N}} \chi_I' \chi_I V_{,\chi_I \chi_I}}{\sum_{I=1}^{\mathbb{N}} \chi_I' V_{,\chi_I}}, \quad (2.63)$$

which, in the two 3-forms case, allows the speed of sound to be explicitly written as

⁶The distinction between adiabatic sound speed and effective sound speed is given for scalar field models in Ref. [142, 143].

$$c_s^2 = \frac{\chi'_1 \chi_1 V_{,\chi_1 \chi_1} + \chi'_2 \chi_2 V_{,\chi_2 \chi_2}}{\chi'_1 V_{,\chi_1} + \chi'_2 V_{,\chi_2}}. \quad (2.64)$$

Unlike the single 3-form sound speed, in a two 3-forms setting the sound speed will depend on χ'_I . For type I inflation, for which we have ($\chi'_I \approx 0$), the speed of sound (2.63) becomes constant during inflation. For the type II solution, where we have $\chi'_I \neq 0$, the speed of sound, c_s^2 , can vary during the inflationary period. This varying speed can subsequently exhibit a peculiar imprint in the primordial power spectrum, scale invariance and bi-spectrum extracted from the CMB data. We are going to explore, in the next two subsections, observational consequences, due to a varying speed of sound, upon important quantities like the tensor-scalar ratio, spectral index and running spectral index, by examining particular type II solutions for suitable choice of potentials.

2.1.3 Isocurvature perturbations and primordial spectra

One important feature of multiple field models is the generation of isocurvature perturbations. In this subsection we examine the effect of these perturbations in the context of two 3-form fields scenario. More concretely, we will distinguish, type I and type II solutions, with respect to the evolution of isocurvature perturbations.

As depicted, in the right panel of Fig. 2.3 type I solutions are characterized by a straight line, whereas type II solutions follow a curved trajectory in field space. In scalar multifield models, a local rotation in the field space is carried to define the adiabatic and entropy modes (or fields [144]). In order to express these adiabatic and entropy fields from two 3-form fields, we use the relation between 3-form field dual scalar field presented in Sec. 2.1.1.1. The motivation to work with the dual action is related to the fact that the general framework of adiabatic and entropy perturbations for the non-canonical multifield model has already been consistently established. In the following we will briefly review and adopt to our case the results described previously in [130, 131, 145-147].

Restricting ourselves now to a two 3-form scenario, and according to [144], we can define the adiabatic and entropy fields through a rotation in the two 3-form dual field space

$$\dot{\sigma} = \sqrt{2X_1} \cos \Theta + \sqrt{2X_2} \sin \Theta, \quad (2.65)$$

$$\dot{s} = -\sqrt{2X_1} \sin \Theta + \sqrt{2X_2} \cos \Theta, \quad (2.66)$$

where $\tan \Theta = \sqrt{X_2}/\sqrt{X_1}$, $X_1 = \frac{1}{2}V_{1,\chi_1}^2$ and $X_2 = \frac{1}{2}V_{2,\chi_2}^2$. Subsequently, the adiabatic and entropy perturbations are

$$Q_\sigma = \delta\phi_1 \cos \Theta + \delta\phi_2 \sin \Theta, \quad (2.67)$$

$$Q_s = -\delta\phi_1 \sin \Theta + \delta\phi_2 \cos \Theta, \quad (2.68)$$

respectively, along and orthogonal to the background classical trajectory in dual field space. Let us assume that the linearly perturbed metric in terms of Bardeen potentials Φ , Ψ which is

given by [15]

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t) (1 - 2\Psi) d\mathbf{x}^2.$$

We choose a flat gauge, where the dynamics of linear perturbations are completely expressed in terms of the scalar field perturbations ($\phi^I \rightarrow \phi_0^I + Q^I$). Moreover, these are defined as gauge invariant combinations given by $Q^I = \delta\phi^I + (\phi^I/H) \Psi$. The comoving curvature perturbation is given by

$$\mathcal{R} \equiv \Psi - \frac{H}{p + \rho} \delta q, \quad (2.69)$$

where $\partial_i \delta q_i = \delta T_i^0$ and \mathcal{R} purely characterizes the adiabatic part of the perturbations. The variation of \mathcal{R} , in the flat gauge, is given by [131]

$$\dot{\mathcal{R}} = \frac{H}{\dot{H}} \frac{c_s^2 k^2}{a^2} \Psi + \frac{H}{\dot{\sigma}} \Xi Q_s \quad \text{with} \quad \Xi = \frac{1}{\dot{P}_{,X}} \left((1 + c_s^2) P_{,s} - c_s^2 \dot{\sigma}^2 P_{,Xs} \right), \quad (2.70)$$

where Ψ is the Bardeen potential and

$$P_{,s} = P_{,X} \dot{\sigma} \dot{\Theta}, \quad \begin{pmatrix} P_{,X\sigma} \\ P_{,Xs} \end{pmatrix} = \begin{pmatrix} \cos \Theta & \sin \Theta \\ -\sin \Theta & \cos \Theta \end{pmatrix} \begin{pmatrix} P_{,\chi_1} \\ P_{,\chi_2} \end{pmatrix}. \quad (2.71)$$

For a two 3-form dual Lagrangian, extracted from (2.17), we can express the above quantities as functions of the 3-form fields, i.e.,

$$P_{,X} \equiv P_{,X_1} + P_{,X_2} = \frac{\chi_1}{V_{1,\chi_1}} + \frac{\chi_2}{V_{2,\chi_2}}. \quad (2.72)$$

Using (2.72) and (2.71) we can simplify Ξ , to obtain,

$$\Xi = H \left((1 + c_s^2) \frac{d\Theta}{dN} - c_s^2 \frac{\dot{\sigma}}{H} \frac{P_{,Xs}}{P_{,X}} \right). \quad (2.73)$$

The function Ξ is a measure of the coupling between the entropy and adiabatic modes.

2.1.3.1 Type I inflation

In type I inflationary scenarios, where $\dot{\Theta} = 0$ (as $\tan \Theta = \lambda_2/\lambda_1 = \chi_2/\chi_1 = \text{constant}$ in the fixed point, cf. (2.49) and see Fig. 2.3), the classical trajectory is a straight line. This fact makes the first term of Ξ , in (2.73), to vanish.

On the other hand, the ratio $P_{,Xs}/P_{,X}$ can be expressed as

$$\frac{P_{,Xs}}{P_{,X}} = \frac{-\chi_1 \sin \Theta + \chi_2 \cos \Theta}{\chi_1 V_{1,\chi_1} + \chi_2 V_{2,\chi_2}}. \quad (2.74)$$

Expression (2.74) vanishes for all type I solutions since $\chi_2 = \chi_1 (\lambda_2/\lambda_1) = \chi_1 \tan \Theta$. In other words, there are no entropy perturbations sourcing the curvature perturbations. We then recover the known relation for a single field inflation

$$\dot{\mathcal{R}} = \frac{H}{\dot{H}} \frac{c_s^2 k^2}{a^2} \Psi \quad (2.75)$$

and we can state that the curvature perturbation is conserved on the large scales. We can,

therefore, compute the power spectrum of curvature perturbations in terms of quantities values at horizon exit.

2.1.3.2 Type II inflation

For type II inflation, the aforementioned effects, namely of entropy perturbations, can be present due to the curved trajectory (cf. the right panel of Fig. 2.3) in field space ($\dot{\Theta} \neq 0$). Due to this the curvature power spectrum could be sourced by entropy perturbations on large scales.

In order to study quantum fluctuations of the system we must consider the following canonically normalized fields defined by,

$$v_\sigma = \frac{a\sqrt{P_{,X}}}{c_s} Q_\sigma, \quad v_s = a\sqrt{P_{,X}} Q_s,$$

we can express the second order action for the adiabatic and entropy modes as

$$S_{(2)} = \frac{1}{2} \int d\tau d^3k \left[v_\sigma'^2 + v_s'^2 - 2\xi v_\sigma' v_s' - k^2 c_s^2 v_\sigma^2 - k^2 v_s^2 + \Omega_{\sigma\sigma} v_\sigma^2 + \Omega_{ss} v_s^2 + 2\Omega_{s\sigma} v_\sigma v_s \right], \quad (2.76)$$

with

$$\xi = \frac{a}{c_s} \Xi, \quad \Omega_{\sigma\sigma} = \frac{z''}{z} \quad \text{and} \quad \Omega_{ss} = \frac{\alpha''}{\alpha} - a^2 \mu_s^2,$$

where z and α are background dependent functions defined by

$$z = \frac{a\dot{\sigma}\sqrt{P_{,X}}}{c_s H}, \quad \alpha = a\sqrt{P_{,X}}.$$

The equations of motion derived from the action (2.76) are given by

$$v_\sigma'' - \xi v_s' + \left(c_s^2 k^2 - \frac{z''}{z} \right) v_\sigma - \frac{(z\xi)'}{z} v_s = 0, \quad (2.77)$$

$$v_s'' + \xi v_\sigma' + \left(k^2 - \frac{\alpha''}{\alpha} + a^2 \mu_s^2 \right) v_s - \frac{z'}{z} \xi v_\sigma = 0, \quad (2.78)$$

where μ_s^2 is the effective mass for the entropy field given by [131]

$$\mu_s^2 = -\frac{P_{,ss}}{P_{,X}} - \frac{1}{2c_s^2(X_1 + X_2)} \frac{P_{,s}^2}{P_{,X}^2} + 2\frac{P_{,Xs}P_{,s}}{P_{,X}^2} \quad (2.79)$$

and

$$\begin{pmatrix} P_{,\sigma\sigma} & P_{,\sigma s} \\ P_{,s\sigma} & P_{,ss} \end{pmatrix} = \begin{pmatrix} \cos \Theta & \sin \Theta \\ -\sin \Theta & \cos \Theta \end{pmatrix} \begin{pmatrix} P_{,X_1 X_1} & P_{,X_1 X_2} \\ P_{,X_2 X_1} & P_{,X_2 X_2} \end{pmatrix} \begin{pmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{pmatrix}. \quad (2.80)$$

The coupling between adiabatic and entropy modes is governed by the parameter ξ . In the cases where this parameter can be assumed to be small (see [131, 145]) at the typical scale of sound horizon exit⁷ the adiabatic and entropy modes decouple and analytical solutions for

⁷In contrast to the inflationary models where a sharp turn in field space occurs during inflation [148-150].

(2.77)-(2.78) can easily be found. In the decoupled case the adiabatic and entropy modes evolve according to the following equations,

$$v''_{\sigma} - \left(c_s^2 k^2 - \frac{z''}{z} \right) v_{\sigma} = 0, \quad (2.81)$$

$$v''_s + \left(k^2 - \frac{\alpha''}{\alpha} + a^2 \mu_s^2 \right) v_s = 0. \quad (2.82)$$

In the slow-roll limit, for a speed of sound that slowly varies while the scales of interest cross out the sound horizon, we can assume $z''/z' = 1/\tau^2$. Using this, we get as a general approximate solutions for the adiabatic and entropy modes with Bunch-Davies vacuum initial conditions,

$$v_{\sigma k} \simeq \frac{1}{\sqrt{2k c_s}} \exp(-ik c_s \tau) \left(1 - \frac{i}{k c_s \tau} \right), \quad (2.83)$$

$$v_{s k} \simeq \frac{1}{\sqrt{2k}} \exp(-ik \tau) \left(1 - \frac{i}{k \tau} \right), \quad (2.84)$$

where we assume $\frac{\mu_s^2}{H^2} \ll 1$ is valid for our case. This means entropy modes get amplified with respect to the adiabatic modes at the sound horizon crossing

$$Q_{\sigma*} \simeq \frac{Q_{s*}}{c_{s*}}. \quad (2.85)$$

The curvature and isocurvature perturbations are respectively,

$$\mathcal{R} = \frac{H}{\dot{\sigma}} Q_{\sigma}, \quad \mathcal{S} = c_s \frac{H}{\dot{\sigma}} Q_s. \quad (2.86)$$

The power spectrum of the curvature perturbation, evaluated at the sound horizon crossing ($c_s k = aH$), is given by

$$\mathcal{P}_{\mathcal{R}*} = \frac{k^3}{2\pi^2} \frac{|v_{\sigma k}|^2}{z^2} \simeq \frac{H^4}{8\pi^2 X P_{,X}} = \frac{H^2}{8\pi^2 \epsilon c_s} \Big|_*, \quad (2.87)$$

which recovers with the single field power spectrum result at horizon crossing [130]. However, in contrast to the single field inflation, the function ξ is not negligible and typically varies with time. This means that there will be a transfer between entropic and adiabatic modes on large scales but the converse is not true. From (2.70) and (2.86), the evolution of the curvature and entropy modes in the long wavelength limit can be approximated as [131]

$$\dot{\mathcal{R}} \approx \alpha H \mathcal{S}, \quad \dot{\mathcal{S}} \approx \beta H \mathcal{S},$$

where the coefficients α and β are taken to be,

$$\alpha = \frac{\Xi}{c_s H}, \quad (2.88)$$

$$\beta \simeq \frac{s}{2} - \frac{\eta}{2} - \frac{1}{3H^2} \left(\mu_s^2 + \frac{\Xi^2}{c_s^2} \right), \quad (2.89)$$

endowed with the definition of an additional slow-roll parameter $s = \frac{\dot{c}_s}{H c_s}$. The evolution of curvature and isocurvature perturbations after horizon crossing can be evaluated using transfer functions defined by

$$\begin{pmatrix} \mathcal{R} \\ \mathcal{S} \end{pmatrix} = \begin{pmatrix} 1 & \mathcal{T}_{\mathcal{R}\mathcal{S}} \\ 0 & \mathcal{T}_{\mathcal{S}\mathcal{S}} \end{pmatrix} \begin{pmatrix} \mathcal{R} \\ \mathcal{S} \end{pmatrix}_*,$$

where

$$\mathcal{T}_{\mathcal{R}\mathcal{S}}(t_*, t) = \int_{t_*}^t dt' \alpha(t') H(t') \mathcal{T}_{\mathcal{S}\mathcal{S}}(t_*), \quad (2.90)$$

and

$$\mathcal{T}_{\mathcal{S}\mathcal{S}}(t_*, t) = \exp \left\{ \int_{t_*}^t dt' \beta(t') H(t') dt' \right\}, \quad (2.91)$$

In addition, the curvature perturbation power spectrum, the entropy perturbation and the correlation between the two can be formally related as

$$\mathcal{P}_{\mathcal{R}} = (1 + \mathcal{T}_{\mathcal{R}\mathcal{S}}^2) \mathcal{P}_*, \quad \mathcal{P}_{\mathcal{S}} = \mathcal{T}_{\mathcal{S}\mathcal{S}}^2 \mathcal{P}_*, \quad (2.92)$$

$$\mathcal{C}_{\mathcal{R}\mathcal{S}} \equiv \langle \mathcal{R}\mathcal{S} \rangle = \mathcal{T}_{\mathcal{R}\mathcal{S}} \mathcal{T}_{\mathcal{S}\mathcal{S}} \mathcal{P}_*. \quad (2.93)$$

In contrast to the power spectrum for the scalar perturbations, the tensor power spectrum amplitude is the same as for a single field,

$$\mathcal{P}_t = \frac{2}{\pi^2} \frac{H^2}{M_{Pl}^2} \Big|_*$$

The tensor to scalar ratio defined in multifield inflation is given by

$$r \equiv \frac{\mathcal{P}_t}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon_* \cos^2 \Delta, \quad (2.94)$$

where Δ is the transfer angle given by

$$\cos \Delta = \frac{1}{\sqrt{1 + \mathcal{T}_{\mathcal{R}\mathcal{S}}^2}}.$$

Similarly, the spectral index also gets a correction, provided by the transfer functions,

$$n_s \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k} = n_s(t_*) + \frac{1}{H_*} \left(\frac{\partial \mathcal{T}_{\mathcal{R}\mathcal{S}}}{\partial t_*} \right) \sin(2\Delta), \quad (2.95)$$

where

$$n_{s*} = 1 - 2\epsilon_* - \eta_* - s_*.$$

The spectral index and the tensor to scalar ratio are the key observables which not only depend

on the slow-roll at horizon crossing, but also depend on the transfer angle Δ . This enables a clear distinction between multifields and single field inflationary scenarios⁸ [151]. The transfer functions defined in (2.90) and (2.91) are allowed to evolve after the Hubble exit, even after inflation, during the reheating and radiation dominated era [151, 153]. However the evolution of isocurvature perturbations, during reheating and radiation dominated era, would depend on the particular final stage of the inflationary scenario. Consider for example, a two field scenario, if one field enters a regime of oscillations while the second field is still inflating the Universe. In such cases the curvature perturbation can be sourced by entropy modes even after inflation [152]. This kind of scenarios are known as ‘curvaton’ or ‘spectator’ field behavior [154, 155] and also found in double quadratic inflation [153]. In the case of two 3-forms inflation, we will assume that entropy perturbations do not grow further after inflation. Therefore we only evaluate transfer functions from horizon exit until the end of inflation and predict the values of n_s and r [149]. We can see from (2.94) and (2.95) that if $\mathcal{T}_{\mathcal{RS}} = 0$ then our predictions match the single field result. From the Sec. 2.1.3.1 and 2.88 it is evident that $\mathcal{T}_{\mathcal{RS}} = 0$ for type I inflation. Therefore to make observational contrast with single 3-form we mainly focus on testing type II inflationary scenario in the following subsection.

2.1.4 Two 3-form fields inflation and Power spectra

Based on the discussion made on the curvature perturbation power spectrum in Sec. 2.1.3, the main objective is to test our two 3-forms model and predicting values of inflationary parameters. We choose suitable potentials and initial conditions, in order to obtain a reasonable fit with present available experimental bounds [132]. The majority of inflationary models with a non canonical kinetic term contain a common feature that the adiabatic fluctuations propagate with a sound speed $c_s^2 < 1$. The recent *Planck* data restricts this speed of sound to be in the interval $0.02 \lesssim c_s^2 < 1$. Multiple field inflation models allow the possibility of having a varying speed of sound, i.e, like for the type II solution in our model (cf. Sec. 2.1.2.2). The speed of sound variation will therefore have implications on the running spectral index and the scale invariance. These peculiar effects, being a consequence of the varying speed of sound, have been studied in a DBI context and also in modified gravity models with an effective inflaton [156-158].

We have examined all the potentials in Table 2.1. We found that $\chi_I^2 + b_i \chi_I^4$ is consistent with observational bounds⁹. It is quite difficult to constrain the speed of sound ($0.02 \lesssim c_s^2 < 1$) during inflation. We found that only type II solutions which are slightly deviated from type I are suitable to maintain consistent speed of sound during inflation. To predict values of inflationary parameters, first we need to compute the transfer functions defined in Sec. 2.1.3 and evaluate their value at the end of inflation.

We can read from (2.95) that the spectral index depends on the derivative of $\mathcal{T}_{\mathcal{RS}}$ at horizon crossing. From the right panel of Fig. 2.4 it is clear that the derivative of $\mathcal{T}_{\mathcal{RS}}$, between $N = 0$ and $N = 60$, is very small and we can, therefore, neglect it. Hence, our prediction of spectral index only depends on the values of the slow-roll parameters at horizon exit.

⁸However tensor to scalar ratio is more constrained by consistency relations in case of inflation with more than two fields [151, 152].

⁹We confront our results with $\chi_I^2 + b_I \chi_I^4$ potential, and one can find make similar predictions with $\chi_I^2 + b_I \chi_I^3$ potential. We have not consider to explore quadratic potential as it is equivalent to inflation with canonical scalar fields (in dual picture).

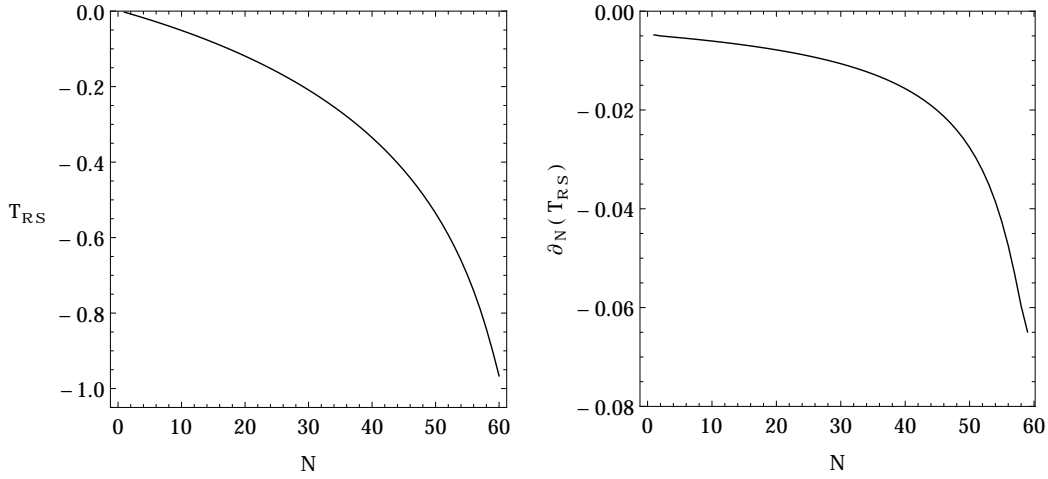


Figure 2.4: Graphical representation of \mathcal{T}_{RS} (left panel) and $\frac{d\mathcal{T}_{RS}}{dN}$ (right panel) until the end of inflation (defined for $\varepsilon = 1$). We have taken $V_1 = V_{01}(\chi_1^2 + b\chi_1^4)$ and $V_2 = V_{02}(\chi_2^2 + b\chi_2^4)$ where $V_{01} = 1$, $V_{02} = 0.93$, $b = -0.35$ and with initial conditions $\theta = \pi/4$.

The running of the spectral index to the lowest order in slow-roll is now given by, regime,

$$\frac{dn_s}{d \ln k} = \left(1 + \varepsilon + \frac{c'_s}{c_s}\right) \bigg|_* \left(n'_{s*} + \frac{\partial \mathcal{T}_{RS}}{\partial N_*} \frac{\partial}{\partial N} \left(\frac{2\mathcal{T}_{RS}}{1 + \mathcal{T}_{RS}^2} \right) + \frac{\partial^2 \mathcal{T}_{RS}}{\partial N_*^2} \sin 2\Delta \right). \quad (2.96)$$

For the choice of potential in Fig. 2.4 we can neglect the transfer function corrections to the running spectral index (2.96). Therefore for this case the additional slow-roll parameter $s = \frac{c'_s}{c_s}$ is of relevance, which enables us to observationally distinguish between two 3-forms and single 3-form inflation¹⁰, with respect to the running of spectral index. Expression (2.96) is expanded up to the first order in the slow-roll parameters. The second order corrections are crucial if there is an abrupt path turn in field space during horizon exit. These types of scenarios are considered in detail in studies related with hybrid inflation and double quadratic inflation [159]. We can neglect these corrections for two 3-form inflation, since the type II solutions herein considered do not exhibit abrupt turns in field space under slow-roll conditions.

To predict tensor to scalar ratio (2.94) for two 3-forms it is required to know the value of \mathcal{T}_{RS} at the end of inflation. From the left panel of Fig. 2.4, \mathcal{T}_{RS} is $\mathcal{O}(1)$ at the end of inflation. Therefore it can reduce the value of tensor to scalar ratio in contrast to the single 3-form case. Evidently two 3-forms inflation can be observationally distinguished from single 3-form inflation, due to the possibility of a varying speed of sound (cf. Sec. 2.1.2.2) and transfer function corrections by the end of inflation. Our method of observational analysis are quite similar to the studies in [146, 149]. In the following we confront our results against *Planck*+WP+BAO data which provides $\frac{dn_s}{d \ln k} = -0.013 \pm 0.009$ for the running of spectral index, and $\frac{d^2 n_s}{d \ln k^2} = 0.017 \pm 0.009$ for the running of running spectral index, both at 95% CL, which rules out exact scale-invariance at more than 5σ level. Our analysis show that for type II solution, a better fit can be achieved given the current observational bounds (ruling out exact scale-invariance).

In Figs. 2.5 and 2.6, obtained through suitable data manipulating programs [160, 161], we have examined various types of potentials for a reasonable fit to the observational constraints from *Planck* data. We found that potentials such as $V_I = V_{0I}(\chi_I^2 + b_I \chi_I^4)$ allow favorable contrast

¹⁰In the single 3-form case [129, 139] and also in the type I solution of two 3-forms case, this additional slow-roll parameter satisfies, $s \equiv \frac{c'_s}{c_s H} = 0$.

of two 3-forms inflation scenario against recent observational data. The parameter b_i , in the mentioned potential, is adequately chosen, so that the speed of sound gets bounded by $0.02 \lesssim c_s^2 < 1$, in order to comply with the *Planck* constraint. We found that type II inflation, obtained through a small asymmetry in the slopes of the potentials (making $V_{01} \neq V_{02}$), is needed to fit the parameters within the bounds of the observational data, especially for the running and running of running spectral indexes. There are two relevant aspects that should be mentioned regarding this comparison; one is related to the property of type II solution for computing the running of the spectral index. This is a consequence of the varying speed of sound, which is natural for this solution. The other aspect is the requirement of the asymmetry between the potentials. This leads to a mild generation of isocurvature perturbations towards the end of inflation, which can accommodate tensor to scalar ratio values within the present bounds of *Planck*. We note that solutions with large curved trajectory in field space can lead to values for inflationary parameters beyond the observational bounds. The presence of curvature, in the field space trajectories, implies a peculiar imprint in the primordial bispectrum during multiple field inflation which we will study in the next section.

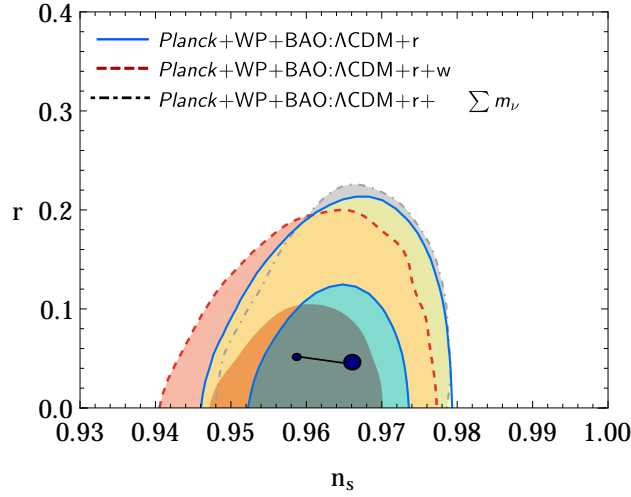


Figure 2.5: Graphical representation of the spectral index versus the tensor to scalar ratio, in the background of *Planck*+WP+BAO data (left panel), for $N = 60$ number of e -folds before the end of inflation (large dot) and $N = 50$ (small dot). We have taken $V_1 = V_{01}(\chi_1^2 + b\chi_1^4)$ and $V_2 = V_{20}(\chi_2^2 + b\chi_2^4)$ where $V_{01} = 1$, $V_{20} = 0.93$, $b = -0.35$ for two 3-form.

2.2 Non-Gaussianities with multiple 3-forms

In the previous section, we have computed the powerspectrum of curvature perturbations and its evolution on superhorizon scales using transfer fuctions. In this section, we compute the Bispectrum and the reduded bispectrum f_{NL} on superhorizon scales using δN formalism [162] which is more convenient method for computing non-Gaussianities with multifields over using transfer functions [146]. However, both of these methods are equivalent and the final results are independent of the formalism we use.

This section is organized as follows. In Sec. 2.2.1 we discuss the bispectrum and describe a procedure to adapt the δN formalism [162] to multiple 3-forms to calculate it. We explain a numerical method for calculating derivatives of the unperturbed number of e -foldings with

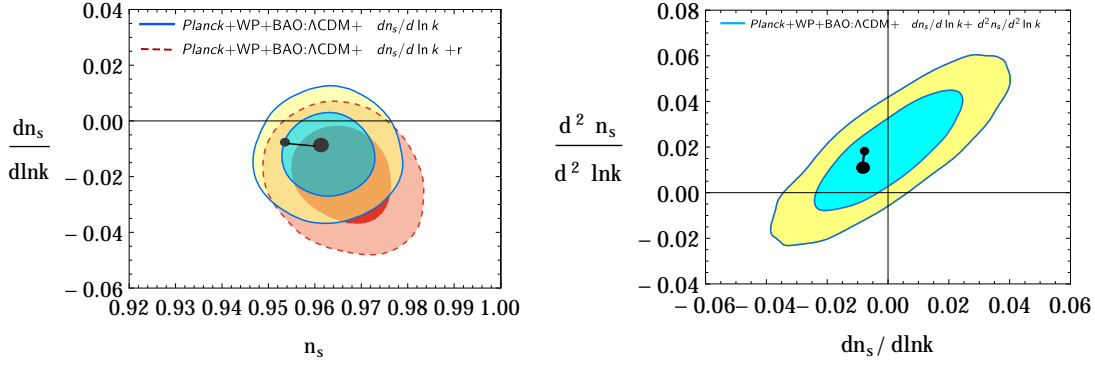


Figure 2.6: Graphical representation of the running of the spectral index versus the spectral index (left panel), and running of the running of the spectral index versus the running of the spectral index (right panel) in the background of *Planck*+WP+BAO data for $N = 60$ number of e -folds before the end of inflation (large dot) and $N = 50$ (small dot). We have taken $V_1 = V_{01}(\chi_1^2 + b\chi_1^4)$ and $V_2 = V_{20}(\chi_2^2 + b\chi_2^4)$ where $V_{01} = 1$, $V_{20} = 0.93$, $b = -0.35$ for two 3-form. This figure was also obtained by taking the initial condition $\theta = \pi/4$.

respect to the unperturbed 3-form field values at sound horizon crossing, and show how these derivatives can be related to those of a dual scalar field description. In turn these can be used in combination with existing results to compute the bispectrum. We stress that although our method utilizes the dual scalar field description, it is not possible in general to simply pass to that description and work solely with a scalar field model. In Sec. 2.2.2 we consider the two 3-form inflation with the same potentials of the previous section that provides a power-spectrum compatible with *Planck* constraints and compute the bispectrum in that model. We quantify and compare the momentum dependent contribution and momentum independent contributions of the reduced bispectrum and plot the shape of the bispectrum.

2.2.1 Non-Gaussianity and the δN formalism

2.2.1.1 The δN formalism

The δN formalism is based on the separate universe assumption [17, 163-167] and provides a powerful tool to evaluate the superhorizon evolution of the curvature perturbation. In the case of multiple 3-forms, however, the direct implementation of the δN formalism would be cumbersome. Using the formal relation between 3-forms and their scalar field duals in Sec. 2.1.1.1, however, one can indirectly implement the δN formalism while still employing only 3-form quantities that are easy to calculate.

The δN formalism allows the evolution of the curvature perturbation to be calculated, on scales larger than the horizon scale where one can neglect spatial gradients, using only the evolution of unperturbed "separate universes". The central result is that the difference in the number of e -folds that occurs from different positions on an initial flat slice of spacetime to a final uniform density slice, when compared with some fiducial value, is related to the curvature perturbation. Writing the number of e -foldings as a function of the initial and final time on the relevant hypersurfaces,

$$N(t, t_i, x) = \int_{t_i}^t dt' H(t', x), \quad (2.97)$$

the primordial curvature perturbation can be expressed as

$$\zeta(t, x) = N(t, t_i, x) - N_0(t, t_i), \quad (2.98)$$

where $N_0(t, t_i) = \int_{t_i}^t dt' H_0(t')$. Taking $t_i = t_*$, the time corresponding to the modes exiting the horizon ($kc_s = aH$), the curvature perturbation on superhorizon scales can be written in terms of partial derivatives of N with respect to the unperturbed scalar field values at horizon exit, while holding the initial and final hypersurface constant. More precisely

$$\zeta(t, x) = \sum_I N_{,I}(t) \delta\phi_*^I(x) + \frac{1}{2} \sum_{IJ} N_{,IJ}(t) \delta\phi_*^I(x) \delta\phi_*^J(x) + \dots, \quad (2.99)$$

where $N_{,I} = \frac{\partial N}{\partial \phi_*^I}$. In momentum space we have

$$\zeta(k) = N_{,I} \delta\phi_*^I(k) + \frac{1}{2} N_{,IJ} [\delta\phi_*^I \star \delta\phi_*^J](k) + \dots, \quad (2.100)$$

where \star indicates a convolution.

2.2.1.2 Calculating the bispectrum with δN

The power spectrum and bispectrum of field fluctuations at horizon crossing follow from the two- and three-point correlations of these perturbations as

$$\langle \delta\phi_*^I(\mathbf{k}_1) \delta\phi_*^J(\mathbf{k}_2) \rangle = (2\pi)^3 G^{IJ} \frac{2\pi^2}{k^3} \mathcal{P}^* \delta(\mathbf{k}_1 + \mathbf{k}_2) \quad (2.101)$$

$$\langle \delta\phi_*^I(\mathbf{k}_1) \delta\phi_*^J(\mathbf{k}_2) \delta\phi_*^K(\mathbf{k}_3) \rangle = (2\pi)^3 \frac{4\pi^4}{\prod_i k_i^3} \mathcal{P}^{*2} A^{IJK}(k_1, k_2, k_3) \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3), \quad (2.102)$$

where $\mathcal{P} = Pk^3/(2\pi^2)$. Employing the δN expansion one finds that

$$P_\zeta(k) = N_I N_I P^* \quad (2.103)$$

and

$$f_{\text{NL}} = f_{\text{NL}}^{(3)} + f_{\text{NL}}^{(4)} + \dots, \quad (2.104)$$

where

$$\begin{aligned} f_{\text{NL}}^{(3)} &= \frac{5}{6} \frac{N_{,I} N_{,J} N_{,K} A^{IJK}}{(G^{IJ} N_{,I} N_{,J})^2 \sum_i k_i^3}, \\ f_{\text{NL}}^{(4)} &= \frac{5}{6} \frac{G^{IK} G^{JL} N_{,I} N_{,J} N_{,KL}}{(G^{IJ} N_{,I} N_{,J})^2}. \end{aligned} \quad (2.105)$$

Here $f_{\text{NL}}^{(3)}$ is momentum dependent, whereas $f_{\text{NL}}^{(4)}$ is momentum independent (which is the definition of local f_{NL})¹¹. In general, the dominant contribution, $f_{\text{NL}}^{(3)}$ or $f_{\text{NL}}^{(4)}$, is model dependent. For example, in the case of multiple canonical scalar fields inflation, $f_{\text{NL}}^{(4)}$ can become significant

¹¹Technically these results are valid only when there is not a large hierarchy between the three wave numbers of the bispectrum and they can all be assumed to cross the horizon at roughly the same time. This provides a good approximation even for large hierarchies as long as there is not a significant evolution between the horizon crossing times of the three modes (see Refs. [168, 169] for a full discussion)

. In contrast, for non-canonical models, $f_{\text{NL}}^{(3)}$ can become large.

For general multi-field non-canonical models in slow-roll (which is the situation relevant to our models), utilising the In-In formalism to calculate the statistics of the scalar field perturbations on flat hypersurfaces at horizon crossing it was found that

$$P_* = \frac{H^2}{2k^3 P_{,X}}, \quad (2.106)$$

and that [170]

$$A_{IJK} = \frac{1}{4} \sqrt{\frac{P_{,X}}{2}} \tilde{A}_{IJK}, \quad (2.107)$$

with

$$\begin{aligned} \tilde{A}^{IJK} = & G^{IJ} \varepsilon^K \frac{u}{\varepsilon} \left[\frac{4k_1^2 k_2^2 k_3^2}{K^3} - 2(\mathbf{k}_1 \cdot \mathbf{k}_2) k_3^2 \left(\frac{1}{K} + \frac{k_1 + k_2}{K^2} + \frac{2k_1 k_2}{K^3} \right) \right] \\ & - G^{IJ} \varepsilon^K \left[6 \frac{k_1^2 k_2^2}{K} + 2 \frac{k_1^2 k_2^2 (k_3 + 2k_2)}{K^2} + k_3 k_2^2 - k_3^3 \right] \\ & + G^{IJ} \left[\left(3 \frac{u}{\varepsilon} + 4u + 4 \right) \tilde{\varepsilon}^K + \tilde{\varepsilon}_{,X}^K \frac{12H^2}{P_{,X}} \right] \times \\ & \left[- \frac{k_1^2 k_2^2}{K} - \frac{k_1^2 k_2^2 k_3}{K^2} + (\mathbf{k}_1 \cdot \mathbf{k}_2) \left(-K + \frac{\sum_{i>j} k_i k_j}{K} + \frac{k_1 k_2 k_3}{K^2} \right) \right] \\ & + \frac{\varepsilon^{IJ}}{\varepsilon} \varepsilon^K \left(\frac{2\lambda}{H^2 \varepsilon^2} - \frac{u}{\varepsilon} \right) \frac{4k_1^2 k_2^2 k_3^2}{K^3} + \text{perms.}, \end{aligned} \quad (2.108)$$

where $K = k_1 + k_2 + k_3$, and the Hubble parameter H , the sound speed squared (c_s^2), and slow-roll parameters ($\varepsilon, \varepsilon^I, \dots$, etc.) are evaluated at sound horizon exit $c_s k = aH$. Expressions for c_s^2 , u and λ are given in Ref. [170] for non-canonical models¹². In this work, we express all of these parameters in terms of 3-form quantities using (2.17) and (2.19). First u is defined as

$$u \equiv \frac{1}{\bar{c}_s^2} - 1, \quad (2.109)$$

where the effective speed of sound¹³ is given by

$$\bar{c}_s^2 = \frac{P_{,X}}{2X P_{,XX} + P_{,X}} = \frac{\sum_I \frac{X_I}{V_{,X_I}}}{\sum_I V_{,X_I}^{-1}}. \quad (2.110)$$

We also define λ , such that

$$\lambda = X^2 P_{,XX} + \frac{2}{3} X^3 P_{,XXX} = - \sum_I \frac{V_{,X_I}^3 V_{,X_I X_I X_I}}{12 V_{,X_I X_I}^3}. \quad (2.111)$$

¹²We have corrected typos in the first and third lines of (2.108) that were present in Ref. [170].

¹³We note that during the slow-roll regime effective sound speed is nearly the same as adiabatic sound speed [143]. Therefore, using the slow-roll approximation, from 2.11 and 2.63 we can deduce $\bar{c}_s^2 \approx c_s^2$.

The various slow-roll quantities are defined by

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2} \frac{\sum_I \chi_I V_{,\chi_I}}{V} \left(1 - \sum_I w_I^2 \right), \quad (2.112)$$

$$\varepsilon^{IJ} = \frac{P_{,X} \dot{\phi}^I \dot{\phi}^J}{2H^2} = \frac{P_{,X} \sqrt{X_I X_J}}{2H^2} = \varepsilon^I \varepsilon^J, \quad (2.113)$$

where

$$\varepsilon^I = \sqrt{\frac{X_I P_{,X}}{2H^2}} = \sqrt{\frac{3V_{,\chi_I}^2}{4V} \left(\sum_I \frac{\chi_I}{V_{,\chi_I}} \right) \left(1 - \sum_I w_I^2 \right)}, \quad (2.114)$$

$$\tilde{\varepsilon}_I = -\frac{P_{,I}}{3\sqrt{2P_{,X}}H^2} = \frac{\sqrt{6}w_I}{3\sqrt{2\sum_I \frac{\chi_I}{V_{,\chi_I}}}H}. \quad (2.115)$$

Using the Friedmann equation in (2.6) we obtain

$$\begin{aligned} \tilde{\varepsilon}_{,X}^I &= -\frac{P_{,XI}}{3\sqrt{2P_{,X}}H^2} + P_{,I} \left[\frac{2XP_{,XX} + P_{,X}}{9\sqrt{2P_{,X}}H^4} + \frac{P_{,XX}}{6\sqrt{2P_{,X}^{3/2}}H^2} \right], \\ &= -\sqrt{6}Hw_I \left[\frac{\sum_I V_{,\chi_I}^{-1}}{\sqrt{2\sum_I \frac{\chi_I}{V_{,\chi_I}}}V} + \frac{\sum_I (V_{,\chi_I}^{-1} V_{,\chi_I}^{-2} - \chi_I V_{,\chi_I}^{-3})}{3\sqrt{2} \left(\sum_I \frac{\chi_I}{V_{,\chi_I}} \right)^{3/2} V} \right] \left(1 - \sum_I w_I^2 \right). \end{aligned} \quad (2.116)$$

Note that the dual scalar field action in (2.17) satisfies $P_{,XI} = 0$.

In the squeezed limit i.e., $k_2 \rightarrow 0$, it can be seen from (2.108) that $f_{\text{NL}}^{(3)}$ reduces to the order of slow-roll parameters. Therefore $f_{\text{NL}}^{(4)}$ is expected to be dominant in this limit if non-Gaussianity is significant.

2.2.1.3 The δN for two 3-forms

The crucial step, when it comes to computing f_{NL} , is the calculation of the derivatives of N with respect to the fields at the sound horizon crossing. In general $N_{,I}$ and $N_{,IJ}$ evolve on superhorizon scales and except in a few cases (see e.g., Ref. [153]) the analytical computation of these quantities is not tractable. For this reason we do our computations numerically using a method that is explained in Sec. 2.2.2.

First of all we must rewrite the derivatives in terms of 3-forms. Here we do this explicitly for two 3-forms. The same procedure can be extended trivially to \mathbb{N} 3-form fields. We can infer the following relations from (2.19) and (2.12) relating two 3-forms to the two non-canonical scalar fields

$$\phi_1 = \sqrt{6}Hw_1 \equiv \phi_1(\chi_1, \chi_2, w_1, w_2), \quad (2.117)$$

$$\phi_2 = \sqrt{6}Hw_2 \equiv \phi_2(\chi_1, \chi_2, w_1, w_2), \quad (2.118)$$

It is highly nontrivial to invert the relations in (2.117) and (2.118). While the fields are slowly rolling, one can verify that the approximation $w_I \approx \sqrt{\frac{3}{2}}\chi_I$ is accurately satisfied. As a consequence, we express the N derivatives $N_{,I}$ and $N_{,IJ}$ in terms of the two 3-forms χ_1, χ_2 as

$$\frac{\partial N}{\partial \phi_1^*} = \frac{\partial N}{\partial \chi_1^*} \frac{\partial \chi_1^*}{\partial \phi_1^*} + \frac{\partial N}{\partial \chi_2^*} \frac{\partial \chi_2^*}{\partial \phi_1^*}, \quad (2.119)$$

$$\begin{aligned} \frac{\partial^2 N}{\partial \phi_1^* \partial \phi_2^*} &= \frac{\partial N}{\partial \chi_1^*} \frac{\partial^2 \chi_1^*}{\partial \phi_1^* \partial \phi_2^*} + \frac{\partial N}{\partial \chi_2^*} \frac{\partial^2 \chi_2^*}{\partial \phi_1^* \partial \phi_2^*} + \frac{\partial^2 N}{\partial \chi_1^{*2}} \frac{\partial \chi_1^*}{\partial \phi_1^*} \frac{\partial \chi_1^*}{\partial \phi_2^*} \\ &\quad + \frac{\partial^2 N}{\partial \chi_2^{*2}} \frac{\partial \chi_2^*}{\partial \phi_1^*} \frac{\partial \chi_2^*}{\partial \phi_2^*} + \frac{\partial^2 N}{\partial \chi_1^* \partial \chi_2^*} \frac{\partial \chi_1^*}{\partial \phi_1^*} \frac{\partial \chi_2^*}{\partial \phi_2^*} + \frac{\partial^2 N}{\partial \chi_1^* \partial \chi_2^*} \frac{\partial \chi_1^*}{\partial \phi_2^*} \frac{\partial \chi_2^*}{\partial \phi_1^*}, \end{aligned} \quad (2.120)$$

$$\frac{\partial^2 N}{\partial \phi_1^{*2}} = \frac{\partial N}{\partial \chi_1^*} \frac{\partial^2 \chi_1^*}{\partial \phi_1^{*2}} + \frac{\partial N}{\partial \chi_2^*} \frac{\partial^2 \chi_2^*}{\partial \phi_1^{*2}} + \frac{\partial^2 N}{\partial \chi_1^{*2}} \left(\frac{\partial \chi_1^*}{\partial \phi_1^*} \right)^2 + \frac{\partial^2 N}{\partial \chi_2^{*2}} \left(\frac{\partial \chi_2^*}{\partial \phi_1^*} \right)^2 + 2 \frac{\partial^2 N}{\partial \chi_1^* \partial \chi_2^*} \frac{\partial \chi_1^*}{\partial \phi_1^*} \frac{\partial \chi_2^*}{\partial \phi_1^*}. \quad (2.121)$$

derivatives of ϕ_2 . These equations define the relations among the N derivatives ($N_{,I}$ and $N_{,IJ}$) with respect to scalar field ϕ_I^* to the N derivatives with respect to 3-form fields at horizon crossing $\frac{\partial N}{\partial \chi_1^*}, \frac{\partial N}{\partial \chi_2^*}, \frac{\partial^2 N}{\partial \chi_1^* \partial \chi_2^*}, \frac{\partial^2 N}{\partial \chi_1^{*2}}, \frac{\partial^2 N}{\partial \chi_2^{*2}}$. In other words, we have indirectly transported the δN formalism from scalar fields to 3-form fields. However, we still need to calculate the derivatives of the 3-form fields with respect to the dual scalar fields. For this purpose we differentiate the relations (2.117) and (2.118) keeping in mind that ϕ_1 and ϕ_2 are independent fields. Then we have that

$$\frac{d\phi_1}{d\phi_1} = \frac{1}{\sqrt{6}w_1} \frac{\partial H}{\partial \phi_1} + \frac{1}{\sqrt{6}H} \frac{\partial w_1}{\partial \phi_1} = 1. \quad (2.122)$$

$$\frac{d\phi_1}{d\phi_2} = \frac{1}{\sqrt{6}w_1} \frac{\partial H}{\partial \phi_2} + \frac{1}{\sqrt{6}H} \frac{\partial w_1}{\partial \phi_2} = 0. \quad (2.123)$$

$$\frac{d\phi_2}{d\phi_1} = \frac{1}{\sqrt{6}w_2} \frac{\partial H}{\partial \phi_1} + \frac{1}{\sqrt{6}H} \frac{\partial w_2}{\partial \phi_1} = 1. \quad (2.124)$$

$$\frac{d\phi_2}{d\phi_2} = \frac{1}{\sqrt{6}w_2} \frac{\partial H}{\partial \phi_2} + \frac{1}{\sqrt{6}H} \frac{\partial w_2}{\partial \phi_2} = 0. \quad (2.125)$$

Solving (2.122)-(2.125) for a potential of the form $V = V(\chi_1) + V(\chi_2)$, we obtain

$$\begin{aligned} \frac{\partial \chi_1}{\partial \phi_1} &= \frac{\chi_2 V_{,\chi_2} + H^2 (6 - 9\chi_1^2)}{3H (6H^2 + \chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2})} \\ \frac{\partial \chi_1}{\partial \phi_2} &= - \frac{\chi_1 (V_{,\chi_2} + 9H^2 \chi_2)}{3H (6H^2 + \chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2})} \end{aligned} \quad (2.126)$$

$$\begin{aligned} \frac{\partial^2 \chi_1}{\partial \phi_1^2} = & \frac{-1}{9H^2 (6H^2 + \chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2})^3} \left\{ \chi_1 V_{,\chi_1}^2 [\chi_2 (\chi_2 V_{,\chi_2 \chi_2} + 2V_{,\chi_2}) + H^2 (9\chi_1^2 - 6)] \right. \\ & - 2V_{,\chi_1} [-3H^2 \chi_2 (3V_{,\chi_2 \chi_2} \chi_1^2 \chi_2 + 6V_{,\chi_2} \chi_1^2 + 4V_{,\chi_2}) - V_{,\chi_2}^2 \chi_2^2 + 18H^4 (3\chi_1^2 - 2)] \\ & + \chi_1 V_{,\chi_1 \chi_1} (\chi_2 V_{,\chi_2} + H^2 (6 - 9\chi_1^2))^2 \\ & \left. - 9\chi_1 H^2 (-3H^2 \chi_2 (3V_{,\chi_2 \chi_2} \chi_1^2 \chi_2 + 12V_{,\chi_2}) - 3V_{,\chi_2}^2 \chi_2^2 + 54H^4 (3\chi_1^2 - 2)) \right\}. \end{aligned} \quad (2.127)$$

$$\begin{aligned} \frac{\partial^2 \chi_1}{\partial \phi_2^2} = & \frac{-1}{9H^2 (6H^2 + \chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2})^3} \left\{ \chi_1 [18V_{,\chi_2} H^2 \chi_2 (V_{,\chi_1 \chi_1} \chi_1^2 - 18H^2) - 2V_{,\chi_2}^3 \chi_2] \right. \\ & + \chi_1 V_{,\chi_2}^2 [\chi_1 (\chi_1 V_{,\chi_1 \chi_1} - 2V_{,\chi_1}) - 3H^2 (3\chi_2^2 + 10)] \\ & + \chi_1 V_{,\chi_2 \chi_2} [V_{,\chi_1} \chi_1 + H^2 (6 - 9\chi_2^2)]^2 \\ & \left. + 9\chi_1 H^2 [3H^2 \chi_1 (3V_{,\chi_1 \chi_1} \chi_1 \chi_2^2 + 4V_{,\chi_1}) + V_{,\chi_1}^2 \chi_1^2 - 18H^4 (9\chi_2^2 - 2)] \right\}. \end{aligned} \quad (2.128)$$

$$\begin{aligned} \frac{\partial^2 \chi_1}{\partial \phi_1 \partial \phi_2} = & \frac{1}{9H^2 (6H^2 + \chi_1 V_{,\chi_1} + \chi_2 V_{,\chi_2})^3} \left\{ \right. \\ & - V_{,\chi_2}^3 \chi_2^2 + V_{,\chi_2}^2 \chi_2 [V_{,\chi_1 \chi_1} \chi_1^2 + 3H^2 (-4 + 3\chi_1^2 - 3\chi_2)] \\ & + V_{,\chi_2} [3H^2 \chi_1 (V_{,\chi_1 \chi_1} \chi_1 (-3\chi_1^2 + 3\chi_2^2 + 2) + 3V_{,\chi_1} (\chi_1^2 - \chi_2^2 + 2)) + V_{,\chi_1}^2 \chi_1^2] \\ & + 36V_{,\chi_2} H^4 (6\chi_1^2 - 3\chi_2^2 - 1) + \chi_2 V_{,\chi_2 \chi_2} V_{,\chi_1}^2 \chi_1^2 \\ & + 3\chi_2 V_{,\chi_2 \chi_2} V_{,\chi_1} H^2 \chi_1 (3\chi_1^2 - 3\chi_2^2 + 2) \\ & + 162\chi_2 H^6 (9\chi_1^2 - 2) \\ & \left. + 27\chi_2 H^4 \chi_1 (\chi_1 (-3V_{,\chi_1 \chi_1} \chi_1^2 + 2V_{,\chi_1 \chi_1} - 3V_{,\chi_2 \chi_2} \chi_2^2 + 2V_{,\chi_2 \chi_2}) + 4\chi_2 V_{,\chi_1}) \right\}. \end{aligned} \quad (2.129)$$

The remaining derivatives can be obtained from these by interchanging $1 \leftrightarrow 2$. Following (2.119)-(2.121) the quantities obtained in (2.126)-(2.129) are to be evaluated at $kc_s = aH$. However, the derivatives of N with respect to the 3-form fields evolve on superhorizon scales. In the squeezed limit i.e., $k_2 \rightarrow 0$, it can be seen from (2.108) that $f_{\text{NL}}^{(3)}$ reduces to the order of slow-roll parameters. Therefore $f_{\text{NL}}^{(4)}$ is expected to be dominant in this limit if non-Gaussianity is significant.

2.2.2 Non-Gaussianities in two 3-form inflation

In this subsection, we aim further update the observational status of two 3-form inflation by means of calculating the reduced bispectrum f_{NL} . We consider, the type II solutions with the potentials, $V(\chi_1, \chi_2) = V_{01}f(\chi_1) + V_{20}f(\chi_2)$, where $f(\chi_I) = \chi_I^2 + b\chi_I^{2n}$, which were studied in Sec. 2.1 and were shown to be consistent with *Planck* data, predicting the scalar spectral index $n_s \sim 0.967$ and the tensor to scalar ratio $r \sim 0.0422$.

The observational prediction of non-Gaussianity for multifield inflation is deeply associated with the evolution of isocurvature perturbations. In the single field inflation the statistics of the curvature perturbation evaluated at horizon exit can be confronted with the observation. This is because the curvature perturbation is conserved on superhorizon scales if the system is adi-

abatic [142, 166, 171]. Whereas for multifield models, the statistics evolve on superhorizon scales and non-Gaussianity can be generated as a consequence of the presence of isocurvature perturbations. This can happen in two regimes, namely, (i) during inflation [23, 149, 172, 173] and (ii) after inflation such as in the curvaton model [174-185]. In general the statistics continue to evolve until all isocurvature perturbations decay, the so-called adiabatic limit [23]. We evaluate f_{NL} at the end of inflation, this is a good approximation as long as reheating proceeds quickly, and curvaton type effects do not occur.

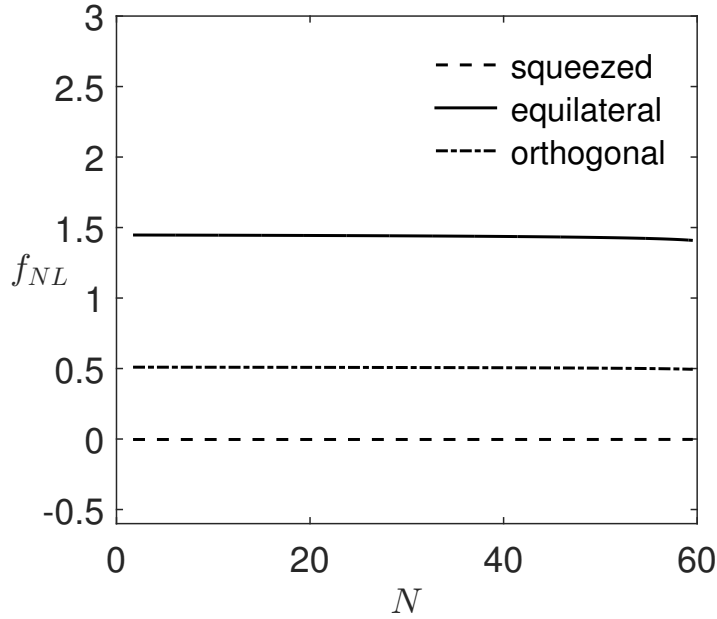


Figure 2.7: In this plot we depict f_{NL} against N for squeezed ($k_2 \ll k_1 = k_3$) equilateral ($k_1 = k_2 = k_3$) and orthogonal ($k_1 = 2k_2 = 2k_3$) configurations. We have considered the potentials $V_1 = V_{01} (\chi_1^2 + b_1 \chi_1^4)$ and $V_2 = V_{20} (\chi_2^2 + b_2 \chi_2^4)$ with $V_{01} = 1$, $V_{20} = 0.93$, $b_{1,2} = -0.35$ and taken the initial conditions $\chi_1(0) \approx 0.5763$, $\chi_2(0) \approx 0.5766$, $\chi'_1(0) = -0.000224$, $\chi'_2(0) = 0.00014$.

To calculate f_{NL} given in (3.6), we need to compute the N derivatives with respect to the initial conditions of 3-form fields defined in (2.119)-(2.121). To compute these numerically, we define the following discrete derivatives that can in principle, be extended to any number of fields,

$$\begin{aligned}
 N_{,\chi_1^*} &= \frac{N(\chi_1^* + \Delta\chi_1, \chi_2^*) - N(\chi_1^* - \Delta\chi_1, \chi_2^*)}{2\Delta\chi_1}, \\
 N_{,\chi_1^* \chi_1^*} &= \frac{N(\chi_1^* + \Delta\chi_1, \chi_2^*) - 2N(\chi_1^*) + N(\chi_1^* - \Delta\chi_1, \chi_2^*)}{\Delta\chi_1^2}, \\
 N_{,\chi_1^* \chi_2^*} &= [N(\chi_1^* + \Delta\chi_1, \chi_2^* + \Delta\chi_2) - N(\chi_1^* + \Delta\chi_1, \chi_2^* - \Delta\chi_2) - \\
 &\quad N(\chi_1^* - \Delta\chi_1, \chi_2^* + \Delta\chi_2) + N(\chi_1^* - \Delta\chi_1, \chi_2^* - \Delta\chi_2)] (4\Delta\chi_1^2)^{-1},
 \end{aligned} \tag{2.130}$$

and similarly we can obtain the remaining derivatives by interchanging $1 \leftrightarrow 2$. In the above expression, $N(\chi_1, \chi_2)$ is the number of e -foldings that occur starting at initial conditions $\{\chi_1^*, \chi_2^*\}$ and ending at a given final energy density. This final energy density is defined by the condition that $N(\chi_1, \chi_2) = 60.35$ at the point $\varepsilon = 1$. That is the central point in the finite difference represents a trajectory that undergoes 60 e -folds of inflation, from the initial field value until inflation ends, and the density at that time is used as the final density for all the other points in the difference scheme. These other points therefore represent slightly different amounts of

inflation, and we note that their associated trajectories do not end exactly at the point $\varepsilon = 1$. In our numerical results we take $\Delta\chi_I \sim 10^{-5}$. Using the N derivatives calculated from (2.130) and evaluating the amplitude given by (2.107), we compute f_{NL} in (3.6). We obtain the momentum independent contribution $f_{\text{NL}}^{(4)}$ in (2.105) to be very small $\mathcal{O}(10^{-3})$. In Fig. 2.7 we plot the total f_{NL} versus N for squeezed ($k_2 \ll k_1 = k_3$), equilateral ($k_1 = k_2 = k_3$) and orthogonal ($k_1 = 2k_2 = 2k_3$) triangles.

It is convenient to express the reduced bispectrum in terms of the following independent variables [186, 187]

$$\alpha = \frac{k_2 - k_3}{k}, \quad \beta = \frac{k - k_1}{k} \quad \text{where} \quad k = \frac{k_1 + k_2 + k_3}{2}, \quad (2.131)$$

where $0 \leq \beta \leq 1$ and, $-(1 - \beta) \leq \alpha \leq (1 - \beta)$. In Fig. 2.8 we depict the shape of a slice through the reduced bispectrum $f_{\text{NL}}(k_1, k_2, k_3)$ at $N = 60$ using these variables. The bispectrum shape reveals details about the dominant interaction contributions [50]. In general, the presence of a signal in the squeezed limit represents the interaction of the long wavelength mode, which already exited the horizon, with the short wavelength modes still being within the horizon. This can happen in the case where more than one light scalar field drives the period of inflation. When, instead, we observe a peak in the equilateral limit, the dominant interaction between the fields occurs when the modes are exiting the horizon at the same time during inflation. This is taken to be the distinctive feature of models with a non-canonical kinetic term or models involving higher derivative interactions [51]. In the case of multiple non-canonical scalar field inflation (which is effectively happening in the two 3-form inflation scenario), it is possible that we would encounter a mixture of shapes [50, 51]. Although in the example we explored there is no significant signal in the squeezed limit.

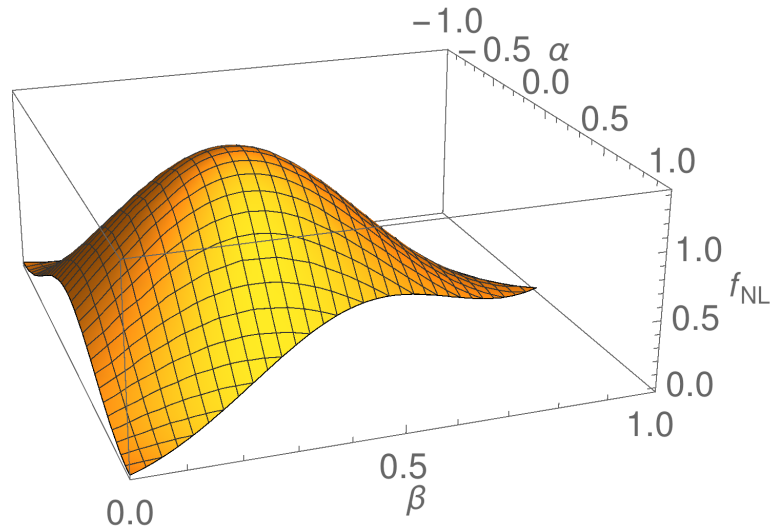


Figure 2.8: Graphical representation of the non-Gaussianity shape $f_{\text{NL}}(\alpha, \beta)$. We have considered the potentials $V_1 = V_{01}(\chi_1^2 + b_1\chi_1^4)$ and $V_2 = V_{20}(\chi_2^2 + b_2\chi_2^4)$ with $V_{01} = 1$, $V_{20} = 0.93$, $b_{1,2} = -0.35$ and taken the initial conditions $\chi_1(0) \approx 0.5763$, $\chi_2(0) \approx 0.5766$, $\chi_1'(0) = -0.000224$, $\chi_2'(0) = 0.00014$.

2.2.3 Summary

Let us summarize our specific results. Inflation driven by a multifield setting, in particular by a couple of 3-form fields, is very much still admissible within current *Planck* data. This is the main assertion that this chapter indicates. Moreover, two 3-form fields with a small asymmetry (in the sense explained in this chapter) produces better results (in terms of fitting within current observational data) for concrete cosmological parameters, in contrast to a symmetric configuration or to a single 3-form setting. This is interesting if we take into consideration, the correspondence (on dualization) between 3-form field and non-canonical (kinetic) scalar field dynamics. In fact, a dual description of two 3-forms assists to relate to k -inflationary models [188]. We have shown that having multiple 3-forms driven inflation brings the inflaton mass to a lower scale, when compared with a single 3-form. We then identified the existence of de Sitter like fixed points, where two 3-forms inflation can mimic single 3-form inflationary scenarios, for a suitable class of potentials. We also did a detailed numerical study of a different type of inflationary dynamics (type II) characterized by the dominance of a non trivial (gravity mediated) coupling, between the two 3-form fields. The type II solution stands physically interesting by its ability to generate substantial isocurvature perturbations at the end of inflation. We have numerically computed the effect of these perturbations via transfer functions. The comparison of selected inflationary parameters against the observational data, in the case where the 3-form fields potential have the form $\chi_I^2 + b_I \chi_I^4$, show that type II solutions, predicting a small variation in the speed of sound, are in excellent agreement with the observational bounds of running spectral indexes.

We presented a generic framework to compute primordial non-Gaussianity in the case of multiple 3-form field inflation. We followed the δN formalism which is a well-known method to study the evolution of curvature perturbations on superhorizon scales in the case of multiple scalar fields. Because of the fact that the 3-form fields are dual to non-canonical scalar fields, which was shown in [130], we developed an indirect methodology to implement δN formalism to 3-form fields. For a specific case of two 3-form fields, we derived a relation between the derivatives of N with respect to unperturbed values of scalar field duals at horizon exit $c_s k = aH$ and the N derivatives with respect to 3-form fields. We employed a numerical finite difference approach for this purpose. We computed the bispectrum at horizon exit for the two 3-form field case using known expressions for 3-point field space correlations for a general multiscalar field model. Then using the N derivatives we determined the complete superhorizon evolution of f_{NL} for squeezed, equilateral and orthogonal configurations until the end of inflation. Considering the potentials $\chi_I^2 + b_I \chi_I^4$ and specific values of model parameters that were consistent with $n_s \sim 0.967$ and $r \sim 0.0422$, we obtained the corresponding f_{NL} predictions for the two 3-form inflationary model as $f_{\text{NL}}^{\text{sq}} \sim -2.6 \times 10^{-3}$, $f_{\text{NL}}^{\text{equi}} \sim 1.409$, $f_{\text{NL}}^{\text{ortho}} \sim 0.495$. Therefore, the model is well within the observational bounds of *Planck* 2015 data and, most important to emphasize, it can be tested with the future probes [29–31].

3

DBI Galileon inflation

If I take the theory as we have it now, literally, I would conclude that extra dimensions really exist. They're part of nature. We don't really know how big they are yet, but we hope to explore that in various ways.

- Edward Witten

In this chapter we explore an observationally consistent inflationary scenario that involves a D-brane setting with an additional effect of induced gravity. In Refs. [189-191], it was observed that the motion of a D-brane in warped space generally causes an effect of induced gravity. This resultant action of D-brane with induced gravity effect comes under a class of generalized Galileon model [49]. Therefore, this new setting is named as DBI Galileon (DBG) model. The studies so far in literature [192-199], are mainly focused to explore the parameter space of the single-field and multifield DBG model with respect to the various types of non-Gaussianities. Furthermore, in Ref. [196] single field DBG inflation is studied in the background of SUGRA under the assumption of a Coleman-Weinberg type of potential. In this chapter, we propose to study single-field DBG inflation without any particular choice of potential. More precisely, our objective is to constrain the parameter space of the DBG model with respect to the inflationary observables of primordial power spectrum in accordance with latest *Planck* 2015 data. We mainly focus our attention in two inflationary regimes. Namely, those with and without a constant warp factor. We aim to identify crucial differences between these two scenarios with respect to the corresponding inflationary predictions. In addition, in each case, we analyze the deviation from the standard slow-roll consistency relation $r = -8n_t$ due to the effect of induced gravity on the D-brane.

The organization of this chapter is as follows. In Sec. 3.1 we briefly describe the model and present the background equations for the DBG inflation with non-trivial warping [195]. In the case of constant sound speed and warp factor, we obtain the exact background solutions. In Sec. 3.2 we study the parameter space of the DBG model by comparing its predictions in different limits with CMB data. In Sec. 3.3 we present general background solutions using two different ansatz to integrate analytically the equations of motion. A detailed computation of the approximate solutions can be found in Appendix C. Finally, we present our conclusions in Sec. 3.4.

3.1 DBI-Galileon inflationary model

We begin by reviewing the DBG inflationary scenario following Ref. [195]. Such a setup considers a D3-brane with tension T_3 evolving in a ten dimensional geometry described by the metric,

$$ds^2 = h^{-1/2} (y^K) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2} (y^K) G_{IJ} (y^K) dy^I dy^J \equiv H_{AB} dY^A dY^B, \quad (3.1)$$

with coordinates $Y^A = \{x^\mu, y^I\}$, where $\mu = 0, \dots, 3$ and $I = 1, \dots, 6$. The induced metric on the D3-brane is given by

$$\gamma_{\mu\nu} = H_{AB} \partial_\mu Y_{(b)}^A \partial_\nu Y_{(b)}^B, \quad (3.2)$$

where the brane is embedded in higher dimensions by means of the functions $Y_{(b)}^A(x^\mu)$, with the x^μ being the space time coordinates on the brane. In brane inflation, the role of the inflaton is played by the radial coordinate (ρ) of the brane that is moving in the extra dimensions. Since we are only considering single-field inflation in this chapter, we choose the brane embedding as $Y_{(b)}^A(x^\mu) = (x^\mu, \varphi(x^\mu))$. Then, the induced metric can be written as

$$\gamma_{\mu\nu} = f^{-1/2} (g_{\mu\nu} + f \partial_\mu \varphi \partial_\nu \varphi), \quad (3.3)$$

where f and φ are the warp factor and the scalar field defined by

$$f = \frac{h}{T_3}, \quad \varphi = \sqrt{T_3} \rho. \quad (3.4)$$

The D3-brane here is embedded in 5D geometry with the induced metric (3.3). This introduces an additional contribution in the action known as Galileon term [200]. The total action is then given by

$$S = \int d^4x \left[\frac{m_P^2}{2} \sqrt{-g} R[g] + \frac{\tilde{m}^2}{2} \sqrt{-\gamma} R[\gamma] + \sqrt{-g} \mathcal{L}_{brane} \right], \quad (3.5)$$

where \tilde{m} is a parameter associated with the induced gravity¹ and

$$\mathcal{L}_{brane} = -\frac{1}{f(\varphi)} (\sqrt{\mathcal{D}} - 1) - V(\varphi), \quad (3.6)$$

where

$$\mathcal{D} \equiv \det(\delta_\nu^\mu + f \partial_\mu \varphi \partial_\nu \varphi).$$

DBIG action belongs to the particular class of generalized G-inflation A.10 [195] with the functions \mathcal{F}_s and \mathcal{G}_s that determine the second order action for scalar perturbations are

$$\begin{aligned} \mathcal{F}_s(c_{\mathcal{D}}, \varepsilon_{\mathcal{D}}, \varepsilon) &= m_P^2 (\varepsilon \mathcal{K} (3\mathcal{K} - 2) + \mathcal{K} - 1) + \frac{\tilde{m}^2}{c_{\mathcal{D}}} \left[(\varepsilon + \varepsilon_{\mathcal{D}}) \mathcal{K} \left(\frac{3\mathcal{K}}{c_{\mathcal{D}}^2} - 2 \right) + \mathcal{K} - c_{\mathcal{D}}^2 \right], \\ \mathcal{G}_s(c_{\mathcal{D}}, \varepsilon_{\mathcal{D}}, \varepsilon) &= \frac{m_P^2}{c_{\mathcal{D}}^2} (\varepsilon \mathcal{K}^2 + 3c_{\mathcal{D}}^2 (1 - \mathcal{K}^2)) + \frac{\tilde{m}^2}{c_{\mathcal{D}}^3} \left[(\varepsilon + \varepsilon_{\mathcal{D}}) \mathcal{K}^2 + 3c_{\mathcal{D}}^2 \left(1 - \frac{\mathcal{K}^2}{c_{\mathcal{D}}^4} \right) \right], \end{aligned} \quad (3.7)$$

where $\mathcal{K} \equiv \frac{m_P^2 + c_{\mathcal{D}}^{-1} \tilde{m}^2}{m_P^2 + c_{\mathcal{D}}^{-3} \tilde{m}^2}$. And the functions corresponds to tensor perturbations are

$$\mathcal{F}_t(c_{\mathcal{D}}) \equiv m_P^2 + \tilde{m}^2 c_{\mathcal{D}} \quad , \quad \mathcal{G}_t(c_{\mathcal{D}}) \equiv m_P^2 + \frac{\tilde{m}^2}{c_{\mathcal{D}}}. \quad (3.8)$$

Assuming the flat FLRW metric and allowing the warp factor f to vary, the gravitational field

¹ \tilde{m} non trivially depends on the warping h , see [195]. In this chapter, \tilde{m} is treated as a model parameter.

equations for the action in (3.5) are [195]

$$3H^2 m_{\text{P}}^2 + 3\hat{H}^2 \frac{\tilde{m}^2}{c_{\mathcal{D}}^3} = \frac{1}{f} \left(\frac{1}{c_{\mathcal{D}}} - 1 \right) + V. \quad (3.9)$$

$$-m_{\text{P}}^2 \dot{H} + \frac{\tilde{m}^2 H^2}{c_{\mathcal{D}}} \left[-\frac{\dot{\hat{H}}}{H^2} - \frac{c_{\mathcal{D}}}{h^{1/4}} \left(\frac{h^{1/4}}{c_{\mathcal{D}}} \right)' \frac{\hat{H}}{H^2} + \frac{3}{2} \left(\frac{1}{c_{\mathcal{D}}^2} - 1 \right) \frac{\hat{H}^2}{H^2} \right] = \frac{\dot{\sigma}^2}{2c_{\mathcal{D}}}, \quad (3.10)$$

where $c_{\mathcal{D}}^2 \equiv 1 - f\dot{\sigma}^2$ is the squared sound speed², $\hat{H} \equiv H - \frac{f}{4f}$ and $\dot{\sigma}^2 \equiv G_{IJ}\dot{\phi}^I\dot{\phi}^J$. The appearance of (3.10) can be simplified to

$$\dot{H} - \lambda_1 H^2 + \lambda_2 = 0 \quad (3.11)$$

after introducing the functions

$$\lambda_1 \equiv \frac{\tilde{m}^2}{m_{\text{P}}^2 c_{\mathcal{D}} + \tilde{m}^2} \left[\frac{\varepsilon_f(\eta_f - \varepsilon)}{4} - \frac{d \ln \left(\frac{h^{1/4}}{c_{\mathcal{D}}} \right)}{d \ln a} \left(1 - \frac{\varepsilon_f}{4} \right) + \frac{3}{2} \left(\frac{1}{c_{\mathcal{D}}^2} - 1 \right) \left(1 - \frac{\varepsilon_f}{4} \right)^2 \right], \quad (3.12)$$

$$\lambda_2 \equiv \frac{1 - c_{\mathcal{D}}^2}{2f(m_{\text{P}}^2 c_{\mathcal{D}} + \tilde{m}^2)}, \quad (3.13)$$

which depend on \tilde{m} and $c_{\mathcal{D}}$. We also introduce the slow-roll parameters

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv \frac{d \ln \varepsilon}{d \ln a}, \quad \varepsilon_{\mathcal{D}} \equiv \frac{d \ln c_{\mathcal{D}}}{d \ln a}, \quad \eta_{\mathcal{D}} \equiv \frac{d \ln \varepsilon_{\mathcal{D}}}{d \ln a}, \quad \varepsilon_f \equiv \frac{d \ln f}{d \ln a}, \quad \eta_f \equiv \frac{d \ln \varepsilon_f}{d \ln a} \quad (3.14)$$

to describe the evolution of the background geometry, the sound speed and the warp factor. Note also that in the above we take the brane tension T_3 to be a constant, as is usually considered.

In the following we obtain solutions to the background equations for the cases when $\lambda_{1,2}$ are constants.

3.1.1 Constant sound speed and warp factor

Whenever the sound speed ($c_{\mathcal{D}} \leq 1$) and the warp factor is constant, i.e., $\varepsilon_{\mathcal{D}} = \varepsilon_f = 0$, the coefficients $\lambda_{1,2}$ in (3.11) are constants. Integrating (3.11) in that case is straightforward. We obtain

$$H^2 = \frac{\lambda_2}{\lambda_1} + \kappa a^{2\lambda_1}, \quad (3.15)$$

where $\kappa \neq 0$ is an arbitrary, dimensionful constant. Writing $H = \dot{a}/a$, the solution to (3.15) is

$$a^{2\lambda_1}(t) = \left(\frac{\lambda_2}{\lambda_1 |\kappa|} \right) \exp[i(1 + \sigma_1)\pi/2] \operatorname{sech}^2 \left[\sqrt{\lambda_1 \lambda_2 \sigma_2} (t - \bar{t}) - i(1 + \sigma_1)\pi/4 \right], \quad (3.16)$$

²Note that the sound speed $c_{\mathcal{D}}$ here depends not only on the brane dynamics, (as in DBI models [112, 113, 115]) but also on the induced gravity [195].

where we introduce

$$\sigma_1 \equiv \text{sign}(\kappa) = \text{sign}(\dot{H}) \quad , \quad \sigma_2 \equiv \text{sign}(\dot{a}) . \quad (3.17)$$

The explicit time-dependence of the Hubble parameter can be obtained from (3.16)

$$H(t) = - \left(\frac{\lambda_2}{\lambda_1} \right)^{1/2} \sigma_2 \tanh \left[\sqrt{\lambda_1 \lambda_2} \sigma_2 (t - \bar{t}) - i(1 + \sigma_1) \pi/4 \right] . \quad (3.18)$$

To study inflation we need to set $\sigma_2 = \text{sign}(\dot{a}) = +1$, regardless of $\sigma_1 = \text{sign}(\dot{H})$. An increasing expansion rate is obtained for $\sigma_1 = +1$ ($\lambda_2 < \lambda_1 H^2$), which corresponds to the singular behaviour of the scale factor and the Hubble parameter at $t \rightarrow \bar{t}$ (purple line) displayed in Fig. 3.1. A decreasing expansion rate corresponds to $\sigma_1 = -1$ ($\lambda_2 > \lambda_1 H^2$), in which case both $a(t)$ and $H(t)$ remain finite throughout the entire evolution (blue line). In the context of inflation, we focus only on the decreasing expansion rate $\sigma_1 = -1$, for which we find a non-singular behaviour for the scale factor $a(t)$.

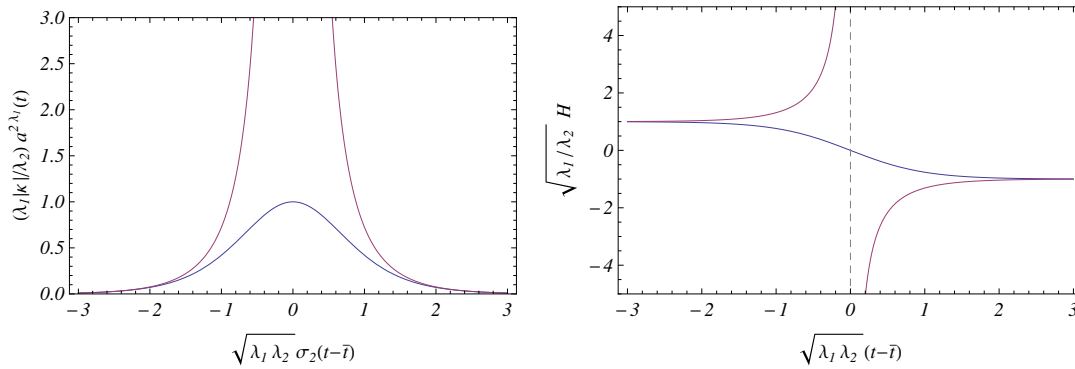


Figure 3.1: Evolution of the scale factor according to (3.16) (left panel) and the Hubble parameter H , according to (3.18) (right panel).

In Sec. 3.2.1 we impose the necessary conditions to obtain an inflationary expansion in agreement with current observations. To do so, in the next section we investigate the scalar and tensor perturbation spectra, which depend on the slow-roll parameters ε and η . Using (3.14) and (3.18) we obtain

$$\varepsilon(t) = \lambda_1 \text{csch}^2 \left[\sqrt{\lambda_1 \lambda_2} \sigma_2 (t - \bar{t}) - i(1 + \sigma_1) \pi/4 \right] , \quad (3.19)$$

$$\eta(t) = 2 \lambda_1 \coth^2 \left[\sqrt{\lambda_1 \lambda_2} \sigma_2 (t - \bar{t}) - i(1 + \sigma_1) \pi/4 \right] , \quad (3.20)$$

from which we arrive at the relations

$$\eta = 2(\varepsilon + \lambda_1) \quad , \quad H^2 = \lambda_2 (\lambda_1 + \varepsilon)^{-1} , \quad (3.21)$$

where we emphasize that the slow-roll parameter η explicitly depends on λ_1 . During inflation, $\eta \ll 1$ implies $\lambda_1 \ll 1$. Therefore, several constraints (to be discussed later on) must be imposed on the model parameters to have $\lambda_1 \ll 1$.

3.2 Comparison to observations

In this section we study in detail the observational predictions of DBIG inflation and examine the status of the tensor consistency relation. We compute n_s , r and n_t by plugging (3.7), (3.8) in the general expressions presented in Appendix. A. We study the different limits of DBIG inflation and evaluate the effect of higher order corrections in slow-roll parameters on the model predictions. We explore the parameter space (c_D, \tilde{m}, f) of DBIG inflation using the *Planck* constraints on (n_s, r) and the observed amplitude of the power spectrum $\mathcal{P}_{\zeta_*} \simeq 2.2 \times 10^{-9}$ at the pivot scale $k_* = 0.002 \text{ Mpc}^{-1}$ [24]. In all cases, we find that the predictions of (n_s, r) do not explicitly depend on the warp factor. Therefore, we first find the range of model parameters (c_D, \tilde{m}) compatible with the observed values of $n_s = 0.968 \pm 0.006$ and $r < 0.1$ at the 95% CL [24]. After that, we calculate the tensor tilt (n_t) for the same parameter space that was previously constrained. We expect to find departures from the consistency relation of single-field inflation, $r = -8n_t$. Finally, we compare our results with the BKP+LIGO constraints on the tensor tilt $n_t = -0.76^{+1.37}_{-0.52}$ at the 68% CL [24, 201].

3.2.1 Constant sound speed and warp factor

Let us examine the parameter space of DBIG inflation with $\varepsilon_D = \varepsilon_f = 0$ in different limits. For this we use the solutions derived in Sec. 3.1.1. We focus only on the decreasing expansion rate $\sigma_1 = -1$, for which we find a non-singular behaviour for the scale factor $a(t)$.

Firstly, the number of e -foldings during inflation can be computed as

$$N = \int_{t_*}^{t_e} H dt, \quad (3.22)$$

where t_* is the time when cosmological scales exit the horizon and t_e signals the end of inflation, set through the condition $\varepsilon(t_e) = 1$. According to observations, the length of the inflationary phase required to solve the flatness and horizon problems is around $N = 50$ to $N = 60$. Using (3.18) and the condition $\varepsilon = 1$ to determine t_e , we integrate (3.22) to obtain

$$N = \frac{1}{\lambda_1} \ln \frac{\cosh [\sqrt{\lambda_1 \lambda_2} \sigma_2 (t_* - \bar{t}) - i(1 + \sigma_1)\pi/4]}{\sqrt{1 + \lambda_1}}, \quad (3.23)$$

which we can relate to the slow-roll parameters ε and $\eta = 2(\varepsilon + \lambda_1)$ at the time of horizon crossing

$$\varepsilon_* = \frac{\lambda_1}{(1 + \lambda_1) \exp[2\lambda_1] - 1}.$$

Using (3.9) and (3.21), we find the scalar potential V in terms of the model parameters

$$V = \frac{3\lambda_2}{\lambda_1 + \varepsilon} \left[m_P^2 + \frac{\tilde{m}^2}{c_D^3} \right] - \frac{1}{f} \left(\frac{1}{c_D} - 1 \right), \quad (3.24)$$

which allows us to find the energy scale of inflation $V_*^{1/4}$ after evaluating at the time of horizon crossing for cosmological scales. Also, we obtain the mass squared of the inflaton

$$m_\phi^2 = V_{,\phi\phi} = \frac{\ddot{V}}{\dot{\phi}^2}, \quad (3.25)$$

where

$$\dot{\phi}^2 = \frac{1 - c_{\mathcal{D}}^2}{f}. \quad (3.26)$$

3.2.1.1 DBI limit: $\tilde{m} \rightarrow 0$

The phenomenology of DBI inflation has been done in recent literature [202, 203] assuming a particular form of potential. We emphasize here, however, that in our study we do not assume any form of the potential.

In this limit $\lambda_1 \rightarrow 0$ (see (3.12)), and we obtain the corresponding background solution from the one obtained in Sec. 3.1.1 as the zeroth order in a series expansion around $\lambda_1 = 0$. Operating similarly for the number of e -foldings in (3.23) we easily obtain

$$\lambda_2 \rightarrow \frac{1 - c_{\mathcal{D}}^2}{2f m_{\text{Pl}}^2 c_{\mathcal{D}}}, \quad H^2 \rightarrow \frac{\lambda_2}{\varepsilon}, \quad \varepsilon \rightarrow \frac{1}{1 + 2N}, \quad \eta \rightarrow 2\varepsilon, \quad \mathcal{P}_{\zeta} \rightarrow \frac{H^2}{8\pi^2 \varepsilon c_{\mathcal{D}}}. \quad (3.27)$$

Fixing the number of e -foldings and the amplitude of the perturbation spectrum we constrain the warp factor f . Since we treat $c_{\mathcal{D}}$ as a model parameter, we obtain its range from the prediction for non-Gaussianity $f_{\text{NL}}^{\text{eq}} = -\frac{35}{108} \left(\frac{1}{c_{\mathcal{D}}^2} - 1 \right)$ in DBI models [110, 111]. Although more accurate expressions exist in the literature [192-195, 197, 198, 204], for our purposes it suffices to consider this simple estimate. This is appropriate since in the absence of a clear detection of non-Gaussianity [25], the use of more elaborate or complicated expressions is, in principle, uncalled for. Therefore, in this chapter we will not be concerned with non-Gaussian computations and will use the above expression to constrain the sound speed $c_{\mathcal{D}}$. The analysis of the *Planck* data on $r < 0.1$ and $f_{\text{NL}}^{\text{equi}} = -4 \pm 43$ allows to set a conservative bound for this $0.087 \leq c_{\mathcal{D}} \leq 0.6$ [24, 25]. Note that larger values of $c_{\mathcal{D}}$, albeit allowed by the bound from non-Gaussianity, are disfavoured as they result in a tensor-to-scalar ratio in excess of the current bound $r < 0.1$. Fig. 3.2 represents the viability of the DBI model. Because of the stringent bound on $f_{\text{NL}}^{\text{equi}}$ the DBI inflation is not capable to induce $r < 0.01$ which is consistent with previous studies [115, 205]. The range of model parameters obtained for $0.087 \leq c_{\mathcal{D}} \leq 0.6$ can be found in Table 3.1. In Fig. 3.2 we depict our results in the DBI limit.

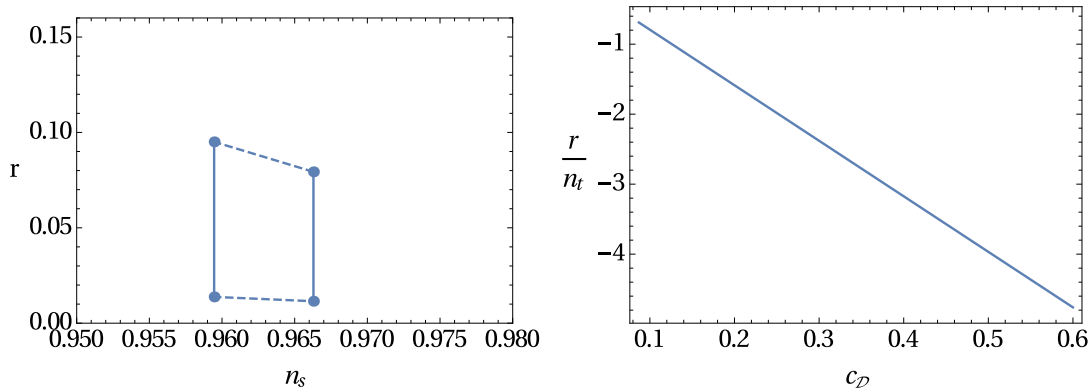


Figure 3.2: In the left panel we depict tensor-to-scalar ratio vs. spectral index where in the plot N varies from 50 to 60 (from left to right) and $c_{\mathcal{D}}$ varies from 0.087 to 0.6 (from bottom to top). In the right panel we plot the ratio r/n_t vs. sound speed $c_{\mathcal{D}}$ for $N = 60$.

3.2.1.2 Galileon limit: $\tilde{m} \gg m_P$

Although studying this limit is not generic with respect to the structure of DBIG, this would nevertheless be useful to understand the role of induced gravity. Since $c_D \lesssim 1$, (3.11) gives

$$\lambda_1 = \frac{3}{2} \left(\frac{1}{c_D^2} - 1 \right) \quad , \quad \lambda_2 \equiv \frac{1 - c_D^2}{2c_D f \tilde{m}^2} . \quad (3.28)$$

The slow-roll parameters in this case which are given below

$$\varepsilon = \frac{3(1 - c_D^2)}{(3 - c_D^2) e^{\frac{3}{2} \left(\frac{1}{c_D^2} - 1 \right) N} - 2c_D^2} \quad , \quad \eta = 3 \left(\frac{1}{c_D^2} - 1 \right) + 2\varepsilon . \quad (3.29)$$

Unlike in the DBI limit (cf. (3.27)), in the Galileon limit, the slow-roll parameters explicitly depend on the sound speed. It is obvious from (3.29) that $c_D \ll 1$ would actually spoil the smallness of η . Therefore, in this case we need to keep the sound speed in the narrow range $0.995 \leq c_D < 1$ for the results to agree with the current *Planck* data. Any value of $c_D < 0.995$ would essentially spoil the prediction of the spectral index and its value would be significantly out of the current bounds $n_s = 0.968 \pm 0.006$. Therefore, observationally viable inflation due to the induced gravity term sets $c_D \lesssim 1$, thus resulting in small non-Gaussianities. This allows to discriminate between the current case and the DBI limit previously studied. Also, the consistency of the predictions with data becomes better as the number of e -foldings reduces. In particular, for $N \sim 50$ our results are perfectly consistent with current data whereas for $N \gtrsim 60$ the model is ruled out. Our results in this case are depicted in Fig. 3.3. The derived model parameters for $0.995 \leq c_D \leq 1$ can be found in Table 3.1.

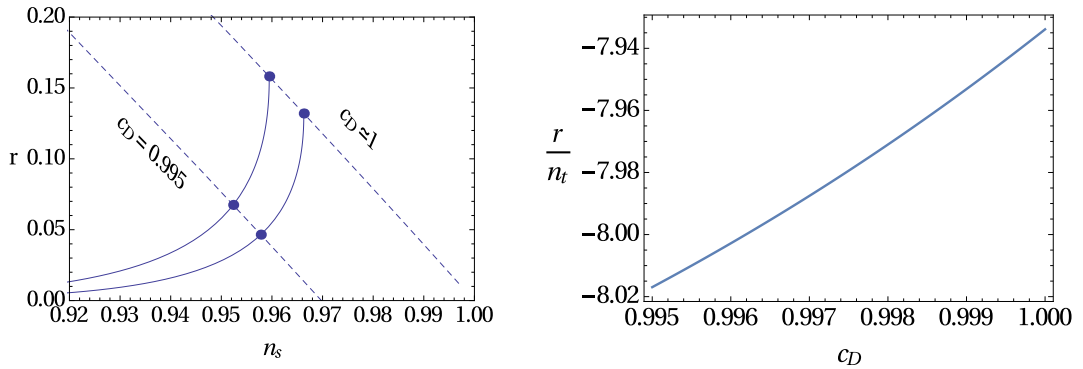


Figure 3.3: Plots of spectral index n_s vs. tensor-to-scalar ratio r (left) and the ratio r/n_t vs. sound speed c_D (right) in the Galileon limit. In the left panel, we take N varying from 50 to 60 (from bottom to top). For the right panel we considered $N = 60$.

3.2.1.3 DBI-Galileon case

In this section we consider Einstein and Galileon gravity are on an equal footing. In this case

$$\lambda_1 \equiv \frac{\tilde{m}^2}{m_P^2 c_D + \tilde{m}^2} \left[\frac{3}{2} \left(\frac{1}{c_D^2} - 1 \right) \right] \quad , \quad \lambda_2 \equiv \frac{1 - c_D^2}{2f(m_P^2 c_D + \tilde{m}^2)} .$$

The corresponding slow-roll parameters are (expressing in the units of $m_P = 1$)

$$\varepsilon = \frac{3(1 - c_D^2) \tilde{m}^2}{[2c_D^3 - (c_D^2 - 3) \tilde{m}^2] e^{\frac{3\left(\frac{1}{c_D^2} - 1\right) \tilde{m}^2 N}{c_D + \tilde{m}^2}} - 2c_D^2(c_D + \tilde{m}^2)}, \quad \eta = \frac{3\left(\frac{1}{c_D^2} - 1\right) \tilde{m}^2}{(c_D + \tilde{m}^2)} + 2\varepsilon. \quad (3.30)$$

Similarly to the Galileon limit studied in Sec. 3.2.1.2, the sound speed needs to be tuned to $c_D \simeq 0.98 - 0.99$ to keep the slow-roll parameter η small enough to have $n_s = 0.968 \pm 0.006$. We find that $c_D < 0.98$ would essentially spoil the prediction of scalar tilt. We also note here that if $c_D = 1$ we obtain exact scale invariance, i.e. $n_s = 1$. Since the slow-roll parameter ε in (3.30) depends on the parameter \tilde{m} , the tensor-to-scalar ratio varies for different values of the induced gravity parameter \tilde{m} . This allows us to identify the range of the parameters consistent with current data. In Fig. 3.4 we study the parameter space (c_D, \tilde{m}) using the bounds on (n_s, r) . The plot shows that, in the limit $\tilde{m} \rightarrow 0$, the model reduces to DBI case. Moreover, unless $\tilde{m} < m_P$, the effect of the induced gravity forces us to constrain the sound speed to $c_D \sim 1$ in order to maintain the agreement with observations.

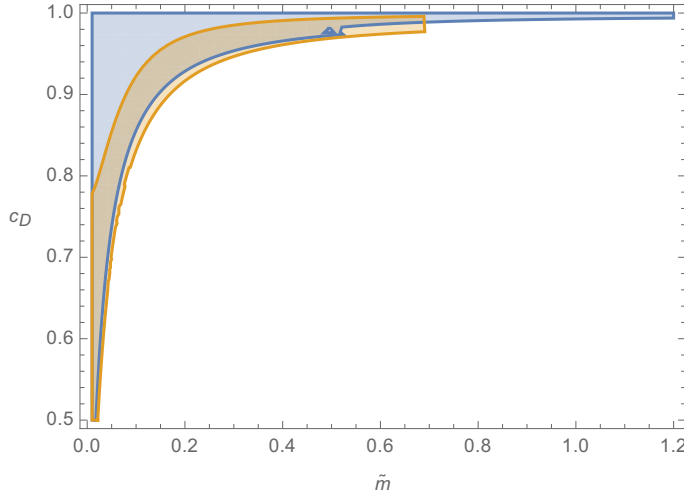


Figure 3.4: Contour plots in the plane (\tilde{m}, c_D) (with \tilde{m} in units of m_P). Blue and orange regions represent the space where $n_s = 0.968 \pm 0.006$ and $0.01 \leq r \leq 0.1$, respectively.

To constrain the model parameters (c_D, \tilde{m}) with the bounds of (n_s, r) it is also necessary to check if non-Gaussianities are large. Since the full study of non-Gaussianity is beyond the scope of this chapter, we use the results in Ref. [195], where the authors study non-Gaussianity in the multifield DBIG inflation model. We adopt their expression for $f_{\text{NL}}^{\text{equi}}$ in the single-field limit, i.e. taking the adiabatic and isocurvature mode transfer function $T_{\sigma s} \rightarrow 0$. We thus constrain our parameter space using the approximate expression [195]

$$f_{\text{NL}}^{\text{equi}} = -\frac{5}{324c_D^2} \frac{21 - 404\alpha + 2233\alpha^2 - 3066\alpha^3}{(1 - 5\alpha)^2(1 - 9\alpha)}, \quad \alpha \equiv \frac{fH^2\tilde{m}^2}{c_D^2}. \quad (3.31)$$

Setting $N = 60$, in Fig. 3.5 we plot the model predictions in the plane (n_s, r) (left panel) for different values of c_D and for different ranges of \tilde{m} , as indicated. In the plotted curves, the tensor-to-scalar ratio decreases as we increase \tilde{m} . Therefore, our results show that an increase of the induced gravity lowers the tensor-to-scalar ratio. In the right panel we plot the ratio r/n_t as a function of \tilde{m} . In the range of values of c_D consistent with the observed value of the

spectral index we find a slight deviation from the standard consistency relation. Nevertheless, such a deviation does not seem to be sufficiently significant to be detected with confidence.

In Fig. 3.6 we plot the mass squared of the inflaton, as obtained from (3.25) evaluated at the time of horizon crossing for cosmological scales (left panel), and $f_{\text{NL}}^{\text{equi}}$ calculated from (3.31) (right panel). From the left plot, we find that the inflaton is tachyonic, whereas for smaller values of \tilde{m} , we recover a potential with positive curvature, in agreement with the DBI case. In this sense, it may be worth mentioning that the authors in Ref. [206] have studied the possibility that the Born-Infeld tachyon be equivalent to a scalar field in an effective field theory in different warped geometries. Moreover, in Ref. [203] the observational constraints on tachyon and DBI inflation were studied, and the authors showed that tachyon inflation fits better with cosmological data than DBI. It is also important to notice that $n_t < 0$ in all cases, which is statistically preferred by data after the *Planck* and BKP joint analysis [24, 26]. Also, the joint analysis of BKP+LIGO indicates a red tensor tilt $n_t = -0.76^{+1.37}_{-0.52}$ at the 68% CL [201]. In Table 3.1 we report the values of the ratio r/n_t , which only results in a slight deviation from the standard consistency relation in most of the cases. We recall that future cosmology probes will be able to discriminate inflationary models by direct detection of primordial B-modes [29]. Finally, from the right panel of Fig. 3.6 we find that the non-Gaussianity parameter $f_{\text{NL}}^{\text{equi}}$ is consistent with the stringent bounds imposed by *Planck* data [25].

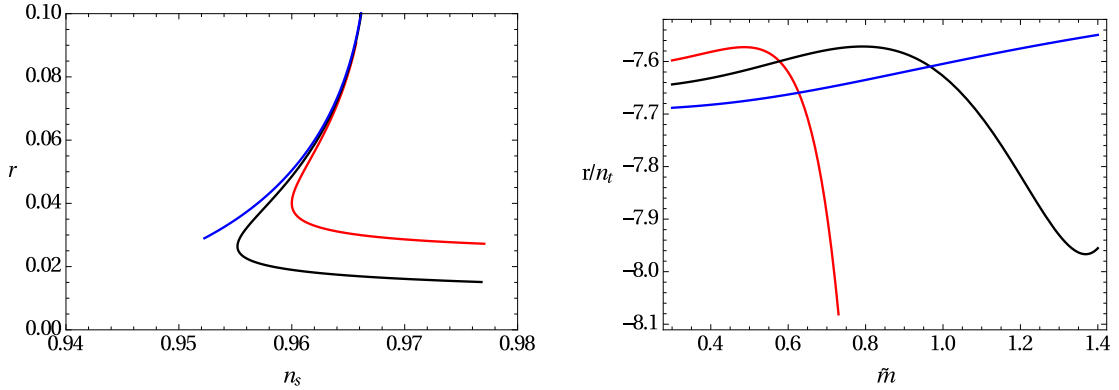


Figure 3.5: Plots of spectral index n_s vs. tensor-to-scalar ratio r (left panel) and the ratio r/n_t vs. \tilde{m} (with \tilde{m} in units of m_P) (right panel) in the DBI model. In the left panel we take $c_D = 0.98$ and $0.3 \leq \tilde{m}/m_P \leq 0.72$ (red), $c_D = 0.985$ and $0.5 \leq \tilde{m}/m_P \leq 1.25$ (black), $c_D = 0.99$ and $0.5 \leq \tilde{m}/m_P \leq 1.25$ (blue). In the plotted curves \tilde{m} increases as r decreases. In the right panel, the plotted curves correspond to $c_D = 0.98$ (red), $c_D = 0.985$ (black) and $c_D = 0.99$ (blue).

3.2.2 Varying both sound speed and warp factor

The cases considered in Sec. 3.2.1 (constant sound speed and constant warp factor) are consistent with observational data. However, it is interesting to understand the cases with varying c_D and f . The questions we can pose in these cases are, can we get a parameter space with $r \sim \mathcal{O}(10^{-3})$? How do the warped geometries and the scale of inflation change when (c_D, f) change with time? What is the nature of inflaton field in such cases? In this section, we obtain exact background solutions in two cases: a slowly varying sound speed at fixed warp factor and a slowly varying warp factor at fixed sound speed.

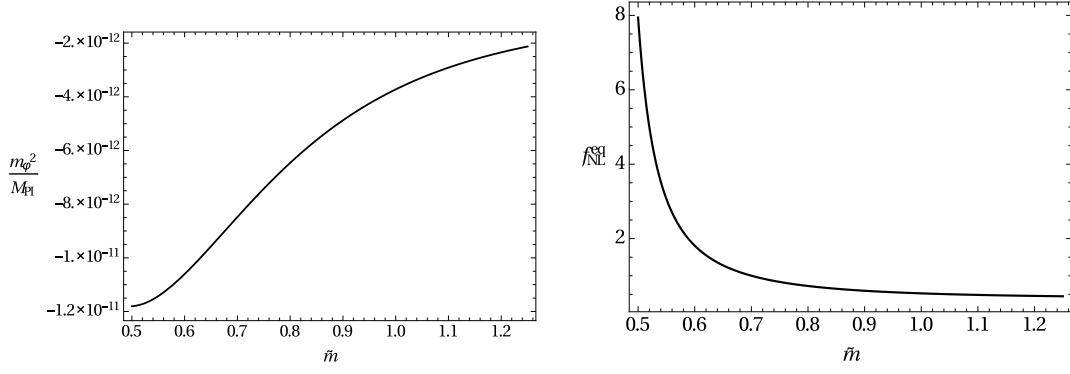


Figure 3.6: Plots of the mass squared of the inflaton field (left panel) and the non-Gaussian parameter $f_{\text{NL}}^{\text{eq}}$ (right panel) as a function of \tilde{m} (with \tilde{m} in units of m_P). In this plot $0.22 \leq \alpha \leq 0.32$ for $0.5 \leq \tilde{m} \leq 1.25$. We take $c_{\mathcal{D}} = 0.985$ to build the plots, hence the depicted behaviour corresponds to the black line in Fig. 3.5.

3.2.2.1 Varying sound speed ($\varepsilon_{\mathcal{D}} \neq 0, \eta_{\mathcal{D}} = 0$) and constant warp factor ($\varepsilon_f = 0$)

We assume a slow variation of the sound speed, i.e. $\varepsilon_{\mathcal{D}} \ll 1$. Using the definition of slow-roll parameters from (3.14), we can approximate $c_{\mathcal{D}}$ in terms of $N = \ln a$ as

$$c_{\mathcal{D}} = c_d \exp(\varepsilon_{\mathcal{D}} N) \simeq c_d (1 + \varepsilon_{\mathcal{D}} N), \quad (3.32)$$

where c_d is a constant whose magnitude is set some four e -foldings after the largest cosmological scales exit the horizon.

To integrate the background (3.11) it is now convenient to rewrite it as

$$H' - \lambda_1 H + \frac{\lambda_2}{H} = 0, \quad (3.33)$$

where $\lambda_{1,2}$ are computed using the approximation in (3.32) and the prime stands for $' \equiv \frac{d}{dN}$. Integrating (3.33) we obtain the solution $H = H(N)$. To fix the integration constant in the solution it suffices to impose that $\varepsilon \equiv -\frac{H'}{H} = 1$ at the end of inflation. We choose not to include here the solution $H = H(N)$ as it is a complicated expression involving imaginary error functions [207]. To constrain the model parameters we proceed as in Sec. 3.2.1. Since in this case (n_s, r) do not depend on warp factor f , we may find the range for $(c_d, \tilde{m}, \varepsilon_{\mathcal{D}})$ using the current bounds on (n_s, r) . Since we assume a slowly varying sound speed, its constraint in this case is not significantly different from the one obtained in Sec. 3.2.1.3. Consequently, we must tune $c_d \simeq 0.98$ so that the spectral index agrees with observations. We also find that consistency with observations demands $\varepsilon_{\mathcal{D}} < 0$. This resembles the result of Ref. [156], where it was shown that DBI inflation with a decreasing sound speed results in an expanding universe, in contrast to the case of increasing sound speed. The observables in this case (n_s, r) are not very different from those obtained for a constant sound speed and warp factor in Sec. 3.2.1.3. In fact, after an extensive numerical study we find it difficult to obtain $r \sim \mathcal{O}(10^{-3})$ in this case. Therefore, from our analysis we conclude that DBIG inflation with a varying sound speed and constant warp factor does not bring any new features.

3.2.2.2 Varying warp factor ($\varepsilon_f \neq 0, \eta_f = 0$) and constant sound speed ($\varepsilon_{\mathcal{D}} = 0$)

In general, the warp factor can depend on fields not stabilised during inflation. Therefore, it is feasible to expect a time-dependent warp factor while cosmological scales are exiting the horizon. For example, in Ref. [208], various solutions for warped geometries were considered in the context of DBI inflation. In the following, we consider a slowly varying warp factor in the DBI-Galileon inflation model and constrain its variation using current data. Therefore, taking $\varepsilon_f \ll 1$ we approximate the warp factor as follows

$$f = f_0 \exp(\varepsilon_f N) \simeq f_0 (1 + \varepsilon_f N), \quad (3.34)$$

where f_0 is the initial value warp factor and ε_f is constant and treated as free parameter. Similarly to the previous case, we set the magnitude of f_0 four e -foldings after the largest cosmological scales exit the horizon.

It is important to remark that, in contrast to the previous case, where $\lambda_{1,2} = \lambda_{1,2}(N)$ and no simple analytical solution can be found for (3.33), using $\varepsilon_{\mathcal{D}} = 0$ and $\varepsilon_f = \text{const.}$ gives $\lambda_1 = \text{const.}$ and only $\lambda_2 = \lambda_2(N)$. In turn, this allows us to find a simple solution to (3.11) in terms of N

$$H^2 = \frac{F_1}{F_3^2} \exp\left(\frac{\tilde{m}^2 N (2c_{\mathcal{D}}^2(\varepsilon_f - 3) - 3\varepsilon_f + 6)}{2c_{\mathcal{D}}^2(c_{\mathcal{D}} m_P^2 + \tilde{m}^2)}\right) C_2 + \frac{F_2(N)}{f_0 F_3^2}, \quad (3.35)$$

where C_2 is an integration constant, determined by the condition $\varepsilon = 1$ at $N = 60$, and

$$F_1 = \tilde{m}^4 [2c_{\mathcal{D}}^2(\varepsilon_f - 3) - 3\varepsilon_f + 6]^2,$$

$$F_2(N) = 2c_{\mathcal{D}}^2 (c_{\mathcal{D}}^2 - 1) \{2c_{\mathcal{D}}^3 m_P^2 \varepsilon_f + 2\tilde{m}^2 c_{\mathcal{D}}^2 [N(\varepsilon_f - 3)\varepsilon_f + 3] - 3(\varepsilon_f - 2)(N\varepsilon_f - 1)\},$$

$$F_3 = \tilde{m}^2 [2c_{\mathcal{D}}^2(\varepsilon_f - 3) - 3\varepsilon_f + 6].$$

In the following we find the range of parameters $(c_{\mathcal{D}}, \tilde{m}, \varepsilon_f)$ using the CMB constraints on (n_s, r) . Firstly, since the sound speed is constant we obtain the same constraint as in Sec. 3.2.1.3, namely $c_{\mathcal{D}} \simeq 0.98$ to keep n_s within its observed range.

In Fig. 3.7 we depict the parameter space $(\tilde{m}, \varepsilon_f)$ consistent with observations of the spectral index and tensor-to-scalar ratio. Taking $c_{\mathcal{D}} = 0.98$ and enforcing $n_s = 0.968 \pm 0.006$, our plot shows that it is indeed feasible to obtain a tensor-to-scalar ratio as low as $r \simeq 6 \times 10^{-4}$. Nevertheless, the plot also evidences that this requires a considerable tuning between \tilde{m} and ε_f . We have checked that using the 2σ interval for the spectral index does not contribute to enlarge significantly the space where $r \sim 10^{-4}$. In the absence of the aforementioned tuning, expected values correspond to the range $10^{-3} \lesssim r \lesssim 3 \times 10^{-3}$. Moreover, we have checked as well that the space where $r \sim 10^{-4}$ becomes incompatible with the observed spectral index even for small deviations away from $c_{\mathcal{D}} = 0.98$. Consequently, finding $r \sim 10^{-4}$ requires the combined tuning of \tilde{m}, ε_f and $c_{\mathcal{D}}$. Nevertheless, it seems fair to say that, despite these tunings, the DBIG model of inflation represents an improvement, albeit a moderate one, with respect to the DBI model studied in Sec. 3.2.1.1.

In addition, we verify the equilateral non-Gaussianity by using the approximate expression for f_{NL}^{equi} in (3.31). Since we consider a tiny variation of the warp factor we can practically neglect its contribution to non-Gaussianity. From Fig. 3.8 we can conclude that the DBIG model with varying warp factor leads to non-Gaussianities within the current observational bounds. Consequently,

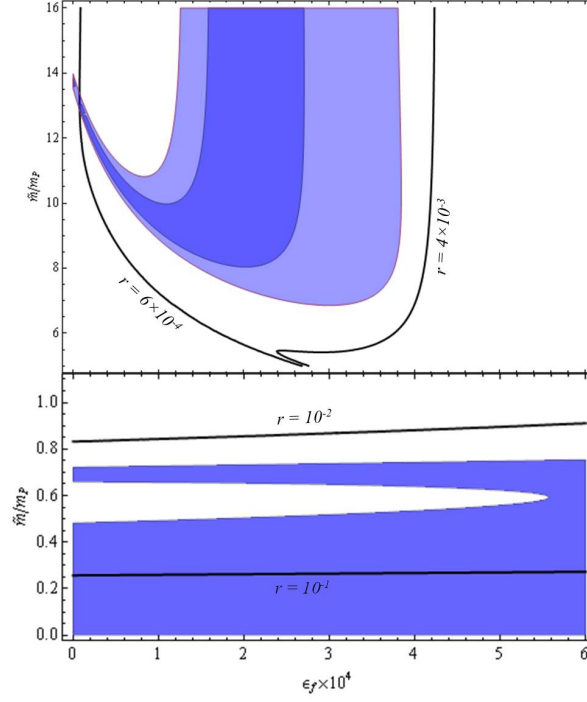


Figure 3.7: Contour plots in the plane (\tilde{m}, ϵ_f) . In the top panel, light and dark blue regions represent the 68% and 95% CL for the spectral index n_s , respectively. Black lines represent contours for different values of the tensor-to-scalar ratio, as indicated. In the bottom panel, the blue region depicts the 95% CL for the spectral index n_s . We use $c_D = 0.980$.

we conclude that after including a varying warp factor the DBIG model of inflation could be of crucial importance with respect to B-mode detection and non-Gaussianities in future CMB experiments [29].

We finish this section by depicting the predictions of DBIG inflation for different sets of values of the model parameters in Fig. 3.9 and by summarizing our results in Table 3.1. We recall that the values collected in the table were obtained taking by enforcing the scalar spectral index to lie within its observed range $n_s = 0.968 \pm 0.006$ at the 95% CL and taking $N = 60$.

Inflation	r	r/n_t	m_ϕ/m_P	$V_*^{1/4}/10^{16} \text{ GeV}$	f/m_P^4
DBI limit	(0.01, 0.1)	(-4.8, -0.7)	6.63×10^{-6}	(0.95, 1.82)	$\sim 10^{12} - 10^{14}$
Galileon limit	(0.13, 0.15)	(-8.1, -7.93)	$m_\phi^2 < 0$	(0.64, 0.70)	$\sim 10^9$
DBIG	(0.01, 0.1)	(-7.95, -7.5)	$m_\phi^2 < 0$	(1.7, 2.1)	$\sim 10^8 - 10^9$
(ii)Varying f	(0.0068, 0.0095)	(-7.95, -7.85)	$(2.41, 2.9) \times 10^{-7}$	(5.9, 6.4)	$(6, 9) \times 10^{10}$
	(0.0018, 0.0027)	(-8.01, -7.95)	$(3.6, 5.2) \times 10^{-8}$	(4.1, 4.6)	$(2.2, 3.4) \times 10^9$
	(0.0006, 0.0007)	(-7.63, -7.52)	$(1.52, 1.58) \times 10^{-8}$	(2.8, 2.85)	$(0.17, 0.18) \times 10^7$

Table 3.1: Inflationary observables in various limits of DBIG inflation.

3.3 On a class of background solutions

Until now, we have explored solutions to the background (3.9) and (3.10) in which the sound speed and warp factor are either constants or time-dependent functions with very slow varia-

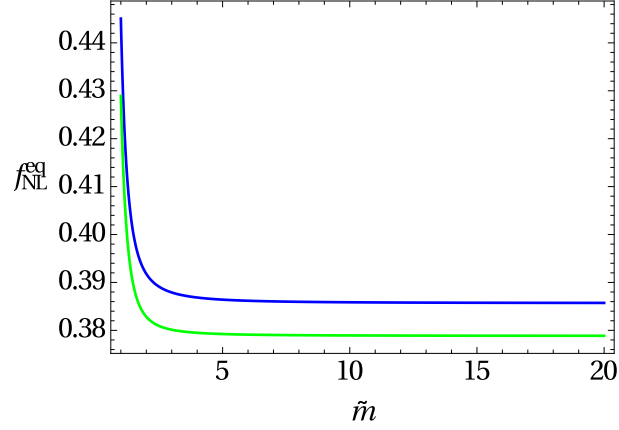


Figure 3.8: In this plot, we depict the non-Gaussian parameter $f_{\text{NL}}^{\text{equi}}$ as a function of \tilde{m} (with \tilde{m} in units of m_P). We take $c_D = 0.98$ and $\varepsilon_f \sim 10^{-4}$ (Blue line) and $\varepsilon_f \sim 10^{-6}$ (Green line). In this plot $0.326 \leq \alpha \leq 0.33$ for $1 \leq \tilde{m} \leq 20$.

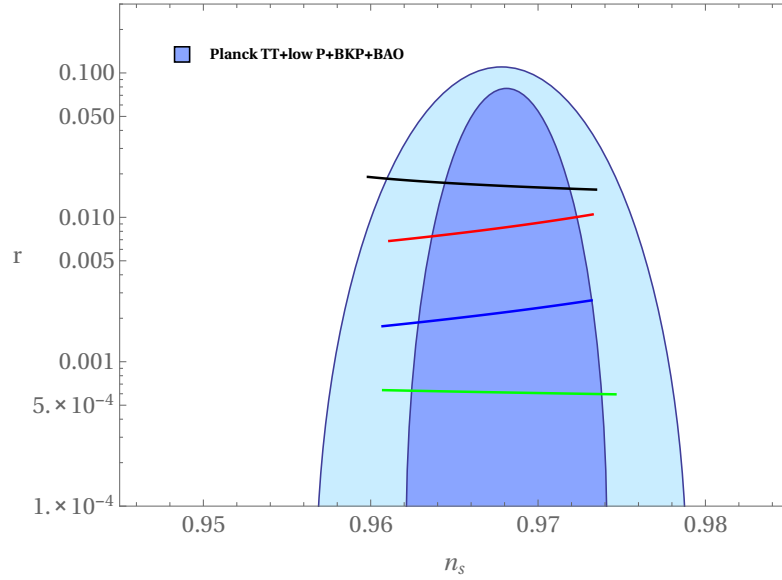


Figure 3.9: Predictions of the DBIG model for $N = 60$ along with the *Planck* TT+lowP+BKP+BAO constraints on the space (n_s, r) at the 68% and 95% CL. The black line represents the case with constant sound speed and warp factor ($c_D = 0.985$, $1 \leq \tilde{m}/m_P \leq 1.25$). Different model predictions for a constant sound speed and varying warp factor are plotted in red ($c_D = 0.985$, $\tilde{m} = 15m_P$ and $5.1 \leq 10^4 \varepsilon_f \leq 8.5$), blue ($c_D = 0.98$, $\tilde{m} = 15m_P$ and $1.5 \leq 10^4 \varepsilon_f \leq 2.6$) and green ($c_D = 0.98$, $\tilde{m} = 13m_P$ and $0.07 \leq 10^4 \varepsilon_f \leq 0.11$).

tion, although not simultaneously time-dependent. This choice is motivated by the simplicity of the perturbation spectrum imprinted in the CMB, which strongly favors the simplest inflationary models. Nevertheless, it is reasonable to conjecture, and to some extent expected, that in the early stages of inflation, when the observable cosmological scales are still deep within the horizon, the background dynamics has been much different from the simple slow-roll evolution supported by CMB observations. Therefore, it is interesting to investigate what kind of inflationary dynamics does the DBI-Galileon model give rise to when the sound speed and warp factor become time-dependent functions simultaneously. In general, however, it is not possible to integrate the equations of motion for general functions $c_{\mathcal{D}}(t)$ and $f(t)$. Owing to this difficulty, in order to find analytical solutions of the background equations we pursue a phenomenological approach in which we consider two different ansatz for the functions λ_1 and λ_2 .

If we allow the sound speed $c_{\mathcal{D}}$ and warp factor f to change ($\varepsilon_{\mathcal{D}}, \varepsilon_f \neq 0$) the coefficients $\lambda_{1,2}$ become time-dependent functions. In such case, (3.11) can be rewritten as

$$\frac{d \ln H}{\lambda_1 - \lambda_2 H^{-2}} = d \ln a. \quad (3.36)$$

In what follows, we discuss two different parameterizations for $\lambda_{1,2}$ to find approximate solutions for $a(t)$.

Parametrization 1

The simplest strategy to integrate (3.36) is to rewrite $\lambda_{1,2}$ as functions of H . Thus, we consider the temporal dependence for λ_i (with $i = 1, 2$) of the form

$$\lambda_i = \bar{\lambda}_i H^{\alpha_i}, \quad (3.37)$$

where $\bar{\lambda}_i, \alpha_i$ are constants. Using this ansatz, (3.36) can be integrated to give

$${}_2F_1 \left(1, 1 + \beta; 2 + \beta; \frac{\lambda_1 H^2}{\lambda_2} \right) H^2 = \lambda_2 (\alpha_2 - 2) \ln |\kappa a|, \quad \beta \equiv \frac{\alpha_1}{\alpha_2 - \alpha_1 - 2}, \quad (3.38)$$

where ${}_2F_1$ is the hypergeometric function and κ is an arbitrary constant. Note that in the limit $\alpha_{1,2} \rightarrow 0$ we can use the identity ${}_2F_1(1, 1; 2; z) z = -\ln |1 - z|$ to arrive at (3.15). Given the complexity of the above solution, substituting $H = \dot{a}/a$ to integrate the resulting differential equation in terms of $a(t)$ is of no practical use. Thus, it is necessary to resort to numerical methods to integrate it. Nevertheless, if $|\beta| < 1$ an approximation to the evolution equation is given by (see Appendix C for details)

$$\ln \left| 1 - \frac{\lambda_1 H^2}{\lambda_2} \right| \simeq \ln |\kappa a|^A \quad \text{with} \quad A \equiv \frac{(2 - \alpha_2)\lambda_1}{1 + \beta} \simeq (2 - \alpha_2)\lambda_1. \quad (3.39)$$

For $\alpha_2 \lesssim \mathcal{O}(1)$, the condition $|\beta| \ll 1$ implies $|\alpha_1| \ll 1$. Provided H does not change exponentially, which can be certainly applied to the regular solution plotted in Fig. 3.1, we can approximate λ_1 by a constant since $\lambda_1 \simeq \bar{\lambda}_1 (1 + \alpha_1 \ln(H/H_*) + \dots)$. This reasoning can be applied to the singular solution as well whenever it finds itself sufficiently away from the singularity at

$t = \bar{t}$. Using (3.11), we rewrite (3.39) as

$$H^{2+\alpha_1-\alpha_2} = \frac{\bar{\lambda}_2}{|\bar{\lambda}_1|} \text{sign}(\lambda_1) \left(1 + \text{sign}(\dot{H}) |\kappa a|^A \right), \quad (3.40)$$

which can be integrated to obtain the scale factor $a(t)$ in terms of hypergeometric functions. The implicit function (for simplicity we present the solution for $\kappa = 1$ and vanishing α_1) which defines the scale factor is given by

$$\bar{\lambda}_1 (t - \bar{t}) \approx -\text{sign}(\dot{H}) \left(a^{(\alpha_2-2)\bar{\lambda}_1} + \text{sign}(\dot{H}) \right) \left(-\text{sign}(\bar{\lambda}_1) \frac{\bar{\lambda}_2 \left(a^{(2-\alpha_2)\bar{\lambda}_1} + \text{sign}(\dot{H}) \right)}{|\bar{\lambda}_1|} \right)^{\frac{1}{\alpha_2-2}} \quad (3.41)$$

$${}_2F_1 \left(1, 1; 1 + \frac{1}{2-\alpha_2}; -\text{sign}(\dot{H}) a^{(\alpha_2-2)\bar{\lambda}_1} \right), \quad \alpha_2 \neq 2.$$

From (3.40) we easily recover the background solution with constant sound speed and constant warp factor, (3.15), in the limit $\alpha_{1,2} \rightarrow 0$. An important aspect of (3.40) is that it only requires $|\alpha_1|$ to be small, whereas $|\alpha_2|$ can be relatively large, thus allowing a significant evolution of λ_2 during inflation. Note that if we consider $c_{\mathcal{D}}$ constant, for consistency with the smallness of α_1 , then from (3.13) it follows that the evolution of λ_2 is to be attributed to the warp factor f . Below we study the behaviour of the computed solution for different values of α_2 . In view of (3.40), we may consider three cases consistent with $H^2 > 0$:

- $\bar{\lambda}_1 > 0$ and $\dot{H} > 0$. This case is illustrated in the left panel of Fig. 3.10, where for $\alpha_2 < 2$ we have a singular solution when $t \rightarrow \bar{t}$. Any other solution with $\alpha_2 > 2$ is regular at $t = \bar{t}$.
- $\bar{\lambda}_1 > 0$ and $\dot{H} < 0$. This regime takes place provided $(|\kappa|a)^A < 1$. A thorough numerical study of this scenario shows that only for a limited range of values of α_2 the integration of (3.40) yields a well behaved physical solution for the scale factor. In the central panel of Fig. 3.10 we depict the solution for a few values of α_2 in the range $3.5 < \alpha_2 < 5$.
- $\bar{\lambda}_1 < 0$ and $\dot{H} < 0$. The constraint now is $(|\kappa|a)^A > 1$. This case, depicted in the right panel of Fig. 3.10, possesses smooth solutions for $\alpha_2 > 2$. Moreover, for large values of α_2 , the scale factor follows approximately a power law $a(t) \sim (t - \bar{t})^{1/|\bar{\lambda}_1|}$.

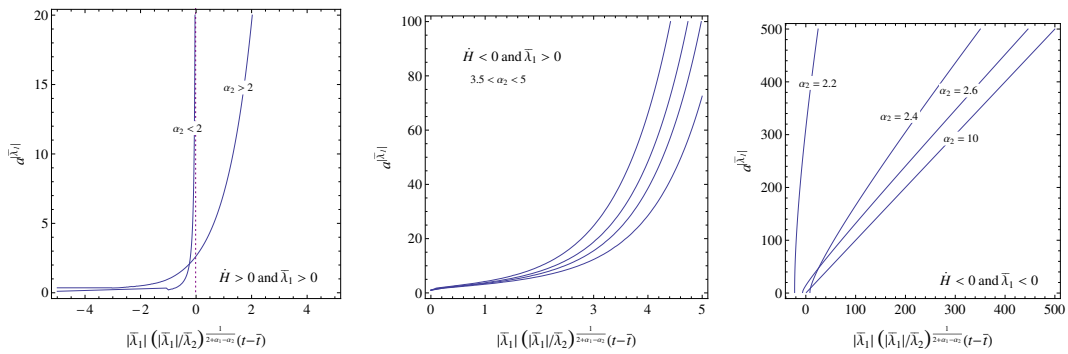


Figure 3.10: Evolution of the scale factor $a(t)$, according to (3.40), for $\bar{\lambda}_1, \dot{H} > 0$ (left panel), for $\bar{\lambda}_1 > 0, \dot{H} < 0$ (central panel) and for $\bar{\lambda}_1, \dot{H} < 0$ (right panel). For simplicity we take $\kappa = 1$.

Parametrization 2

A second, simple alternative to solve (3.11) with time-dependent $\lambda_{1,2}$ is to parametrize their dependence as

$$\lambda_i = \lambda_{i*} (a/a_*)^{\alpha_i}, \quad (3.42)$$

where λ_{i*}, α_i are constants and a_* is the scale factor when the largest cosmological scales exit the horizon. Defining

$$z \equiv \ln H, \quad y \equiv \ln(a/a_*), \quad (3.43)$$

we find the exact solution to (3.11) (see Appendix C)

$$e^{2z} = \frac{2\lambda_2}{\alpha_1} \left(\frac{\alpha_1}{2\lambda_1} \right)^{\frac{\alpha_2}{\alpha_1}} \exp \left(\frac{2\lambda_1}{\alpha_1} e^{y\alpha_1} \right) \Gamma \left(\frac{\alpha_2}{\alpha_1}, \frac{2\lambda_1}{\alpha_1} e^{y\alpha_1} \right) + \kappa \exp \left[\frac{2\lambda_1}{\alpha_1} (e^{y\alpha_1} - 1) \right], \quad (3.44)$$

where $\Gamma(s, x)$ is the incomplete Gamma function [207]. In the limit $\alpha_{1,2} \rightarrow 0$ we easily recover (3.15), whereas for $\alpha_{1,2} \neq 0$ we can use the asymptotic formula $\Gamma(s, x) \approx x^{s-1} e^{-x}$ when $x \gg 1$. In such case, the above equation becomes

$$e^{2z} \simeq \left(\frac{\lambda_{2*}}{\lambda_{1*}} \right) e^{y(\alpha_2 - \alpha_1)} + \kappa \exp \left[\frac{2\lambda_{1*}}{\alpha_1} (e^{y\alpha_1} - 1) \right]. \quad (3.45)$$

If we focus on the background evolution while cosmological scales are exiting the horizon then $0 \leq y \lesssim 9$, and $y\alpha_1 \ll 1$ provided $|\alpha_1| \ll 1$. Neglecting higher orders in $y\alpha_1$ we obtain

$$H^2 \simeq \left(\frac{\lambda_{2*}}{\lambda_{1*}} \right) a^{\alpha_2 - \alpha_1} + \kappa a^{2\lambda_{1*}}, \quad (3.46)$$

which can be integrated to obtain $a(t)$ in terms of hypergeometric functions, and also gives (3.15) in the limit $\alpha_{1,2} \rightarrow 0$. The implicit function (again, and for simplicity, we present the solution for $\kappa = 1$ and vanishing α_1) that determines the scale factor $a(t)$ is given by

$$(t - \bar{t}) \approx - \frac{2\lambda_{1*} a^{-\alpha_2} \sqrt{\frac{\lambda_{2*} a^{\alpha_2}}{\lambda_{1*}} + a^{2\lambda_{1*}}} {}_2F_1 \left(1, \frac{\lambda_{1*} - \alpha_2}{2\lambda_{1*} - \alpha_2}; \frac{-\alpha_2}{4\lambda_{1*} - 2\alpha_2} + 1; -\frac{a^{2\lambda_{1*} - \alpha_2} \lambda_{1*}}{\lambda_{2*}} \right)}{\alpha_2 \lambda_{2*}}. \quad (3.47)$$

Similarly to (3.40), $|\alpha_2|$ is allowed to take on relatively large values in (3.46).

Notice that in (3.47) the hypergeometric function ${}_2F_1$ is undefined when $\left(\frac{-\alpha_2}{4\lambda_{1*} - 2\alpha_2} + 1 \right)$ is a negative integer. Therefore, from a formal point of view, by taking $\alpha_2 < 0$ we avoid the regions where (3.47) is undefined. In addition we must also impose that $\alpha_2 \neq 2\lambda_{1*}$. When $\alpha_2 < 0$, we have from (3.46) that H becomes singular if the scale factor a goes to zero. It can be checked in (3.47) that $(t - \bar{t})$ is zero whenever a is zero, which amounts to have an undesirable singular solution for H when $t = \bar{t}$.

In view of our results, it seems reasonable to conclude that the solutions obtained using the ansatz in (3.37) for $\dot{H} < 0$ (with either sign of $\bar{\lambda}_1$) provide a more appropriate qualitative evolution for $a(t)$ than those described by the ansatz in (3.42). Therefore, our analysis *demonstrates* that, within the context of DBI-Galileon inflation, it is possible to envisage an early inflationary stage during which the warp factor undergoes a significant variation. The relevance of this result is that such phase can be smoothly connected to the last phase of slow-roll while allowing a

marginal variation of the warp factor and agreeing with current CMB observations. In this sense, it is very suggestive to imagine that the early phase of rapidly evolving geometric structure could be connected to the very beginning of inflation.

3.4 Summary

In this chapter we studied the DBI-Galileon inflationary scenario, which constitutes a generic extension of the DBI model involving an induced gravity, and obtain the gravitational field equations allowing the sound speed c_D and warp factor f that the model depends on to be time-dependent. We find exact solutions to the background (3.9) and (3.10) when c_D and f are constant. We obtain a singular behaviour at finite time for the scale factor and Hubble parameter when $\lambda_2 < \lambda_1 H^2$, and also a regular behaviour when $\lambda_2 > \lambda_1 H^2$ (see Fig. 3.1). We focused on inflationary scenarios under the slow-roll approximation and constrain the model parameters using the *Planck* 2015 results. In addition, we constrain the warp factor in the different inflationary regimes using CMB data. Notice that the warp factor scale might be important, regarding warped string phenomenology, to understand extra dimensions and warped geometries arising from string theory. We found that, in general, different warped geometries give rise to distinct inflationary predictions. In the case of constant c_D and f (see Fig. 3.4), the tensor-to-scalar ratio is $r \gtrsim \mathcal{O}(10^{-2})$. Later, we considered the DBI-Galileon model with a slowly varying warp factor and find that the tensor-to-scalar ratio can be as low as $r \simeq 6 \times 10^{-4}$ (see Figs. 3.7 and 3.9). However, we find that this requires the combined tuning of \tilde{m} , ε_f and c_D . In any case, a varying warped geometry brings the predictions of the DBIG model closer to those of the Starobinsky model.

Another aspect of our study is the violation of the standard consistency relation of single-field inflation, $r = -8n_t$. Since DBIG inflation is a class of generalized G-inflation, we find deviations away from the standard consistency relation $r = -8n_t$. However, with the exception of the DBI limit (see Fig. 3.2), the deviations found in the rest of cases under study are quite small (see Table. 3.1). This result is consistent with the status about the tensor consistency relation in Galileon models as it is described in Ref. [209]. We emphasize that a prominent detection of the B-modes, within future CMB probes devised with a greater sensitivity [29-31], can discriminate DBIG inflation.

Finally, we aimed at describing an early stage of inflation taking place well before cosmological scales exit the horizon, we obtain general background solutions allowing an arbitrary time dependence for c_D and f by promoting the coefficients λ_1 and λ_2 in the background (3.11) to time-dependent functions. To integrate the background equations analytically we pursue a phenomenological approach, making use of the ansatz in (3.37) and (3.42). The validity of our approximations demands that λ_1 remains approximately constant ($\alpha_1 \simeq 0$) for both ansatz, whereas λ_2 can have substantial variation since α_2 is not constrained to be small (see Fig. 3.10). This variation of λ_2 , in turn, can be attributed to a variation of the geometric warp factor f since c_D remains approximately constant. From our numerical exploration of the approximate solution we conclude that the ansatz in (3.37) provides a more appropriate, qualitative evolution for the scale factor. Our analysis thus provides the intriguing possibility to consider an early stage of DBI-Galileon inflation (may be even connected to its very beginning) with a significantly varying geometric structure that gives way, once the geometric structure becomes

approximately stabilized, to a final phase of slow-roll in perfect agreement with current CMB observations.

4

Effective models of inflation from SFT framework

Quantum mechanics brought an unexpected fuzziness into physics because of quantum uncertainty, the Heisenberg uncertainty principle. String theory does so again because a point particle is replaced by a string, which is more spread out

- Edward Witten

Accounting string theory as a key player in cosmological inflation, we take an inspiration from string field theory (SFT) [122, 210] and construct successful effective models of inflation¹. Our model is based on the system of open string tachyon and closed string dilaton including the concepts of non-locality and tachyon condensation (c.f., appendix D for a brief review). In this chapter, we consider a system of closed string dilaton and open string tachyon, present in the low energy limit and assume any higher excitations are either stabilized or not relevant for our purposes. The open string tachyon is known to condense rolling to its potential minimum due to brane (or brane-anti-brane pair) instability, present in the system as it decays [211]. This phenomenon is called as the tachyon condensation (TC) process. It is important to understand that the Sen conjecture about TC i.e., the compensation of the brane tension by the negative vacuum energy of the tachyon in the minimum of its potential, was considered in Minkowski as the target spacetime for strings. The TC process itself does not require a dynamical departure from Minkowski background. This is supported by explicit papers [212, 213] and related studies. Therefore, the rolling tachyon process does not have to be necessarily associated with an inflation or other spacetime dynamics. The models [214, 215] which treat the open string tachyon as the inflaton are often effective phenomenological constructions without a computational support in the SFT framework.

The novel step in this chapter is that we assume a system of the dilaton and the open string tachyon near TC. In this regime, non-locality enters through the tachyon potential, without introducing any dynamics to the tachyon itself (see Appendix D where we presented some review of SFT, TC and non-locality). In the low energy limit of SFT, the dilaton and the open string tachyon are coupled through the metric and the dilaton (see c.f. [216] for the so-called “linear dilaton” model). We will show that this regime can only support Minkowski backgrounds as long as the brane tension of the decaying brane is compensated by the open string tachyon vacuum energy and the dilaton is stabilized. Notwithstanding this and motivated by the fact that any higher energy modification of this theory introduces higher order couplings between tachyon and dilaton we claim that in general the model can support an anti-de Sitter or de Sitter (AdS/dS) backgrounds. We continue by introducing an action that accounts the higher order couplings of the dilaton and the open string tachyon system near the TC point. Although, our proposed action is not systematically derived within SFT, it is supported by current developments beyond the linear dilaton model [217].

¹Note that our study is different from the early attempts of considering inflation in SFT studied with p -adic strings [123, 124]

We study the quadratic variations of our newly introduced action around dS background which is possible in our model. We observe that dilaton perturbations acquire non-locality from the infinite derivative terms in the tachyon potential. This is one of the significant result of this chapter that we attach the features of non-locality to the dilaton. Here the non-locality of dilaton is characterized by the function $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ where \Box is the d'Alembertian. Depending on the number of roots of the characteristic equation $\mathcal{F}(z) = 0$, following the studies of [216, 218, 219], we can write the effective actions that are equivalent to our proposed action up to the quadratic perturbations. More specifically, if $\mathcal{F}(\Box)$ has only one real root at z_1 , the corresponding effective action contains just one propagating scalar where the kinetic term contains the parameter $\mathcal{F}'(z_1)$ and any higher derivatives can be neglected assuming the field slow-rolls on a sufficiently flat potential. As a consequence we obtain a successful single field inflation with controlled slow-roll dynamics through the parameter $\mathcal{F}'(z_1)$, which leads to the universal prediction of $r < 0.09$ without changing $n_s = 0.967$ for 60 e -foldings. If $\mathcal{F}(\Box)$ has a complex root the corresponding effective action contains two real scalar fields, which we show to bear conformal invariance. In this case, the two scalar fields share an opposite sign of kinetic terms. With spontaneous breaking of conformal symmetry, we gauge fix one of the scalar field and obtain a Starobinsky like inflation, accompanied with a non-trivial uplifting of the inflaton potential at the minimum.

This chapter is organized as follows. In Sec. 4.1 we discuss the low energy SFT model and show that it can only provide a Minkowski background, upon consideration of TC due to brane decay supported by the details presented in Appendix D. Then we provide SFT heuristic motivations for a more generalized action which can support AdS/dS backgrounds. In Sec. 4.2, given an action that can support dS solution, we perform perturbations around it and prove that dilatonic perturbations acquire non-local properties from the tachyonic part. Then we prescribe a method to write effective actions depending on the structure of non-locality. We study in detail two particular effective actions which leads to interesting inflationary scenarios. In Sec. 4.3 we summarize and discuss open questions which follow from our postulated SFT action and followed by corresponding inflationary scenarios.

4.1 Introducing a framework of SFT for AdS/dS backgrounds

Before we proceed we refer to the Appendix D for some review of SFT and tachyon condensation (TC). In this section we start with the well-known action of a low energy open-closed SFT coupling obtained in the framework of the linear dilaton conformal field theory (see for instance [220]). We will show by means of a simple computation that this regime yields only a Minkowski space-time background as long as the open string tachyon in the minimum of its potential compensates the decaying brane tension and the dilaton field itself is stabilized. Then we will provide generic SFT motivations to propose a generalized action which supports AdS/dS solutions which make it possible to construct effective models of inflation.

4.1.1 Low energy open-closed SFT coupling

From closed SFT, the massless part of action containing dilaton and graviton is given by [221, 222]

$$S_c = \int d^4x \sqrt{-g} \frac{m_P^2}{2} e^{-2\phi} (R + 4\partial_\mu \phi \partial^\mu \phi). \quad (4.1)$$

Here m_P is the reduced Planck mass such that $8\pi G = \frac{1}{m_P^2}$, with G being the Newtonian constant. The dilaton field ϕ is dimensionless. Notice that it is the correct sign for the dilaton kinetic term as it appears in a closed string spectrum. Action (4.1) is the zero mass level of the closed strings. We can add to consideration a p -form but it enters the action quadratically and we put it to zero using its equations of motion. Direct SFT based computation can be done to support the latter action [223, 224].

We however do not include neither the closed string tachyon, nor any potential for the dilaton. Closed string tachyon, even though it is in the spectrum of closed strings, seems to condense to a point where the value of the field is infinite but the potential is zero (not only its derivative) [225]. Additionally, it was shown in [225] that this vacuum is background independent exactly due to the fact that the field takes an infinite value. In such a way, a closed string tachyon does not contribute to our consideration of subsequent inflation. Regarding the dilaton potential, it was suggested in [226] that apparently no dilaton potential is generated in the string frame. This claim finds supporting computations in the same paper and this is known as the dilaton theorem.

Considering the open SFT sector we immediately make use of formula (D.4) which is relevant to describe the open string effects close to the end of an unstable D -brane decay² due to the open string tachyon \mathcal{T} . If we couple (D.4) to dilaton field in a minimal way supported by the linear dilaton conformal field theory (see for instance [220]), we obtain

$$S_o = -\frac{T}{2} \int d^4x \sqrt{-g} e^{-\phi} [v(\square, \mathcal{T}) + 1]. \quad (4.2)$$

The unit term represents the brane tension. This would exactly compensate the value of the potential at the minimum in a pure open SFT in Minkowski background where all the computation regarding the Sen's conjecture were done in a standard SFT approach. However, since the value of the open string tachyon field in the minimum of the potential is finite, the minimum should be background-dependent. This means that in a curved background the energy may not (and most likely will not) be compensated exactly.

Proceeding with a minimal gravitational coupling of (4.1) and (4.2) we get

$$S = S_c + S_o = \int d^4x \sqrt{-g} \left[\frac{m_P^2}{2} (\Phi^2 R + 4\partial_\mu \Phi \partial^\mu \Phi) - \frac{T}{2} \Phi [v(\square, \mathcal{T}) + 1] \right]. \quad (4.3)$$

Here we have redefined the dilaton field as $\Phi = e^{-\phi}$. Dilaton gravity on its own is a well developed subject already for a long time. See, for instance, [228] for a review. A careful but quick analysis immediately shows that this latter action does not support dS background. We can easily see that the Minkowski background is the only option here that corresponds to an exact compensation of the tension of the initial D -brane by the tachyon energy at the bottom

²To avoid confusions we notice that this is in no way the so called Vacuum SFT (VSFT) [227] but rather a linearization of the spacetime action derived in *perturbative* SFT near the bottom of the tachyon potential. VSFT on contrary is a whole new construction involving a different BRST operator.

of the potential and the dilaton is a constant. Indeed, varying with respect to Φ and seeking for a constant dilaton solution (which cannot be zero as the true dilaton is $\phi = -\log(\Phi)$) we get

$$m_{\text{P}}^2 R = \frac{T}{2} [v(\square, \mathcal{T}) + 1]. \quad (4.4)$$

This latter equation together with the trace of Einstein equations gives rise to the result that the brane tension must compensate exactly the tachyon potential value in the minimum and consequently we are left with $R = 0$. This further yields $R_{\mu\nu} = 0$.

4.1.2 Action beyond the low-energy open-closed SFT coupling

In the previous section we learned that the low-energy set-up articulated in (4.3) is not suitable to produce inflation, in which case we essentially require a presence of a nearly dS background. However, including further terms in (4.3) we may expect that other and in particular constant curvature backgrounds are possible. Such terms may arise from a number of sources:

- Once a general (not linear) conformal field theory of the dilaton is considered the above analysis would not work. New interactions will be generated since the BRST algebra of the primary fields will get modified.
- Open-closed string interactions in general contain higher vertexes beyond the action above. These contributions generate new vertexes involving graviton, dilaton and open string tachyon.
- The so called “marginal deformation” [224] excitation in the closed strings. This operator is also of a weight zero but in fact is non-dynamical at a low-level considerations. However, its exclusion by equations of motion will generate additional terms to an effective action as well.

We propose a generalized action that includes new possible interactions of tachyon of open string and the dilaton of closed string:

$$S = \int d^4x \sqrt{-g} \left[\frac{m_{\text{P}}^2}{2} (\Phi^2 R + 4\partial_\mu \Phi \partial^\mu \Phi) - \frac{T}{2} \sum_{n=0}^{\infty} \Phi^{n+1} v_n(\square, \mathcal{T}) \right]. \quad (4.5)$$

where R is Ricci scalar, T is the tension of the D-brane. Here, the term for v_0 is the one appearing in (4.3), i.e. $v_0 = v(\square, \mathcal{T}) + 1$. The other terms $v_n(\square, \mathcal{T})$ for $n \geq 1$ correspond to the higher order couplings of the tachyon potential to the dilaton which in general depends on infinite number of d'Alembertian operators (\square) based on the concepts of SFT (cf., Appendix D). We assume here it is possible to organize a dimensional reduction with all moduli fields stabilized so that we are left with an action (4.5) in $(1+3)$ -dimensional space time. The dimensional reduction of this kind such that an impact of the compactification is absorbed in the overall action normalization can generically be done in a straightforward way [222, 225]. The low-energy p -brane action obtained from SFT is a good example here [122, 210]. Also p -adic string theory is a model worth mentioning in this regard. It reproduces SFT properties up to and including the tree-level scattering and can be formulated in any dimension [229, 230].

This latter action is different from (4.3) by new terms involving coupling of dilaton and tachyon. First we stress that we aim at establishing whether an inflation is possible in this framework

keeping dilaton constant in the vacuum and as such we hunt for constant curvature solutions. This makes irrelevant to consider higher curvature terms. We will comment on this below in the next Section. Second, appearance of an explicit dilaton potential does not contradict the “dilaton theorem” claim as this claim was developed in pure closed string framework. Moreover, results of [217] indicate that the open-closed SFT coupling will waive the “dilaton theorem” statement. As such, the latter action is a viable attempt to account in full the open-closed strings coupling during the TC process. Explicit computation of all such extra terms in the action within the pure SFT considerations is beyond the scope of our present analysis.

The dilaton is a natural candidate for the inflaton as the present day understanding of inflation from the point of view of collected CMB data significantly favours models where the inflaton is coupled non-minimally to the Ricci scalar in the action. Inflation via dilaton in (4.5) can be achieved given that the string scales are higher than the brane tension which in turn is higher than the scale of inflation. In this hierarchy, inflation would start at the final stage of the brane decay.

To support this idea we have to show that action (4.5) indeed may have a constant curvature (in particular dS) background solution when dilaton field takes a constant value and the open string tachyon condenses to its minimum. Varying (4.5) with respect to the metric $g_{\mu\nu}$, \mathcal{T} and Φ we can show that the following configuration is a solution

$$\Phi = \Phi_0 = 1, \mathcal{T} = \mathcal{T}_0, g_{\mu\nu} \text{ is dS with } R = R_0 = 2\frac{T}{m_{\text{P}}^2} \sum_n v_{n,0}, \quad (4.6)$$

together with the following relations fulfilled

$$\sum_n v'_{n,0} = \sum_n v_{n,0}(3 - n) = 0, \quad (4.7)$$

where prime ' is the derivative with respect to an argument and the subscript 0 means that the function is evaluated at $\mathcal{T} = \mathcal{T}_0$. We note that Φ_0 can be any value and is irrelevant as long as it is finite, so we took $\Phi_0 = 1$ for simplicity. We will pay the special account to the question how generic such configurations (4.6) satisfying (4.7) arise in SFT in a separate forthcoming study [231]. We recall from (4.3) and (4.4) that having just a single component $v_0(\square, \mathcal{T}) = v(\square, \mathcal{T}) + 1$ ends up with necessity with a Minkowski spacetime. Thus, in order to generate dS spacetime we need at least two terms with different powers of Φ in the action.³

Hence our proposed modification of linear dilaton in (4.5) supports dS solution (4.6). We stress that our main goal is the retrieval of satisfactory inflation and subsequently computation of inflationary observables. In the next section we study the quadratic perturbations of the action (4.5) and find the effective models of inflation.

4.2 Retrieving effective models of inflation

The quadratic variation of our background action (4.5) can be written as two parts in the following way

³Moreover, we notice that the generality of our construction implies that an appearance of AdS spacetime in which the quantization of strings is well-defined [232] also requires dilaton potential terms like in the (SFT inspired) action (4.5).

$$\delta^{(2)}S = \delta^{(2)}S_{m_P^2} + \delta^{(2)}S_{int}. \quad (4.8)$$

The perturbative modes are $\varphi = \delta\Phi$, trace of the metric perturbations h (we define $\delta g_{\mu\nu} = h_{\mu\nu}$, $h = h^\mu_\mu$) and $\tau = \delta\mathcal{T}$. Generically, different spins do not mix in the quadratic action i.e., tensor modes do not mix with scalar modes. Therefore, the first part of the quadratic action reads

$$\delta^{(2)}S_{m_P^2} = \int d^4x \sqrt{-g} \frac{m_P^2}{2} \left[\varphi^2 R_0 + 4\partial\varphi^2 - \frac{3}{32}h \left(\square + \frac{R_0}{3} \right) h - \frac{3}{2}\varphi \left(\square + \frac{R_0}{3} \right) h \right]. \quad (4.9)$$

From the above action we can exclude h from its equation of motion. Due to the fact that differential operators acting on h and φ are identical, we have $h = -8\varphi + h_{hom}$ where $(\square + R_0/3)h_{hom} = 0$. Substituting this h back in the quadratic action yields

$$\delta^{(2)}S_{m_P^2} = \int d^4x \sqrt{-g} \frac{m_P^2}{2} \varphi (2\square + 3R_0) \varphi. \quad (4.10)$$

The second part of the quadratic action after a Taylor expansion of the tachyon potential $v(\square, \mathcal{T})$ around $\mathcal{T} = \mathcal{T}_0$ reads

$$\delta^{(2)}S_{int} = -\frac{T}{2} \int d^4x \sqrt{-g} \sum_n \left[(n+1)n\varphi^2 v_{n,0} + nv'_{n,0}\varphi f(\square)\tau + \frac{v''_{n,0}}{2}\tau e^{\gamma(\square)}\tau \right], \quad (4.11)$$

where we have used (D.5). Accounting the fact that the open string tachyon on its own is not dynamical, the function $\gamma(\square)$ in the exponent must be an entire function but the operator $f(\square)$ may have eigenvalues. Excluding τ by its equation of motion is dictated by $\tau = -\frac{\sum_n (nv'_{n,0})}{\sum_n v''_{n,0}} f(\square) e^{-\gamma(\square)} \varphi$. Substituting this back into action (4.11) yields

$$\delta^{(2)}S_{int} = -\frac{T}{2} \int d^4x \sqrt{-g} \varphi \left[\sum_n ((n+1)nv_{n,0}) - \frac{(\sum_n nv'_{n,0})^2}{2\sum_n v''_{n,0}} f(\square)^2 e^{-\gamma(\square)} \right] \varphi. \quad (4.12)$$

It is clear from the above formulae that higher curvature corrections are not relevant for us. Indeed, suppose there is a term in the action like $\sqrt{-g}\Phi^2 R^2$, such a term would produce contributions to h^2 and φh but as long as our background has constant scalar curvature and constant dilaton field the final effect of such an additional term would be just renormalization of constants in action (4.10). We see that both the spin-0 excitation of the metric and the dilaton field are combined into one joint scalar mode. Again, we can show by explicit computation that including other interactions, like for instance $\sqrt{-g}\Phi^2 R^2 w(\square, \tau)$, will result in the same net result when all but one scalar fields can be excluded by equations of motion which finally results in a single (non-local) scalar excitation.⁴

We established above why our proposal (4.5) provides a framework to generate a dS background and we will demonstrate how it can describe inflationary effects, which require the second

⁴We here note that additional contributions to scalar and tensor modes can be generated by means of adding the curvature squared corrections, like $R^2_{\mu\nu}$ or C^2 where C is the Weyl tensor. Moreover, following the recent studies performed in [233, 234] one has to pay special attention in order to maintain unitarity upon inclusion of terms which modify the Lagrangian for tensor modes beyond the Einstein's gravity. A standard minimal structure like C^2 in the action will generate a massive spin-2 ghost (see [235] for the first comprehensive study of this question). We therefore leave the full consideration as an open question.

variation of the action around such a background. We recall here that the open string sector contains only the tachyon, since higher mass fields have been integrated out, in the course of the brane decay consideration (cf. the Appendix D). Thus in the nearly dS phase when the scalar curvature does not change considerably, we get from (4.8), (4.10) and (4.12) the following action that describes the propagation of scalar perturbations

$$\delta^{(2)}S = \frac{1}{2} \int d^4x \sqrt{-g} \varphi \mathcal{F}(\square) \varphi, \quad (4.13)$$

where

$$\mathcal{F}(\square) = m_{\text{P}}^2 (2\square + 3R_0) - T \left[\sum_n ((n+1)nv_{n,0}) - \frac{(\sum_n nv'_{n,0})^2}{2\sum_n v''_{n,0}} f(\square)^2 e^{-\gamma(\square)} \right]. \quad (4.14)$$

To generate inflation we must have an appropriate potential in our set-up. The linearization of (4.5) and corresponding analysis do not shed light on the form of the potential though. Rigorously speaking, a potential would follow from SFT provided we have computational abilities to extract one. At present, the state of the art of the knowledge in SFT lacks established methods to do so. In the course of this chapter we will continue by assuming potentials which do not violate general principles of SFT construction (cf. the Appendix D for more discussions on this issue). This strategy can be reversed and be used to constrain perhaps certain parameters in SFT, given we will reach eventually the ability to do such computations directly in the SFT framework.

Considering action (4.13) for a general operator function $\mathcal{F}(\square)$ we cannot convey inflationary physics straightforwardly. In general, $\mathcal{F}(\square)$ being considered as an algebraic function may have many roots. That is, equation

$$\mathcal{F}(z) = 0 \quad (4.15)$$

can have more than one solution. We name it a characteristic equation. Because of that, the propagator for the field φ will have more than one pole. As such, it is equivalent to multiple degrees of freedom. Let us therefore write a local realization of (4.13). Originally, this was done in [216] and then formalized in [218, 219, 236]. We use the Weierstrass factorization [216] which prescribes that any entire function (we recall that SFT ensures that operators $\mathcal{F}(\square)$ are analytic functions and in all existing computations they appear to be entire functions) can be written as

$$\mathcal{F}(z) = e^{\gamma(z)} \prod_j (z - z_j)^{m_j}, \quad (4.16)$$

where z_j are roots of the characteristic equation and m_j are their respective multiplicities. We assume hereafter that all $m_j = 1$ for simplicity. $\gamma(z)$ is an entire function and as such its exponent has no roots on the whole complex plane. It was shown in [216] that for a quadratic Lagrangian of the type (4.13), local equivalent quadratic Lagrangian can be constructed as

$$\delta^2 S_{\text{local}} = \frac{1}{2} \int d^4x \sqrt{-g} \sum_j \mathcal{F}'(z_j) \varphi_j(\square - z_j) \varphi_j \quad (4.17)$$

where prime means derivative with respect to the argument z with the further evaluation at the point z_j . It is said to be equivalent, thanks to the fact that solution for φ which can be obtained

from equations of motion following from (4.13) is connected to solutions for φ_j simply

$$\varphi = \sum \varphi_j. \quad (4.18)$$

Roots z_j become the most crucial objects in classifying our model. Several comments are in order here:

- Note that roots z_j can be complex in general. One real root z_1 is the simplest situation. In this case, we have just a Lagrangian for a massive scalar. It is acceptable if $\mathcal{F}'(z_1) > 0$ in order to evade a ghost in the spectrum.
- More than one real root apparently seems not to be a promising scenario. Since the function $\mathcal{F}(z)$ is analytic (and therefore continuous), neighbouring real roots will be accompanied with $\mathcal{F}'(z_j)$ of opposite signs. In other words, one root is normal and the next to it is a ghost.

4.2.1 Effective model of single field inflation

If $\mathcal{F}(z)$ has one real root, then (4.17) contains a single scalar degrees of freedom

$$\delta^2 S_{local} = \frac{1}{2} \int d^4x \sqrt{-g} \mathcal{F}'(z_1) \varphi (\Box - z_j) \varphi. \quad (4.19)$$

The effective action which is perturbatively equivalent up to quadratic order to (4.19) around dS background, looks like (taking $m_P = 1$)

$$S_1 = \int d^4x \sqrt{-g} \left[\frac{1}{2} \tilde{\Phi}^2 R - \frac{A}{2} \partial \tilde{\Phi}^2 - V(\tilde{\Phi}) \right], \quad (4.20)$$

where $\tilde{\Phi}$ is an effective dilatonic field and the respective correspondence is

$$\begin{aligned} \mathcal{F}'(z_1) &= 6 + A \\ \mathcal{F}'(z_1) z_1 &= 3R_0 - V''(\tilde{\Phi}_0). \end{aligned} \quad (4.21)$$

Here R_0 is scalar curvature of the dS vacuum solution for a constant $\tilde{\Phi}$. Assuming the generalized structure of from the proposed action (4.5), the potential $V(\tilde{\Phi})$ can be taken to be arbitrary. If we consider a potential $V_J(\tilde{\Phi}) = V_0 (-\tilde{\Phi}^2 + \tilde{\Phi}^4)^2$ which looks in the Einstein frame as

$$V_E = \tilde{V}_0 \left(1 - e^{-\sqrt{\frac{2}{3[\mathcal{F}'(z_1)/6]}} \tilde{\phi}} \right)^2, \quad (4.22)$$

where $\tilde{\phi}$ is canonically normalized field by definining $\tilde{\Phi} = e^{-\sqrt{\frac{1}{A+6}} \tilde{\phi}}$. The inflationary predictions corresponding to the potential in (4.22) are well known [81, 84, 237, 238] and in particular we retrieve

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{2\mathcal{F}'(z_1)}{N^2},$$

where we consider $N = 60$ number of e -foldings. We therefore conclude that provided the non-local operator $\mathcal{F}(\square)$ contains one real root, it gives a successful inflation with a universal prediction of $n_s = 0.967$ and the tensor to scalar ratio $r < 0.09$. The value of r can be varied to any value by varying the non-local parameter $\mathcal{F}'(z_1)$.

4.2.2 Effective model of conformal inflation

If $\mathcal{F}(z)$ has a complex root then we should write action (4.17) for a scalar field and also for its complex conjugate. So considering such a pair of complex conjugate roots, we have

$$\delta^2 S_{local} = \frac{1}{2} \int d^4x \sqrt{-g} [\mathcal{F}'(z_1) \varphi_1 (\square - z_1) \varphi_1 + \mathcal{F}'(\bar{z}_1) \bar{\varphi}_1 (\square - \bar{z}_1) \bar{\varphi}_1], \quad (4.23)$$

where a bar over represents the complex conjugates. To maintain the connection with the original action (4.13) we should consider complex conjugate solutions to equations of motion, such that $\varphi = \varphi_1 + \bar{\varphi}_1$ is real. The important feature is that the quadratic form of fields is already diagonal. Introducing $\varphi_1 = \chi + i\sigma$, $z_1 = \alpha + i\beta$, $\mathcal{F}'(z_1) = c + is$ we can rewrite action (4.23) in terms of real components as

$$\delta^2 S_{local} = \int d^4x \sqrt{-g} [\chi(c\square - c\alpha + s\beta)\chi - \sigma(c\square - c\alpha + s\beta)\sigma - 2\chi(s\square - s\alpha - c\beta)\sigma]. \quad (4.24)$$

The above action is inevitably non-diagonal and features a cross-product of real fields $\sim \chi\sigma$. In the formulation above, note that the two fields χ, σ share a opposite sign of kinetic term [239]. We will show that the following effective action of two fields with conformal invariance can be perturbatively equivalent up to quadratic order to (4.24) around dS background

$$S_2 = \int d^4x \sqrt{-g} \left[\frac{m_P^2}{2} [\tilde{\alpha}\tilde{\Phi}_1^2 - \tilde{\alpha}\tilde{\Phi}_2^2 - 2\tilde{\beta}\tilde{\Phi}_1\tilde{\Phi}_2] f\left(\frac{\tilde{\Phi}_2}{\tilde{\Phi}_1}\right) R + \frac{A}{2} [\tilde{\alpha}\partial\tilde{\Phi}_1^2 - \tilde{\alpha}\partial\tilde{\Phi}_2^2 - 2\tilde{\beta}\partial_\mu\tilde{\Phi}_1\partial^\mu\tilde{\Phi}_2] f\left(\frac{\tilde{\Phi}_2}{\tilde{\Phi}_1}\right) - V(\tilde{\Phi}_1, \tilde{\Phi}_2) \right]. \quad (4.25)$$

where $\tilde{\Phi}_1, \tilde{\Phi}_2$ are effective dilatonic fields.

We can write the quadratic Lagrangian for the spin-0 part which contains 2 components $\tilde{\chi} = \delta\tilde{\Phi}_1$ and $\tilde{\sigma} = \delta\tilde{\Phi}_2$ (i.e. again the spin-0 metric perturbation is excluded by equations of motion), as

$$\delta^2 S_2 = \frac{1}{2} \int d^4x \sqrt{-g} [\tilde{\chi}\Delta_{\tilde{\chi}}\tilde{\chi} + \tilde{\sigma}\Delta_{\tilde{\sigma}}\tilde{\sigma} + \tilde{\chi}\Delta_{\tilde{\chi}\tilde{\sigma}}\tilde{\sigma}], \quad (4.26)$$

where

$$\begin{aligned} \Delta_{\tilde{\chi}} &= \frac{m_P^2}{2} \left(\frac{(\partial_{\tilde{\Phi}_1} I_0)^2}{I_0} (3\square + R_0) + \frac{\partial^2 I_0}{\partial \tilde{\Phi}_1^2} R_0 \right) - A\tilde{\alpha}f_0\square - \frac{\partial^2 V_0}{\partial \tilde{\Phi}_1^2}, \\ \Delta_{\tilde{\sigma}} &= \frac{m_P^2}{2} \left(\frac{(\partial_{\tilde{\Phi}_2} I_0)^2}{I_0} (3\square + R_0) + \frac{\partial^2 I_0}{\partial \tilde{\Phi}_2^2} R_0 \right) + A\tilde{\alpha}f_0\square - \frac{\partial^2 V_0}{\partial \tilde{\Phi}_2^2}, \\ \Delta_{\tilde{\chi}\tilde{\sigma}} &= \frac{m_P^2}{2} \left(\frac{\partial_{\tilde{\Phi}_1} I_0 \partial_{\tilde{\Phi}_2} I_0}{I_0} (3\square + R_0) + \frac{\partial^2 I_0}{\partial \tilde{\Phi}_1 \partial \tilde{\Phi}_2} R_0 \right) - A\tilde{\beta}f_0\square - \frac{\partial^2 V_0}{\partial \tilde{\Phi}_1 \partial \tilde{\Phi}_2}, \end{aligned}$$

where R_0 is the scalar curvature of dS vacuum for constant dilatonic fields $\tilde{\Phi}_1 = \tilde{\Phi}_{1,0}$, $\tilde{\Phi}_2 = \tilde{\Phi}_{2,0}$. Here we define $I(\tilde{\Phi}_1, \tilde{\Phi}_2) = [\tilde{\alpha}\tilde{\Phi}_1^2 - \tilde{\alpha}\tilde{\Phi}_2^2 - 2\tilde{\beta}\tilde{\Phi}_1\tilde{\Phi}_2] f(\tilde{\Phi}_2/\tilde{\Phi}_1)$ and $I_0 \equiv I(\tilde{\Phi}_{1,0}, \tilde{\Phi}_{2,0})$, $\partial_{\tilde{\Phi}_1} I_0 \equiv$

$\partial I(\tilde{\Phi}_1, \tilde{\Phi}_2)/\partial \tilde{\Phi}_1$ are the quantities evaluated at the values of fields at dS vacuum and so on for analogous terms.

We can make use of (4.24), which is the case of two complex conjugate roots with the Lagrangian written in real fields. Hence, we can try to juxtapose (4.24) and (4.26). The motivation for doing this is to establish a more fundamental correspondence for the effective model (4.25). This is, however much more involved than in the previous Section with a single field. Essentially, the most important is to establish $\Delta_{\tilde{\chi}} = -\Delta_{\tilde{\sigma}}$. On this way, we can neglect the second derivatives of the potential V . However, we must satisfy a number of constraints, namely, all parameters and vacuum fields values must be real and I_0 strictly positive. And we want to have $\tilde{\beta} \neq 0$, which we will explain why in the following. The greatly simplifying point is that we must require such an adjustment of coefficients of Δ -s only in a single point ($\tilde{\Phi}_1 = \tilde{\Phi}_{1,0}$, $\tilde{\Phi}_2 = \tilde{\Phi}_{2,0}$). On top of this we emphasize once again that we aim at retrieving a nearly dS phase, not an exact one. These requirements are generically satisfied altogether with the presence of a function $f\left(\frac{\tilde{\Phi}_2}{\tilde{\Phi}_1}\right)$ (apart from special situations which we discuss shortly). It is important that being a function of the ratio of fields it cannot spoil a possible conformal invariance.

Let us recall that our main purpose in this Section is to establish an effective setting which can emulate (4.24). We claim that we have such an effective model as long we can match quadratic actions for scalar modes around a dS background. We can thus establish a correspondence between (4.24) and (4.26) by means of the following:

- During inflationary expansion we can assume that the scalar fields varies slowly and the kinetic terms can be neglected. We are thus mainly interested in whether $\Delta_{\tilde{\chi}} = -\Delta_{\tilde{\sigma}}$ for the terms proportional to R_0 . To have this we should require

$$\frac{(\partial_{\tilde{\Phi}_1} I_0)^2}{I_0} + \frac{\partial^2 I_0}{\partial \tilde{\Phi}_1^2} + \frac{(\partial_{\tilde{\Phi}_2} I_0)^2}{I_0} + \frac{\partial^2 I_0}{\partial \tilde{\Phi}_2^2} \approx 0. \quad (4.27)$$

- We can check that even in the very simple case of $\tilde{\beta} = 0$, a non-constant function f is required to satisfy the above relation. A simple choice like

$$f = 1 + f_1 \tilde{\Phi}_2 / \tilde{\Phi}_1, \quad (4.28)$$

with just one free parameter f_1 is sufficient. Otherwise, for $f = \text{const}$ a condition $\tilde{\Phi}_{1,0} = \pm i \tilde{\Phi}_{2,0}$ arises from (4.27). Therefore to build such an effective model the function $f\left(\frac{\tilde{\Phi}_2}{\tilde{\Phi}_1}\right)$ is very useful and important. The cross-product of fields may arise for $\tilde{\beta} = 0$ but a quite involved non-polynomial function f is required.

- For a non-trivial $\tilde{\beta}$ the same function f as above in (4.28) is enough to arrange the condition (4.27). Moreover $\tilde{\beta} \neq 0$ generates a cross-product of fields.
- In complete analogy we can consider the coefficients of the kinetic terms. We have to require a non-constant function f . We note that having opposite coefficients in front of d'Alembertian operators for different fields essentially means that one of these fields is a ghost.

Recalling expressions (4.17) and (4.24), we see that the presence of a cross-product is a special feature related to a complex root of the function $\mathcal{F}(z)$ (which defines the non-local operator $\mathcal{F}(\square)$). This means that the parameter β found in (4.24) is essentially non-zero (notice that

there is no a direct simple relation between $\tilde{\beta}$ and β). In the limiting case of $\beta \rightarrow 0$, we should see the cross-product disappearing and this corresponds to $\tilde{\beta} \rightarrow 0$ in the effective model (4.25). Another way to recognize the effective model (4.25) without a cross-product of fields is to consider directly (4.17) with two specially tuned real roots. This means that these roots are related as $z_2 = -z_1$ and moreover $\mathcal{F}'(z_2) = -\mathcal{F}'(z_1)$.

To resolve the issue of a ghost in the spectrum requires an extra symmetry in order to gauge the ghost away. The most natural candidate is the conformal symmetry used in the building of similar models in [83, 84, 240]. The conformal invariance is restored in (4.25) if we assume $A = 6$. Our model without a cross-product resembles the conformal models studied in [241, 242]. We stress that the cross-product appeared for the first time in the cosmological models and we have here provided an imperative explanation through the non-local dilaton.

Assuming $f\left(\frac{\tilde{\Phi}_2}{\tilde{\Phi}_1}\right) \approx \text{constant}$ during inflation (4.25) can be written as

$$S_2 = \int d^4x \sqrt{-g} \left[\left(\tilde{\alpha} \tilde{\Phi}_1^2 - \tilde{\alpha} \tilde{\Phi}_2^2 - 2\tilde{\beta} \tilde{\Phi}_1 \tilde{\Phi}_2 \right) \frac{R}{12} + \frac{\tilde{\alpha}}{2} \partial \tilde{\Phi}_1^2 - \frac{\tilde{\alpha}}{2} \partial \tilde{\Phi}_2^2 - \tilde{\beta} \partial_\mu \tilde{\Phi}_1 \partial^\mu \tilde{\Phi}_2 - V_J(\tilde{\Phi}_1, \tilde{\Phi}_2) \right], \quad (4.29)$$

where we have set $m_P = 1$ for simplicity and use the subscript J for the Jordan frame as before. Since the field $\tilde{\Phi}_1$ has a wrong sign kinetic term (assuming $\tilde{\alpha} > 0$), we can eliminate it by the choice of conformal gauge $\tilde{\Phi}_1 = \sqrt{6}$ which spontaneously breaks the conformal invariance. To obtain a consistent inflation within this model we consider the following potential

$$V_J(\tilde{\Phi}_1, \tilde{\Phi}_2) = \frac{\lambda}{4} \left(\gamma_1 \tilde{\Phi}_2^2 + \gamma_2 \tilde{\Phi}_1 \tilde{\Phi}_2 + \gamma_3 \tilde{\Phi}_1^2 \right) \left(\tilde{\Phi}_2 - \tilde{\Phi}_1 \right)^2, \quad (4.30)$$

where $\gamma_1, \gamma_2, \gamma_3$ are arbitrary constant parameters. The potential (4.30) is motivated from [83], which we generalize here to our conformal model with a term containing the cross-product of fields. The importance of this generalization will be explained in what follows. Note that if $\tilde{\beta} = \gamma_2 = \gamma_3 = 0$, the model reduces to the conformal model without a cross-product of fields studied in [83].

Rescaling the fields as $\tilde{\Phi}_1 \rightarrow \frac{\tilde{\Phi}_1}{\sqrt{\tilde{\alpha}}}$ and $\tilde{\Phi}_2 \rightarrow \frac{\tilde{\Phi}_2}{\sqrt{\tilde{\alpha}}}$ in action (4.29) and using the gauge $\tilde{\Phi}_1 = \sqrt{6}$ we yield

$$S_2 = \int d^4x \sqrt{-g} \left[\frac{R}{2} \left(1 - \frac{\tilde{\Phi}_2^2}{6} - \frac{2\tilde{\beta}}{\sqrt{6}\tilde{\alpha}} \tilde{\Phi}_2 \right) - \frac{1}{2} \partial_\mu \tilde{\Phi}_2 \partial^\mu \tilde{\Phi}_2 - \frac{\lambda}{4\tilde{\alpha}^2} \left(\gamma_1 \tilde{\Phi}_2^2 + \gamma_2 \tilde{\Phi}_1 \tilde{\Phi}_2 + \gamma_3 \tilde{\Phi}_1^2 \right) \left(\tilde{\Phi}_2 - \sqrt{6} \right)^2 \right]. \quad (4.31)$$

Performing the conformal transformation $g_{\mu\nu} \rightarrow \left[1 + \frac{\tilde{\beta}^2}{\tilde{\alpha}^2} - \frac{1}{6} \left(\tilde{\Phi}_2 + \frac{\tilde{\beta}}{\tilde{\alpha}} \sqrt{6} \right)^2 \right]^{-1} g_{\mu\nu}$ and shifting the field $\tilde{\Phi}_2 \rightarrow \tilde{\Phi}_2 + \frac{\tilde{\beta}}{\tilde{\alpha}} \sqrt{6}$, we arrive to the Einstein frame action

$$S_{2E} = \int d^4x \sqrt{-g_E} \left[\frac{R_E}{2} - \frac{\omega}{2 \left(\omega - \frac{\tilde{\Phi}_2^2}{6} \right)^2} \partial_\mu \tilde{\Phi}_2 \partial^\mu \tilde{\Phi}_2 - V_E(\tilde{\Phi}_2) \right], \quad (4.32)$$

where $\omega = 1 + \frac{\tilde{\beta}^2}{\tilde{\alpha}^2}$ and

$$V_E(\tilde{\Phi}_2) = \frac{9\lambda}{\tilde{\alpha}^2} \frac{\left[\gamma_1 \tilde{\Phi}_2^2 + \left(\gamma_2 - 2\gamma_1 \frac{\tilde{\beta}}{\tilde{\alpha}} \right) \sqrt{6} \tilde{\Phi}_2 + 6 \left(\gamma_1 \frac{\tilde{\beta}^2}{\tilde{\alpha}^2} - \gamma_2 \frac{\tilde{\beta}}{\tilde{\alpha}} + \gamma_3 \right) \right] \left(\tilde{\Phi}_2 - \sqrt{6} \frac{\tilde{\beta}}{\tilde{\alpha}} - \sqrt{6} \right)^2}{\left(6\omega - \tilde{\Phi}_2^2 \right)^2}. \quad (4.33)$$

If γ_i are chosen such that $\gamma_2 = 2\gamma_1 \frac{\tilde{\beta}}{\tilde{\alpha}}$ and $\gamma_1 \frac{\tilde{\beta}^2}{\tilde{\alpha}^2} - \gamma_2 \frac{\tilde{\beta}}{\tilde{\alpha}} + \gamma_3 \gtrsim 0$, we can obtain inflation with an uplifting of the potential at the minimum.

For example, let us consider a simple case with $\gamma_1 = 1$, $\gamma_2 = 2\frac{\tilde{\beta}}{\tilde{\alpha}}$ and $\gamma_3 = 2\frac{\tilde{\beta}^2}{\tilde{\alpha}^2}$, for which (4.33) reduces to the following form in terms of canonically normalized field $\tilde{\Phi}_2 = \sqrt{6\omega} \tanh\left(\frac{\tilde{\phi}}{\sqrt{6}}\right)$ as

$$V_E(\tilde{\phi}) = \mu^2 \left[\sinh^2\left(\frac{\tilde{\phi}}{\sqrt{6}}\right) + \frac{\tilde{\beta}^2}{(\tilde{\alpha}^2 + \tilde{\beta}^2)} \cosh^2\left(\frac{\tilde{\phi}}{\sqrt{6}}\right) \right] \left[\cosh\left(\frac{\tilde{\phi}}{\sqrt{6}}\right) - \frac{1}{1 + \frac{\tilde{\beta}}{\tilde{\alpha}}} \sqrt{1 + \frac{\tilde{\beta}^2}{\tilde{\alpha}^2}} \sinh\left(\frac{\tilde{\phi}}{\sqrt{6}}\right) \right]^2 \quad (4.34)$$

where $\mu^2 = \frac{9\lambda(\tilde{\alpha} + \tilde{\beta})^2}{\tilde{\alpha}^2(\tilde{\alpha}^2 + \tilde{\beta}^2)}$. In the limit $\frac{\tilde{\beta}}{\tilde{\alpha}} \ll 1$, the first term in (4.34) dominates during inflation while the second term is negligible. The potential (4.34) is always positive and in particular has a non-zero value at the minimum at $\tilde{\phi} \approx 0$. In general the shape of the potential is similar to the Starobinsky-like models in no-scale SUGRA [81]. In Fig. 4.1 we depict the shape of the potential for various values of $\tilde{\beta}$. This corresponds to different values of vacuum energy (Λ) after inflation. We can see that the smaller the value of $\tilde{\beta}$, the greater the chance of approaching the plateau region of the Starobinsky model, and eventually the smaller will be the value of the vacuum energy.

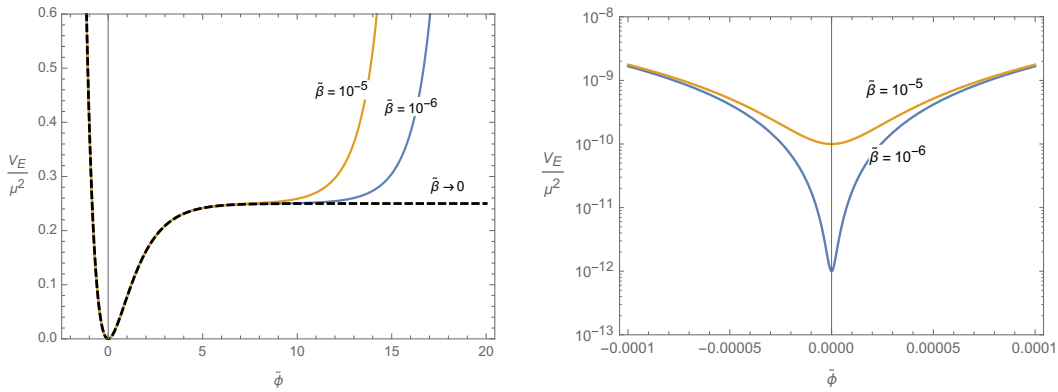


Figure 4.1: In the left panel we plot the potential $V_E(\tilde{\phi})$ for values of $\tilde{\beta} = 10^{-5}$, 10^{-6} and $\tilde{\alpha} = 1$. In the right panel, we depict the corresponding minimum of the potential around $\tilde{\phi} \approx 0$.

Setting $\tilde{\alpha} = 1$, in the limit $\tilde{\beta} \ll 1$, we can approximate the potential in (4.34) as

$$V_E(\tilde{\phi}) \approx \frac{\mu^2}{4} \left(1 - e^{-\sqrt{\frac{2}{3}}\tilde{\phi}} \right)^2 + \frac{\mu^2 \tilde{\beta}^2}{4} \left(1 + e^{-\sqrt{\frac{2}{3}}\tilde{\phi}} \right)^2, \quad (4.35)$$

where the first term dominates when $\tilde{\phi} \gg 1$ and leads to a Starobinsky like inflation i.e., $n_s \sim 0.967$, $r \sim 0.0033$ for $N = 60$ and the second term gives a non-zero vacuum energy at the

minimum of the potential⁵ near $\tilde{\phi} = 0$. Here $\mu \approx 2 \times 10^{-5}$ (in Planck units as we have set $m_P = 1$) which can be determined from the observed amplitude of scalar perturbations $A_s = 2.2 \times 10^{-9}$ at the horizon exit [24]. In particular $\tilde{\beta} \sim 10^{-55}$ gives a vacuum energy that reproduces the present day cosmological constant $\Lambda \sim 10^{-120}$. Therefore, we conclude that a non-locality induced cross-product of the fields $\tilde{\Phi}_1$ and $\tilde{\Phi}_2$ in (4.29) naturally uplifts the inflaton potential at the minimum and possibly explain the present day dark energy (assuming it is Λ CDM).

4.3 Summary

In this chapter, we have investigated effective models of inflation emerging from a framework motivated from SFT. Our models of inflation are essentially an aftermath of TC being possible since not all the brane tension is compensated by the tachyon in a curved background. For the inflation to happen we assume that the inflation scale is below the brane tension. In our setup, we proposed an action beyond the low-energy open-closed strings coupling in SFT containing closed string dilaton and open open string tachyon near the tachyon condensation. We observed that this action can contain (A)dS as background solutions. We have studied the quadratic perturbations of this action and have shown that the infinite derivative operators associated with tachyon induce non-locality dilaton perturbations characterized by $\mathcal{F}(\square)$. The cornerstone technical question is about the roots z_j of the characteristic equation $\mathcal{F}(z) = 0$. Moreover, the derivatives $\mathcal{F}'(z_j)$ play an important role. This is seen from action (4.17), which describes the evolution of scalar perturbations around a dS vacuum within a non-local context, SFT being a guide in this process. Its importance is obvious as inflation is a dS like expansion and all the observable quantities related to scalars can be obtained from exploring the action for linear perturbations. A very important restriction is that no ghosts must be in the spectrum. This selects two configurations of roots.

First, there is a situation with one real root z_1 accompanied with a correct sign of $\mathcal{F}'(z_1)$. In this case there is one scalar perturbative degree of freedom. Such a configuration can be obtained from the effective model description (4.20). It is important that coefficients in front of the Einstein-Hilbert term and the kinetic term of a scalar field are independent. We therefore conclude that provided the non-local operator $\mathcal{F}(\square)$ contains one real root, it gives a successful inflation with a universal prediction of $n_s = 0.967$ and tensor to scalar ratio as in (4.2.1) which can be adjusted to any value $r < 0.09$ by means of the parameter $\mathcal{F}'(z_1)$. A future more accurate detection of parameter r from CMB [29] would indicate the values of z_1 and $\mathcal{F}'(z_1)$.

Second, there was a case with two roots. They can be complex conjugate and then we should look at (4.24) which is written in manifestly real components. In this scenario, we inevitably get a quadratic cross-product of fields. Moreover, one field looks like a ghost. However, kinetic and mass terms have exactly opposite signs. This suggests that a conformal symmetry may help exorcising the ghost. Indeed, building an effective model (4.25) we have taken the conformal symmetry into account and have shown that we indeed can make use of it to remove the unwanted degrees of freedom. The cross-product of fields naturally leads to an uplifting of the potential in the reheating point. In principle one can get a similar two-field model starting with two real roots which are related as $z_1 = -z_2$ and $\mathcal{F}'(z_1) = -\mathcal{F}'(z_2)$. This latter case has no

⁵A potential of similar kind can be found in the α -attractor models where the inflaton potential was uplifted due to the effect of a SUSY breaking mechanism [238].

cross-product of fields and falls into the considerations of [241, 242]. The novel feature here is that the conformally invariant models with a quadratic cross-product of scalar fields appear for the first time in a cosmological setup and can be naturally explained using the non-locality of a dilaton.

5

Non-slow-roll dynamics in α -attractors

I'm a fan of supersymmetry, largely because it seems to be the only route by which gravity can be brought into the scheme. If you have supersymmetry, then there are more of these particles. That would be my favourite outcome.

- Peter Higgs

Since the first release of *Planck* 2013 data, two scenarios (Starobinsky model and Higgs inflation) started to attract a lot of attention. They have been extensively studied and realized in the context of conformal symmetries [241, 242], later generalized as α - and non-minimal (or) ξ -attractors. In addition, these models have been embedded in SUGRA through the use of superconformal symmetries [84, 240, 243-245]. Recently, α - attractor models were also realized by means of the inclusion of an auxiliary vector field for the Starobinsky model [87]. These two classes of models have also, a posteriori, been unified as cosmological attractor models (CAM) [119, 125, 246]. By varying the parameters (α, ξ) in CAM, on the one hand, it leads to the predictions of Starobinsky inflation and on the other hand it also reproduces the chaotic inflation predictions with the $m^2\phi^2$ potential. In particular, for $\alpha = \frac{1}{9}$, we retrieve the first model of chaotic inflation in SUGRA proposed in 1983, which is known as the Goncharov-Linde (GL) model, and it is well consistent with the present data [247-249]. CAM were embedded in $\mathcal{N} = 1$ SUGRA using superconformal symmetries by introducing a 3 chiral super multiplets: a conformon X^0 , an inflaton $X^1 = \Phi$ and a sGoldstino $X^3 = S$ [84, 240, 243]. In this set up, single field inflation is achieved at the minimum of the superpotential by the requirement that the fields S and $\text{Im } \Phi$ remain heavy during inflation¹. In recent studies, α - attractors were realized in SUGRA² by only requiring a single chiral superfield [256, 257]. A generalization of Kähler potentials for viable single field models with respect to *Planck* data, plus their connection to open and closed string sector has been investigated in [88].

In this chapter, we study non-slow-roll inflaton dynamics in the α -attractor model using the recently proposed approach of Gong and Sasaki (GS) [127], which constitutes, to our knowledge, a new strategy. More concretely, we focus on the non-canonical aspect of the α - attractor model. We start with the assumption of GS [127], where the number of e -foldings N which is counted backward in time is assumed to be a function of the inflaton field ϕ during inflation. We retrieve the local shape of the potential during inflation which can be steep and allowing for 60 e -foldings to occur. More precisely, we restrict our study to the region of the potential where inflation is occurring. We emphasize that both the pre- and post-inflationary dynamics are beyond the scope of this chapter. Afterwards, we explore the GS parametrization within our chosen inflaton dynamics showing that inflation occurs for a wider class of potentials. We further show that we can maintain the predictions of the α -attractor model displayed in [84],

¹This mechanism has also envisaged the multifield inflation with a curvaton, i.e, where we can have generation of isocurvature perturbations when S or $\text{Im } \Phi$ are light and play the role of curvaton during or after the end of inflation [250-252]

²Obtaining inflation from SUGRA also brings other benefits such as, exploring SUSY breaking sector and the presence of dark energy [80, 238, 253-255].

but now herein retrieved alternatively within a non-slow-roll. Finally, we study the possibility of realizing this model within $\mathcal{N} = 1$ SUGRA. We explore the relation between the inflaton dynamics and the corresponding Kähler geometry curvature. We also comment on the stability of inflaton trajectory during inflation.

The chapter is organized as follows: In Sec. 5.1, we revise the α -attractor model and present arguments supporting a non-slow-roll approach for these models. In Sec. 5.2, we describe GS parametrization and implement the non-slow-roll dynamics in the context of α -attractors. In Sec. 5.3, we present predictions for a specific case of the GS parametrization. In Sec. 5.4, we complement the previous predictions for a wider class of non-slow-roll dynamics and discuss on large and small field inflation. We show that these scenarios exhibit an attractor in the (n_s, r) plane and discuss the (dis)similarities with standard slow-roll inflaton dynamics. In Sec. 5.5, we review the SUGRA realization of this scenario and verify the stabilization of the inflaton trajectory during inflation.

5.1 α -attractor model

In this section, we revise the essentials of α -attractor models which have been studied under slow-roll frameworks so far as in [84, 125, 254] and provide a baseline for our interest on these models which we will be exploring in the rest of the manuscript from a new perspective and methodology.

The Lagrangian for α -attractor models, in the Einstein frame, is given by³ [254]

$$\mathcal{L}_E = \sqrt{-g} \left[\frac{R}{2} - \frac{1}{(1 - \phi^2/6\alpha)^2} \frac{(\partial\phi)^2}{2} - f^2 \left(\phi/\sqrt{6\alpha} \right) \right], \quad (5.1)$$

where $\alpha = 1$ leads to the same prediction of the Starobinsky model (in the Einstein frame), $\alpha = 1/9$ corresponds to GL model [247], and for large α this model is equivalent to chaotic inflation with quadratic potential [9]. In order to prevent negative gravity in the Jordan frame it is required to have $|\phi| < \sqrt{6\alpha}$ [84, 245]. Furthermore, in the SUGRA embedding of this model, the parameter α is shown to be related to the curvature of Kähler manifold as

$$\mathcal{R}_K = -\frac{2}{3\alpha}. \quad (5.2)$$

The Lagrangian (5.1) is a subclass of k -inflationary model where the kinetic term is linear⁴ in X , i.e.,

$$P(X, \phi) = K(\phi) X - f^2 \left(\phi/\sqrt{6\alpha} \right), \quad (5.3)$$

where $K(\phi) = \frac{1}{(1 - \phi^2/6\alpha)^2}$ and $X = -\frac{(\partial\phi)^2}{2}$. The speed of sound for these class of models is $c_s^2 = 1$ [258], therefore these models are not expected to show large non-Gaussianities [36].

In this theory, the Friedmann equation is

³We assume the units $m_P = 1$.

⁴ $K(\phi) = 1$ gives the canonical kinetic term.

$$H^2 = \frac{1}{3} \left[XK(\phi) + f^2 \left(\frac{\phi}{\sqrt{6\alpha}} \right) \right]. \quad (5.4)$$

The Raychaudhuri equation is

$$\dot{H} = -XP_{,X} \quad \text{with} \quad P_{,X} = \frac{\partial P}{\partial X}, \quad (5.5)$$

and the equation of motion for the scalar field is given by

$$\frac{d}{dt} \left(K(\phi) \dot{\phi} \right) + 3HK(\phi) \dot{\phi} - P_{,\phi} = 0. \quad (5.6)$$

In the literature it is found that inflation in the α -attractor model has been realized in terms of a canonically normalized field (φ) as

$$\frac{d\varphi}{d\phi} = \frac{1}{\left(1 - \frac{\phi^2}{6\alpha}\right)} \Rightarrow \frac{\phi}{\sqrt{6\alpha}} = \tanh \frac{\varphi}{\sqrt{6\alpha}}. \quad (5.7)$$

In this case, flat potentials are natural and subsequent slow-roll dynamics of φ lead to viable inflationary scenario with respect to the observational data. The predictions of (n_s, r) for these models are shown to be solely determined by the order and residue of the Laurent series expansion leading pole in the kinetic term [125]. The slow-roll inflationary predictions of α -attractor models are

$$n_s = 1 - \frac{2}{N} \quad r = \frac{12\alpha}{N^2}. \quad (5.8)$$

In terms of this canonically normalized field (φ) the equation of motion (5.6) becomes

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0. \quad (5.9)$$

Therefore, under slow-roll assumption this reduces to

$$3H\dot{\varphi} \simeq V_{,\varphi}. \quad (5.10)$$

Our purpose is to obtain viable inflationary predictions, by means of extending α -attractors towards non-slow-roll dynamics. Therefore, in the present work, we restrict ourselves to the range $\phi^2 < 6\alpha$. We will emphasize similarities and of course the differences with the (canonically normalized field) slow-roll inflation case. In the following section we unveil the context of non-slow-roll towards α -attractors.

5.2 Non-slow-roll dynamics

The recent work by Gong & Sasaki (GS) [127] points out a cautionary remark on applying slow-roll approximation in the context of k -inflation. The argument, presented there, lies in the fact that the second derivative term in the equation of motion (5.6) may not be negligible in general. In this regard, the authors introduce a new parameter

$$p = \frac{\dot{P}_{,X}}{HP_{,X}}, \quad (5.11)$$

which could bring significant differences in the local non-Gaussianity. They have illustrated the role of this new parameter and observationally viable inflationary scenarios in the context of some non-trivial examples.

Let us implement the aforementioned procedure here in the context of α -attractors as ϕ is a non-canonical scalar field given by (5.3). This new approach enable us to study the α -attractors in the context of non-slow-roll by assuming that the inflaton field during inflation behaves as⁵

$$\phi = n \exp(\beta N), \quad (5.12)$$

where $N = \ln a(t)$ is the number of efoldings counted backward in time from the end of inflation and n is treated as a free parameter that specifies the value of the field at $N \rightarrow 0$. We assign (5.12) as GS parametrization for subsequent reference. This parametrization is particularly useful in the cases of non-canonical scalar field models, whereas in Refs. [259, 260] a different parametrization was applied to the case of canonical scalar field inflation. We declare here that our study of inflation in α -attractor model is based on the dynamics for the inflaton assumed in (5.12) parametrized by (n, β) . Therefore, we label our approach for the α -attractor framework as non-slow-roll, following the same terminology used in Ref. [127]. Being more precise, in this chapter we do not impose any slow-roll approximation in particular. We note at this point that non-slow-roll does not mean a non-smallness of conventional parameters ε, η (see Ref. [127] for more details). Moreover, and we stress that this is a most important point in our study, we completely relax the choice of the inflaton potential and rather concentrate on the inflaton dynamics that can give rise to viable observational predictions.

Substituting ϕ from (5.12) in the Raychaudhuri equation we obtain

$$H' = \frac{\alpha^2 H(N)}{2} \phi^2 K(\phi), \quad (5.13)$$

where the prime ' denotes differentiation with respect to N . Integrating (5.13), we get

$$H(N) = \lambda e^{-\frac{9\beta\alpha^2}{\phi^2 - 6\alpha}}, \quad (5.14)$$

where λ is the integration constant. At this point, we should mention that our calculations are similar to the Hamilton-Jacobi like formalism found in [126, 260, 261].

Inserting the aforementioned solution in (5.4), we can express the local shape of the potential

⁵We start with a similar parametrization as the one used in Sec.3.2 of [127].

during inflation as

$$f^2 \left(\frac{\phi}{\sqrt{6\alpha}} \right) = \lambda \exp \left(-\frac{18\beta\alpha^2}{\phi^2 - 6\alpha} \right) \left[3 - \frac{\beta^2\phi^2}{2 \left(1 - \frac{\phi^2}{6\alpha} \right)^2} \right]. \quad (5.15)$$

It should therefore be noted that the suitable choice of potentials considered in the case of slow-roll α -attractors are quite different, namely, power law type $V \sim \phi^{2n}$ in terms of original scalar field (or) T-models, i.e., $V \sim \tanh^{2n} \frac{\varphi}{\sqrt{6\alpha}}$ in terms of canonically normalized field [84, 125, 254]. In Ref. [245] the power law potentials were generalized to the following form of power series

$$f^2 \left(\frac{\phi}{\sqrt{6\alpha}} \right) = \sum_n c_n \phi^n, \quad (5.16)$$

where c_n are non-zero constants and it was argued to be $c_0 \ll 1$. In this class of potentials the inflaton slow-rolls towards the potential minimum⁶ which is located at $\phi = 0$.

In the subsequent sections, with the assumed GS parametrization, we will show that non-slow-roll inflation occurs to be near the pole of the kinetic term i.e., $|\phi| \rightarrow \sqrt{6\alpha}$. Therefore, we can observe from (5.15) that the local shape of the potential in the non-slow-roll approach is different from the power-law (or) T-models and also the power series form given in (5.16). In this regard, our study about the non-slow-roll approach widens the scope for different shapes of inflationary potentials in α -attractors.

Subsequently, for the conventional parameters general definitions⁷

$$\varepsilon = \frac{H'}{H}, \quad \eta = -\frac{\varepsilon'}{\varepsilon}, \quad (5.17)$$

substituting the Hubble parameter from (5.14) and demanding the end of inflation $\varepsilon = 1$ at $N = 0$ we get

$$\alpha = \frac{n^2}{3\sqrt{2}\beta n + 6}. \quad (5.18)$$

Consequently, constraining the parameter space (n, β) automatically gives the values of α . In the next sections we show that the β parameter determines the value of scalar spectral index n_s , whereas as the parameter n , which indicates the value of inflaton field at the end of inflation, regulates the tensor to scalar ratio r . From (5.12), (5.15) and (5.18), we can say that the local shape of the potential, the inflaton dynamics and the parameter α are interconnected. In other words, identifying α as the curvature of Kähler geometry given by (5.2), we can establish a web of relations,

⁶It has been studied in the Ref. [262] that the slow-roll inflation in T-models can be interrupted abruptly in some cases of matter couplings to inflaton field.

⁷The sign difference in the definition of parameters ε, η is due to N which is counted backward in time from the end of inflation (see (5.17)).

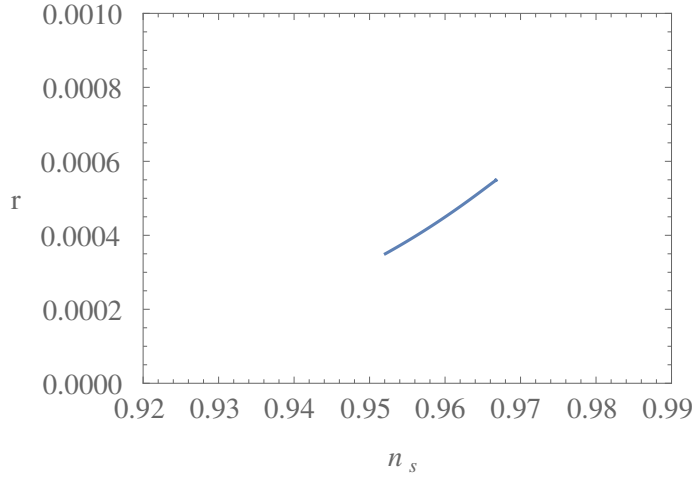
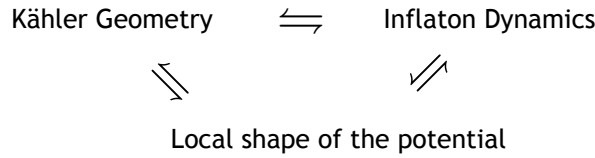


Figure 5.1: Parametric plot of spectral index (n_s) versus tensor scalar ratio (r). We have considered 60 number of efoldings with $n = 1$, $-0.03 < \beta < -0.001$ (or equivalently $0.166 \lesssim \alpha \lesssim 0.17$).



From the above schematic diagram we can decipher that the class of potentials which are obtained by allowing different values for (n, β) is related to the family of Kähler geometries, which determine the dynamics of inflaton during inflation. In the next section, we derive the scalar and tensor power spectrum for this model.

5.3 Inflationary predictions for $n = 1$

In this section, we study the inflationary predictions of the model taking $n = 1$. We constrain the parameter β to obtain the predictions of (n_s, r) within current observational range.

Imposing the spectral index $n_s = 0.968 \pm 0.006$, we obtain the constraint $|\beta| \sim \mathcal{O}(10^{-3})$ (or equivalently, from (5.18), $\alpha \sim \mathcal{O}(10^{-1})$). However, we verify that the inflaton dynamics for the case $\beta > 0$ violates the requirement that $\phi^2 < 6\alpha$. Therefore, we only consider the case with $\beta < 0$ as a viable inflationary paradigm complying with $\phi^2 < 6\alpha$ during inflation. In this case, we find that inflation occurs while approaching asymptotically the kinetic term pole at $|\phi| \rightarrow \sqrt{6\alpha}$. The predictions of (n_s, r) are depicted in the Fig. 5.1.

The left panel of Fig. 5.2 depict the shape of the potential during which inflation is happening in the non-slow-roll context. In the right panel of Fig. 5.2, we plot the parameter ε versus N for a particular value of α corresponds to $n = 1$.

In addition, we compute the energy scale of inflation and mass of the inflaton (m_ϕ^2) by computing the $V_*^{1/4}$ and the $\partial_\phi^2 V_*$ where V_* is the the potential evaluated at horizon exit. In this context, the shape of the potential during inflation is given by (5.15), consequently we obtain,

$$f_*^{1/2} \sim 1.2 \times 10^{17} \text{ GeV} \quad , \quad m_\phi^2 < 0. \quad (5.19)$$

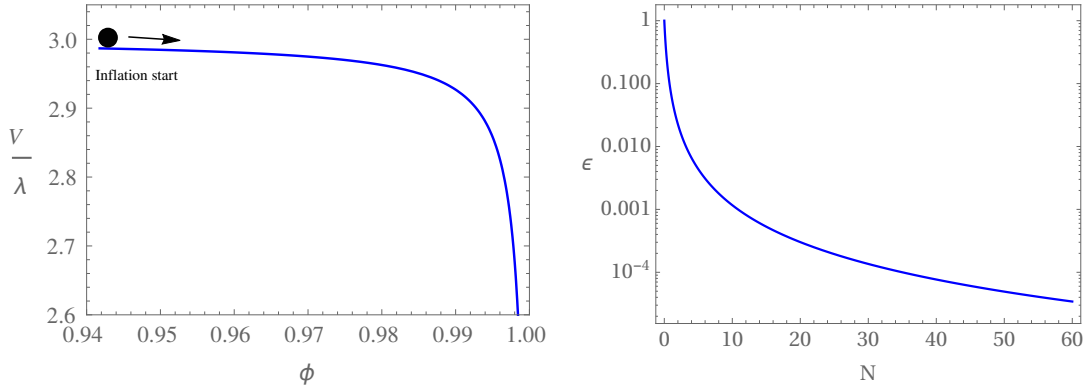


Figure 5.2: The left panel is the graphical presentation of the local shape of the potential versus scalar field during inflation. The right panel depicts the parameter ϵ versus N . We have taken $\beta = -0.001$ (or equivalently $\alpha = 0.167$) for both plots.

Therefore, since the energy scale of inflation appears to be greater than GUT scale but still below Planck scale, this naturally justify the embedding of this model in SUGRA. Since the mass squared of the inflaton is negative, inflation is driven by a tachyonic field.

5.4 Non-slow-roll α -attractor

In Sec. 5.3, we have studied non-slow-roll inflation with GS parametrization and $n = 1$, in this case we obtained $r \sim \mathcal{O}(10^{-4})$. The objective, at this point, is to assess inflationary scenarios with any value of $r < 0.09$, by allowing $n \neq 1$ in (5.18).

5.4.1 Conditions for small field and large field inflation

In this section, we study the parameter space of the model allowing the inflaton to do large and small field excursions during inflation. We address the possibility of large and small field inflation in the context of non-slow-roll dynamics in α -attractors.

Using the parametrization from (5.12) the field excursion during the period of inflation is given by

$$\Delta\phi = n(1 - \exp(60\beta)) . \quad (5.20)$$

The above relation allows us to identify the parameter space of (n, β) to explicit the region of large field ($\Delta\phi > 1$) and small field ($\Delta\phi < 1$) inflation (see Fig. 5.3). We further constrain the parameter space, by imposing $0.962 < n_s < 0.974$ which is the 95% CL region given by *Planck* 2015. This constraint on spectral index confine $-0.001 < \beta < -0.01$, and precisely $\beta \sim -0.002$ corresponds to the central value of $n_s \sim 0.967$.

The relation between tensor to scalar ratio and field excursion during the period of inflation is defined by the Lyth bound [39] which is

$$\Delta\phi > \sqrt{\frac{r}{8}} \left(\frac{N}{60} \right) . \quad (5.21)$$

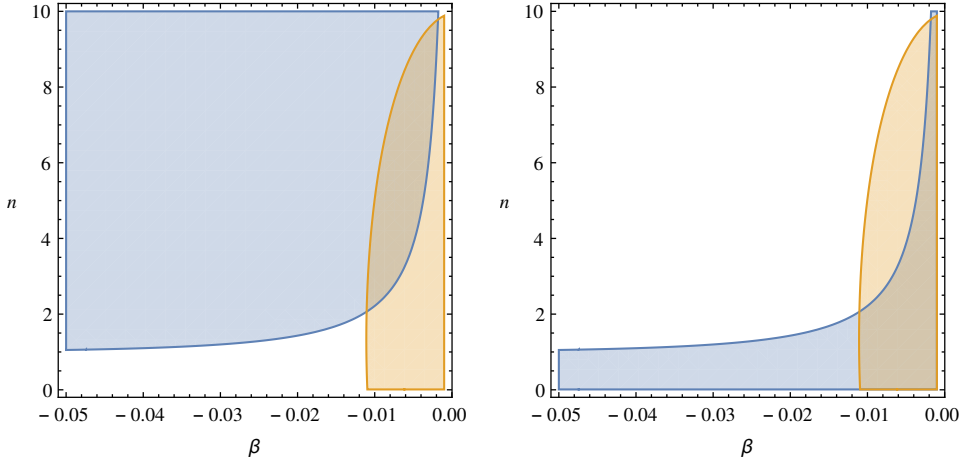


Figure 5.3: In both plots orange shaded region corresponds to the constraint $0.962 < n_s < 0.974$. The blue shaded region in the left panel is for large field $\Delta\phi > 1$ whereas in the right panel is for small field $\Delta\phi < 1$. We have considered $N = 60$.

We can see from the above relation that $r > 0.002$ implies $\Delta\phi > M_{\text{Pl}}$, i.e, large field inflation. However, this bound gets modified for the k -inflationary models [115]. In this case, the generalization of (5.21) is given by

$$\Delta\phi > \int_0^{N_e} \sqrt{\frac{r}{8} \frac{1}{c_s P_{,X}}} dN. \quad (5.22)$$

where the sound speed $c_s = 1$ in the case of α -attractor model. In (5.22) the term $P_{,X} = \left(1 - \frac{\phi^2}{6\alpha}\right)^{-2}$ affects Lyth bound depending on the value of the parameter α . From (5.18) we know that the α parameter is directly related to the inflaton dynamics. In Fig. 5.3, we depict the parameter space for large and small field inflation overlapped on the region where $0.962 < n_s < 0.974$. Here, we explicitly characterize the possibility of super planckian excursion of the field ϕ attributing to the field value at the end of inflation $n \gtrsim 2$ and the parameter $\beta \sim -0.01$ (see left panel of Fig. 5.3). The field ϕ is sub planckian for $0 < n < \mathcal{O}(10)$ and the parameter $\beta \sim -0.002$ (see right panel of Fig. 5.3). We present the corresponding predictions in Fig. 5.4, where we found that the large field inflation in the non-slow-roll context can give rise to the tensor to scalar ratio $0.003 \lesssim r < 0.09$ and the spectral index $0.955 \lesssim n_s \lesssim 0.964$. Whereas in the case of small field we obtain $0 \lesssim r < 0.09$ and the spectral index $0.96 \lesssim n_s \lesssim 0.967$.

The parametrization used in (5.12) leads to an attractor starting at $r \sim 5.5 \times 10^{-4}$ which is the prediction for $n = 1$. We find that $r \rightarrow 0$ as $n \rightarrow 0$ (or equivalently $\alpha \rightarrow 0$). We depict this behavior in Fig. 5.5. This attractor behaviour resembles with the recently studied E-models [263]. The most interesting feature of our study is that, even with non-slow-roll dynamics of the inflaton, α -attractors still appear to be the most promising models in the (n_s, r) plane. Including the higher order corrections in (A.24) and (A.34) we have undetectably small deviation from the standard consistency relation $r = -8n_t$ as presented in the right panel of Fig. 5.4. However, the validity of the standard consistency relation remains an open question and not even expected to be tested in any future CMB observations [30].

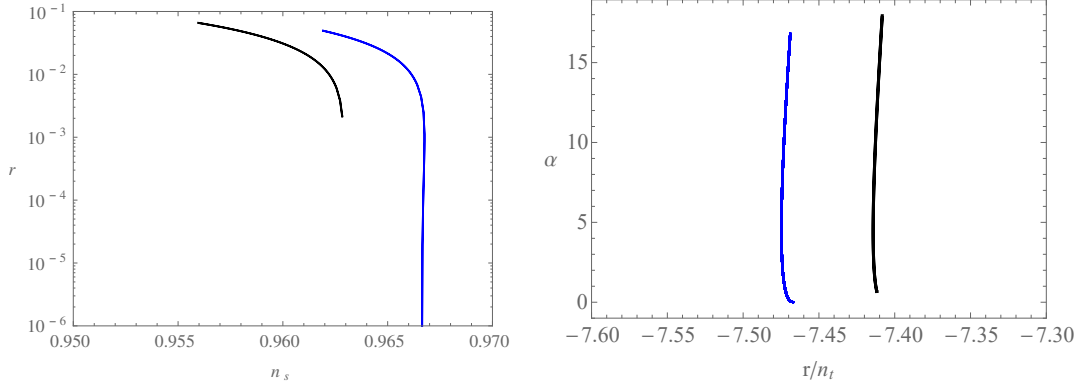


Figure 5.4: Parametric plots of spectral index (n_s) versus tensor scalar ratio (r) (left panel), α versus the ratio of tensor scalar ratio and tensor tilt (right panel). In these plots the blue line denote predictions for small field inflation for which we take $\beta \sim -0.002$ and $0 < n < 10$. In this case $r \rightarrow 0$ as $n \rightarrow 0$ (equivalently $\alpha \rightarrow 0$). The black line denote predictions for large field inflation for which $\beta \sim -0.01$ and $2 < n < 10$. In this case $r \gtrsim \mathcal{O}(10^{-3})$. We have considered $N = 60$.

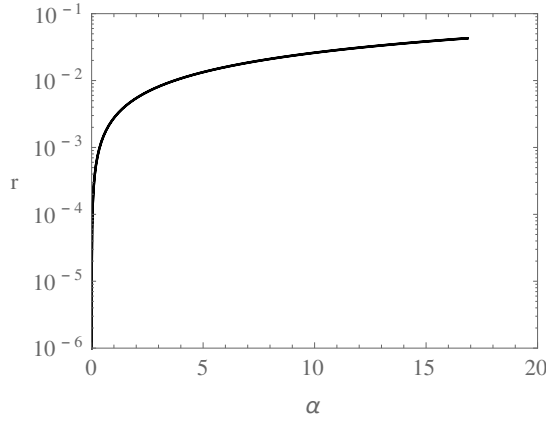


Figure 5.5: Plot of tensor scalar ratio (r) versus α . Here we have taken $\beta \sim -0.002$ and $0 < n < 10$. This plot is for $N = 60$.

5.5 Embedding in $\mathcal{N} = 1$ SUGRA

In this section, we revise the embedding of α -attractor within $\mathcal{N} = 1$ SUGRA [84] and verify the stability of inflaton trajectory [250, 251] in the context of non-slow-roll dynamics.

The α -attractor model can be embedded in SUGRA using 3 chiral multiplets: a conformon X^0 , an inflaton $X^1 = \Phi = \frac{\phi + i\sigma}{\sqrt{2}}$ and a sGoldstino $X^2 = S$. In order to extract a Poincaré SUGRA conformon is gauge fixed as $X^0 = \overline{X^0} = \sqrt{3}$. We write the Kähler and superpotential in the similar way as studied in Refs. [84, 254],

$$\mathcal{K} = -3\alpha \log \left(1 - Z\overline{Z} - \frac{S\overline{S}}{3\alpha} + \frac{g}{3\alpha^2} \frac{(S\overline{S})^2}{(1 - Z\overline{Z})} - \frac{\gamma}{3\alpha^2} \frac{S\overline{S}(Z - \overline{Z})^2}{(1 - Z\overline{Z})^2} \right), \quad (5.23)$$

$$W = Sf(Z)(1 - Z^2)^{(3\alpha-1)/2}, \quad (5.24)$$

where $Z = \frac{X^1}{X^0} = \frac{\Phi}{\sqrt{6\alpha}}$ and $f(Z)$ is an arbitrary function and the square of which serves as the

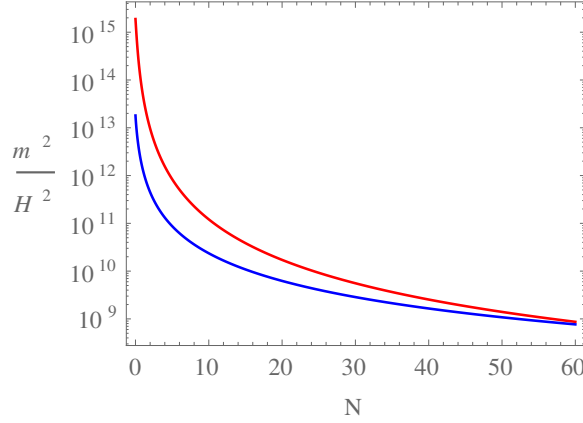


Figure 5.6: In this figure we depict the ratio of the square of masses to the square of Hubble parameter H^2 . The red line indicates for $\text{Im}\Phi$ and the blue line is for S . We have taken $n = 1$, $\alpha = 0.167$, $g = 0.5$ and $\gamma = 0.2$.

inflaton potential along $S = \text{Im}\Phi = 0$. In the Kähler potential in (5.23) we added an extra term $\frac{S\bar{S}(Z-\bar{Z})^2}{(1-Z\bar{Z})^2}$ in order to stabilize the inflaton trajectory in the direction of $\text{Im}\Phi$ for any value of α . Although in some cases it is not required to add this extra term [254, 263]. In our case, we only focus our attention to the form of Kähler potential given by (5.23).

The mass squares of S and $\text{Im}\Phi$ for a given Kähler potential are given by [251],

$$\begin{aligned} m_\sigma^2 &= 2(1 - \mathcal{K}_{\Phi\bar{\Phi}S\bar{S}})f^2 + (\partial_\Phi f)^2 - f\partial_\Phi^2 f, \\ m_s^2 &= -\mathcal{K}_{S\bar{S}S\bar{S}}f^2 + (\partial_\Phi f)^2, \end{aligned} \quad (5.25)$$

where all the terms in (5.25) are to be evaluated along the inflaton trajectory $S = \text{Im}\Phi = 0$. And here $\mathcal{K}_{a\bar{b}c\bar{d}} = \partial_a \partial_{\bar{b}} \partial_c \partial_{\bar{d}} \mathcal{K}$. For the stability of the inflaton trajectory it is required to have $m_\sigma^2, m_s^2 \gg H^2$ during inflation, in order to ensure the absence of isocurvature perturbations and therefore to have inflation solely driven by a single field [251].

For the Kähler potential given by (5.23) we obtain

$$\mathcal{K}_{\Phi\bar{\Phi}S\bar{S}} = -\frac{36\alpha^2(6(\alpha - 2\gamma) + \phi^2)}{(\phi^2 - 6\alpha)^3}, \quad \mathcal{K}_{S\bar{S}S\bar{S}} = \frac{24\alpha(1 - 6g)}{(\phi^2 - 6\alpha)^2}. \quad (5.26)$$

Evaluating the masses m_σ^2 and m_s^2 for the local shape of inflaton potential given by (5.15) for $n = 1$, we obtain $m_s^2, m_\sigma^2 \gg H^2$ for $g, \gamma \geq 0.2$ and for $\alpha \sim 0.17$. For example, in Fig. 5.6, we depict the ratio of inflaton mass square to Hubble parameter square during inflation for a chosen values of (g, γ) .

We can similarly verify the stability of the inflaton trajectory for $n \neq 1$ by appropriate choice of free parameters (g, γ) .

5.6 Summary

In this chapter we have considered the α -attractor models from a new perspective, more precisely, employing the framework of non-slow-roll approach in the way it was recently proposed

by Gong and Sasaki [127]. We found that the α -attractor models are quite compatible in the (n_s, r) plane of *Planck* 2015 within non-slow-roll inflaton dynamics. We showed that such a particular inflationary scenario predicts an attractor at $n_s \approx 0.967$ and $r \approx 5.5 \times 10^{-4}$. We further found that the model can in principle predict any $r < 0.09$. In addition, we have extracted relation (5.18) between the α - parameter, to the curvature of Kähler geometry, and to the inflaton dynamics. In other words, in our model, the curvature of the Kähler geometry defines the local shape of the inflaton potential during inflation. This constitutes an interesting phenomenon which might be useful to understand the pre-inflationary physics. Furthermore, we also studied the possibility of large and small field inflation in the non-slow-roll context and contrasted them in terms of the predictions of the tensor to scalar ratio.

6

Conformal GUT inflation

*The history of the Universe is co-determined by the basic mathematical law of beauty
and an unimaginably long sequence of accidents*

- Murray Gell-Mann

Since the inflationary scale is in general expected to be $\sim 10^{16}$ GeV, it is natural to consider the inflaton to be a scalar field associated with grand unified theory (GUT) groups, such as $SU(5)$ and $SO(10)$. Shafi-Vilenkin (SV) model [89] is one of the first realistic model of inflation which was based on $SU(5)$ GUT [264]. In this framework, inflation is a result of the spontaneous breaking of $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ by a GUT field (**24-plet** adjoint Higgs) and a inflaton, which is a $SU(5)$ singlet that rolls down to a vacuum expectation value (VEV). The success of the SV model is that it can lead to a successful baryogenesis after inflation and predicts proton life time above the current lower bound [128, 265]. In this model, the scalar field potential is of a Coleman-Weinberg (CW) form, according to which primordial gravitational waves are constrained by $0.02 \leq r \leq 0.1$ [266]. Although the SV model is well within the current bounds of *Planck* 2015, several extensions of this model were studied to get smaller values of tensor to scalar ratio. In [267-269], CW inflation was studied in the context of induced gravity, non-minimal coupling and brane-world scenario, where the tensor to scalar ratio was obtained to be $r \sim \mathcal{O}(10^{-2}) - \mathcal{O}(10^{-3})$. After all these modifications necessarily introduce an additional parameter into the theory that is responsible for the flatness of the potential.

Moreover, extensions of the SV model within particle physics offer rich physics beyond the SM. Therefore, the SV model is embedded in a higher gauge group as $SO(10)$, which can be broken to SM via an intermediate group $G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$ [95, 270]. Obtaining successful inflation in $SO(10)$, is more realistic with additional benefits to explain physics beyond SM, such as neutrino physics, matter anti-matter asymmetry through non-thermal leptogenesis, monopoles and dark matter (DM) [128]. For example, Ref. [271] considered a complex singlet scalar being coupled to RHNs followed by implementing type I seesaw mechanism. This approach unified inflation with Majorana DM together with the scheme of generating neutrino masses. In [272] an additional $U(1)_{B-L}$ symmetry was considered in the SM i.e., $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, where¹ $B - L$ symmetry can be spontaneously broken when the scalar field takes the VEV. In this setup, we can explain baryon asymmetry of the Universe through non-thermal leptogenesis [95, 273-275]. Recently, CW inflation was studied in an extension with $SO(10)$ and E_6 groups, pointing out the possibilities of observing primordial monopoles [276].

The main goal of this chapter is to generalize the SV model in order to achieve $r \sim \mathcal{O}(10^{-3})$ without introducing any additional parameters for inflaton potential flatness². Instead, we consider an additional conformal invariance (or local scale invariance) in our GUT model. It was long ago shown by Wetterich [278] that scale symmetries play a crucial role in the construction of

¹Here B, L stands for Baryon number and Lepton number respectively.

²Our construction is different from the models with non-minimally coupled scalars where a flat potential comes from requiring $\xi \gg 1$ [277].

realistic cosmological models based on particle physics. Moreover, scale symmetries successfully explain the hierarchy of different scales such as the Planck and Higgs mass [279-282]. Therefore, it is natural to consider scale invariance in constructing an inflationary scenario, through which we can obtain dynamical generation of the Planck mass, inflationary scale and particle physics scales beyond SM. In this regard, we introduce two complex singlet fields (\bar{X}, Φ) of $SU(5)$ or $SO(10)$ and couple them to Ricci scalar and adjoint Higgs field (Σ) such that the total action would be conformally invariant. We promote inflation as a result of spontaneous breaking of conformal and GUT symmetries. The former occurs due to gauge fixing of one singlet field to a constant for all spacetime and the latter occurs due to Σ field takes its GUT VEV. Here the inflaton is identified with the real part of the second singlet ($\phi = \sqrt{2}\Re[\Phi]$), whereas the imaginary part is the corresponding Nambu-Goldstone boson, is assumed to pick up a mass due to the presence of small explicit soft lepton number violation terms in the scalar potential [271]. Here, we assume Φ carries two units of lepton number and coupled to the right handed neutrinos (RHNs) in such a way that the coupling is highly suppressed during inflation³. Near the end of inflation, the inflaton is supposed to reach its VEV and also the global lepton number is violated. Thereafter, we study the dominant decay of inflaton into heavy RHNs producing non-thermal leptogenesis. We compute the corresponding reheating temperature and also discuss the issue of producing observed baryon asymmetry. We provide an observationally viable inflationary scenario, predicting proton life time, neutrino masses and producing non-thermal leptogenesis from heavy RHNs.

The chapter is briefly organized as follows. In Sec. 6.1, we describe toy models with conformal and scale invariance. We identify the interesting aspects of spontaneous symmetry breaking leading to viable inflationary scenario. In Sec. 6.2, we briefly present the SV model and the computation of proton life time. In Sec. 6.3 we propose our generalization of SV model by introducing an additional conformal symmetry. We report the inflationary predictions of the model together with estimates of proton life time. In Sec. 6.4 we later explore the nature of inflaton couplings to the SM Higgs, singlet RHNs through type I seesaw mechanism. We constrain the Yukawa couplings of the inflaton field compatible with the generation of light neutrino masses. In Sec. 6.5 we implement non-thermal leptogenesis and compute the reheating temperatures corresponding to the dominant decay of inflaton to heavy RHNs. We additionally comment on the necessary requirements for the production of observed baryon asymmetry through CP violation decays of RHNs. In Sec. 6.6 we summarize our results pointing future steps.

6.1 Conformal vs Scale invariance

Models with global and local scale invariance (Weyl invariance (or) conformal invariance) are often very useful to address the issue of hierarchies in both particle physics and cosmology [279-281, 283-285]. Models with these symmetries contains no mass input mass parameters. The spontaneous breaking of those symmetries induced by the VEV's of the scalar fields present in the theory, generates a hierarchy of mass scales e.g., Planck mass, GUT scale and neutrino masses⁴. Moreover, it is a generic feature that scale or conformal symmetry breaking induce a

³This will be explained in detail in the due course of this chapter.

⁴For example, single scalar field models with the spontaneously broken scale invariance due to the 1-loop corrections to the tree level potential were studied in [286-288]. In [289] two field model with the spontaneously broken scale invariance was studied to generate hierarchy of mass scales and the dynamical

flat direction in the scalar field potential [278] which makes these models even more interesting in the context of inflation. Another motivation to consider scale invariance for inflationary model building comes from CMB power spectra which is found to be nearly scale invariant [24]. In this section, we discuss firstly a toy model (with two fields) that is (global) scale invariant and present the generic form of (scale invariant) potentials and their properties. We review the presence of massless Goldstone boson that appears as a result of spontaneous breaking of global scale invariance. In the following, we discuss the two field conformally invariant model, in which case the presence of a massless Goldstone boson can be removed by appropriate gauge fixing. The resultant Spontaneous Breaking of Conformal Symmetry (SBCS) turns to be very useful to obtain a Starobinsky like inflation⁵. We will later explore the role of SBCS in a more realistic inflationary setting based on GUTs.

6.1.1 Scale invariance

Here we discuss a toy model with two scalar fields (in view of Refs. [278, 289, 292, 293]) and point out interesting features that we later utilize in our construction.

A generic two field global scale invariant action can be written as

$$S_{global} = \int d^4x \sqrt{-g} \left[\frac{\alpha}{12} \phi^2 R + \frac{\beta}{12} \chi^2 R - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} \partial^\mu \chi \partial_\mu \chi - \phi^4 f(\rho) \right], \quad (6.1)$$

where α, β are constants and $\rho = \frac{\phi}{\chi}$, the generic function $f\left(\frac{\phi}{\chi}\right)$ here can be treated as quartic self coupling of the field ϕ [278, 293]. The action (6.1) is scale invariant, i.e., invariant under global scale transformations $g_{\mu\nu} \rightarrow e^{-2\lambda} g_{\mu\nu}$, $\phi \rightarrow e^\lambda \phi$, $\chi \rightarrow e^\lambda \chi$ for any constant λ (dilatation symmetry).

Since the potential $V(\phi, \chi) = \phi^4 f(\rho)$ is homogeneous, it must satisfy the following constraint [289, 293]

$$\phi \frac{\partial V}{\partial \phi} + \chi \frac{\partial V}{\partial \chi} = 4V. \quad (6.2)$$

The extremum conditions for V , i.e., $\partial_\phi V = \partial_\chi V = 0$ can also be written as $f(\rho) = f'(\rho) = 0$. One of the conditions fix the ratio of VEV's of fields, while the other gives a relation between couplings (if $\langle \phi \rangle \neq 0$ and $\langle \chi \rangle \neq 0$). The most important and crucial point here is that if $\langle \phi \rangle \propto \langle \chi \rangle$ there exists a flat direction for the field ϕ (see [278] for detailed analysis). This will be more clearer in the due course of this chapter, when we show this property turns out to be maintained and more useful in the context of local scale invariant model.

Lets consider a scale invariant potential of the form

$$V_1 = \frac{\lambda_\phi}{4} \phi^4 + \frac{\lambda_m}{2} \phi^2 \chi^2 + \frac{\lambda_\chi}{4} \chi^4, \quad (6.3)$$

generation of the Planck mass from the VEV's of the scalar fields. Recently in [290], some constraints were derived on these models from Big Bang Nucleosynthesis (BBN).

⁵Toy models of conformal inflation were studied in [242, 243] and were embedded in $\mathcal{N} = 1$ SUGRA. Furthermore, in a recent study conformal models were shown to be motivated in the context of string field theory [291].

where the couplings can in general depend on the ratio of two fields i.e., ϕ/χ . If for example, we assume the couplings are independent of the ratio of two fields and consider the spontaneous breaking of scale symmetry i.e., the case with $\langle\phi\rangle \neq 0$, $\langle\chi\rangle \neq 0$, thus, as a result of minimizing the potential, we arrive at [293]

$$\frac{\langle\phi\rangle}{\langle\chi\rangle} = -\frac{\lambda_m}{\lambda_\phi} \quad , \quad V = \frac{\lambda_\chi}{4} \left(\chi^2 + \frac{\lambda_m}{\lambda_\chi} \phi^2 \right)^2 \quad , \quad (6.4)$$

with $\lambda_m^2 = \lambda_\phi \lambda_\chi$ and $\lambda_m < 0$.

In (6.4) we can re-define the coupling as

$$\bar{\lambda}_\chi = \lambda_\chi \left(1 + \frac{\lambda_m}{\lambda_\chi} \frac{\phi^2}{\chi^2} \right)^2 \quad , \quad (6.5)$$

then the potential (6.4) looks like a simple quartic potential

$$V_1 = \frac{\bar{\lambda}_\chi}{4} \chi^4 \quad . \quad (6.6)$$

We can also alternatively have the potential of the form

$$V_2 = \frac{\tilde{\lambda}_\phi}{4} \phi^4 \quad , \quad \tilde{\lambda}_\phi = \lambda_\phi \left(1 - \frac{\phi^2}{\chi^2} \right)^2 \quad , \quad (6.7)$$

which also satisfies the constraint (6.2) and is slightly different from (6.3). We will later see that the form of potential in (6.7) gives viable inflationary scenario. From (6.4) -(6.7) we can crucially learn that how to define couplings as a function of ratio of two fields in a scale invariant model. Of course, we only considered here a simple toy model. However, we note that such field dependent couplings can be expected to arise in string theory and were applied in the context of early Universe [294].

The spontaneous breaking of scale symmetry occurs when one of the fields develops a VEV (let us take the field χ). This leads to an emergence of a corresponding massless Goldstone boson (dilaton) defined by $\tilde{\chi} = \sqrt{6}M \ln \left(\frac{\chi}{\sqrt{6}M} \right)$ with an arbitrary mass scale $M \propto m_P$ [278]. By performing a Weyl rescaling of the metric $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \left(\frac{\chi}{\sqrt{6}M} \right)^2 g_{\mu\nu}$ and $\phi \rightarrow \tilde{\phi} = \frac{M}{\sqrt{6}\chi} \phi$ we indeed observe that the field $\tilde{\chi}$ is massless since the potential becomes independent of the field $\tilde{\chi}$

$$V(\phi, \chi) = \phi^4 f \left(\frac{\phi}{\chi} \right) = \tilde{\phi}^4 f \left(\frac{\tilde{\phi}}{M} \right) \quad . \quad (6.8)$$

Although interesting cosmology and particle physics can be developed based on the scale invariant models, we need to constrain the implications of the massless dilaton present in the system [282]. It was shown that the dilaton can be gauged away if we consider a model with local scale symmetry [295].

6.1.2 Conformal invariance

A general action that is invariant under local scale transformations $g_{\mu\nu} \rightarrow \Omega^{-2}(x) g_{\mu\nu}$, $\phi \rightarrow \Omega(x)\phi$, $\chi \rightarrow \Omega(x)\chi$ can be written as

$$S_{local} = \int d^4x \sqrt{-g} \left[\frac{(\chi^2 - \phi^2)}{12} R + \frac{1}{2} \partial^\mu \chi \partial_\mu \chi - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \phi^4 f\left(\frac{\phi}{\chi}\right) \right], \quad (6.9)$$

where the potential in the above action should also satisfy the condition (6.2).

From the above action we can define an effective Planck mass $m_{eff}^2 = \frac{\chi^2 - \phi^2}{6}$ which evolves with time. In these theories, we would recover the standard Planck scale m_P when the fields reach their VEV. Note that the field χ contains a wrong sign for kinetic term but it is not a problem as we can gauge fix the field at $\chi = \text{constant} = \sqrt{6}M$ for all spacetime where $M \sim \mathcal{O}(m_P)$. This particular gauge choice is called *c-gauge*⁶ which spontaneously breaks the conformal symmetry. It was argued that the theories in this gauge are of interest especially in cosmological models based on particle physics [282]. In the inflationary models based on GUTs it natural that the field ϕ takes a non-zero VEV, i.e., $\langle \phi \rangle \neq 0$ in which case it is useful to assume $6M^2 - \langle \phi \rangle^2 = 6m_P^2$ in order to generate Planck mass. Moreover, it is also necessary to keep the evolution of the field $\phi \lesssim \sqrt{6}M$ in order to avoid an anti-gravity regime.

Considering $f\left(\frac{\phi}{\chi}\right) = \lambda \left(1 - \frac{\phi^2}{\chi^2}\right)^2$ in (6.9), SBCS via gauge fixing $\chi = \sqrt{6}m_P$ leads to the Einstein frame action in terms of a canonically normalized field $\phi = \sqrt{6}m_P \tanh\left(\frac{\varphi}{\sqrt{6}m_P}\right)$ and it is written as

$$S_{local} = \int d^4x \sqrt{-g} \left[\frac{m_P^2}{2} R - \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi - \lambda m_P^4 \tanh^4\left(\frac{\varphi}{\sqrt{6}m_P}\right) \right]. \quad (6.10)$$

We can see that the above action leads to a Starobinsky like inflation as the potential acquires a plateau when $\varphi \gg m_P$ (i.e., $\phi \rightarrow \sqrt{6}m_P$). In this case the inflaton rolls down to zero VEV by the end of inflation and consequently, because of the gauge fixing $\chi = \sqrt{6}m_P$, Einstein gravity is recovered.

In the next sections, we will study realistic GUT inflationary models where inflaton rolls down to non-zero VEV and sources interesting implications in particle physics sector.

6.2 Coleman-Weinberg GUT inflation

In this section, we briefly review the Shafi-Vilenkin model [89, 90]. It is one of the first realistic model of inflation which was based on SU(5) GUT. In this framework a new scalar field ϕ , a SU(5) singlet was considered and it weakly interacts with the GUT symmetry breaking field (adjoint) Σ and fundamental Higgs field H_5 . The tree level scalar potential is given by

⁶It was first realized in the SUGRA models [295] and shown to be useful to gain geodesic completeness of the theory.

$$V(\phi, \Sigma, H_5) = \frac{1}{4}a(\text{Tr}\Sigma^2)^2 + \frac{1}{2}b\text{Tr}\Sigma^4 - \alpha(H_5^\dagger H_5)\text{Tr}\Sigma^2 + \frac{\beta}{4}(H_5^\dagger H_5)^2 \\ + \gamma H_5^\dagger \Sigma^2 H_5 + \frac{\lambda_1}{4}\phi^4 - \frac{\lambda_2}{2}\phi^2\text{Tr}\Sigma^2 + \frac{\lambda_3}{2}\phi^2 H_5^\dagger H_5. \quad (6.11)$$

where the coefficients a, b, α and β are taken to be of the order of g^2 , therefore the radiative corrections in (Σ, H_5) sector can be neglected. The coefficient γ takes a relatively smaller value and $0 < \lambda_i \ll g^2$ and $\lambda_1 \lesssim \max(\lambda_2^2, \lambda_3^2)$.

The GUT field Σ which is a 5×5 matrix can diagonalized as

$$\Sigma_i^j = \delta_i^j \sigma_i \\ \sum_{i=1}^5 \sigma_i = 0. \quad (6.12)$$

where $i, j = 1, \dots, 5$.

Various symmetry breaking patterns of $SU(5)$ were studied in [296], among which the one with $SU(5)$ symmetry is broken to $SU(3)_c \times SU(2)_L \times U(1)_Y$ corresponds to

$$\langle \Sigma \rangle = \sqrt{\frac{1}{15}} \sigma \cdot \text{diag} \left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2} \right), \quad (6.13)$$

where σ is scalar field that emerges from spontaneous breaking of $SU(5)$. Substituting it in (6.11) the equations of motion for the σ field reads as

$$\square \sigma + \frac{\lambda_c}{4} \sigma^3 - \frac{\lambda_2}{2} \sigma \phi^2 = 0, \quad (6.14)$$

where $\lambda_c = a + \frac{7}{15}b$. Taking $\lambda_2 \ll \lambda_c$, the σ field quickly evolves to its local minimum of the potential given by

$$\sigma^2 = \frac{2\lambda_2}{\lambda_c} \phi^2, \quad (6.15)$$

Adding the radiative corrections due to the couplings $-\frac{\lambda_2}{2}\phi^2\text{Tr}\Sigma^2$ and $\frac{\lambda_3}{2}\phi^2 H_5^\dagger H_5$, the effective potential of ϕ gets to the CW form given by [89, 90]

$$V_{eff}(\phi) = A\phi^4 \left[\ln \left(\frac{\phi}{\mu} \right) + C \right] + V_0, \quad (6.16)$$

where

$$A = \frac{\lambda_2^2}{16\pi^2} \left(1 + \frac{25}{16} \frac{g^4}{\lambda_c^2} + \frac{14}{9} \frac{b^2}{\lambda_c} \right). \quad (6.17)$$

The (ϕ, σ) sector of effective potential is given by

⁷The field Σ interacts with vector boson X with a coupling constant g .

$$V_{eff} = \frac{\lambda_c}{16}\sigma^4 - \frac{\lambda_2}{4}\sigma^2\phi^2 + A\phi^4 \left[\ln\left(\frac{\phi}{\mu}\right) + C \right] + V_0. \quad (6.18)$$

and $\mu = \langle \phi \rangle$ denotes the VEV of ϕ at the minimum. $V_0 = \frac{A\mu^4}{4}$ is the vacuum energy density i.e., $V(\phi = 0)$. C is a constant which we can chose such that $V(\phi = \mu) = 0$. Therefore, the effective potential (6.18) can be written as

$$V_{eff} = A\phi^4 \left[\ln\left(\frac{\phi}{\mu}\right) - \frac{1}{4} \right] + \frac{A\mu^4}{4}. \quad (6.19)$$

Following (6.15) the GUT field σ reaches its global minimum only when the inflaton field reach its VEV by the end of inflation. The inflationary predictions of this model were reported in detail in [128, 265]. This model was shown to be in good agreement with spectral index $n_s = 0.96 - 0.967$ and the tensor to scalar ratio $0.02 \leq r \leq 0.1$, which is well consistent with the *Planck* 2015 data [24, 266].

From the VEV of the singlet field ϕ we can compute the masses of superheavy gauge bosons as

$$M_X = \sqrt{\frac{5\lambda_2 g^2}{3\lambda_c A^{1/2}}} V_0^{1/4}. \quad (6.20)$$

Taking $A \sim \frac{\lambda_2^2}{16\pi^2}$ the mass of gauge bosons are approximately 2-4 times larger than the scale of vacuum energy ($V_0^{1/4}$). The key prediction of GUT models is proton decay ($p \rightarrow \pi^0 + e^+$) mediated by X, Y gauge bosons. The life time of proton can be computed using

$$\tau_p = \frac{M_X^4}{\alpha_G^2 m_{pr}^5}, \quad (6.21)$$

where m_{pr} is proton mass and $\alpha_G \sim 1/40$ is the GUT coupling constant. The current lower bound on proton life time is given by $\tau_p > 1.6 \times 10^{34}$ years indicates $M_X \sim 4 \times 10^{15}$ GeV [297, 298].

6.3 GUT inflation with conformal symmetry

As discussed in Sec. 6.1, conformal symmetry is useful to generate flat potentials and the hierarchy of mass scales. Therefore, embedding conformal symmetry in GUT inflation is more realistic and helpful to generate simultaneously a Planck scale m_P along with the mass scale of X Bosons $M_X \sim 10^{15}$ GeV that sources proton decay. In this section, we extend the previously discussed CW inflation by means of introducing conformal symmetry in SU(5) GUT theory. We then obtain an interesting model of inflation by implementing spontaneous breaking of conformal symmetry together with GUT symmetry⁸. We start with two complex singlet fields⁹ of SU(5) (Φ, \bar{X}) where the real part of Φ ($\phi = \sqrt{2}\Re[\Phi]$) is identified as inflaton. Gauge fixing the field \bar{X} causes SBCS

⁸We note that conformal symmetry was considered in GUT inflation [299-301] but in those models inflaton was fundamental Higgs field of SU(5) whereas in our case inflaton is GUT singlet weakly coupled to fundamental Higgs.

⁹Complex singlet is required to implement type I mechanism which we later explain in Sec. 6.4.

as discussed in Sec. 6.1. It is worth to note that the same framework we study here based on $SU(5)$ GUT can be easily realized in the $SO(10)$ GUT. Therefore, the two complex singlets of $SU(5)$ considered here are also singlets of $SO(10)$ [95, 128].

The conformally invariant action with complex $SU(5)$ singlet fields (Φ, \bar{X}) can be written as

$$S_G = \int d^4x \sqrt{-g} \left[(|\bar{X}|^2 - |\Phi|^2 - \text{Tr}\Sigma^2) \frac{R}{12} - \frac{1}{2} (\partial\Phi)^\dagger (\partial\Phi) + \frac{1}{2} (\partial\bar{X})^\dagger (\partial\bar{X}) - \frac{1}{2} \text{Tr} \left[(D^\mu \Sigma)^\dagger (D_\mu \Sigma) \right] - \frac{1}{4} \text{Tr} (F_{\mu\nu} F^{\mu\nu}) - V(\Phi, \bar{X}, \Sigma) \right], \quad (6.22)$$

where $D_\mu \Sigma = \partial_\mu \Sigma - ig[A_\mu, \Sigma]$, A_μ are the 24 massless Yangmills fields with Field strength defined by $F_{\mu\nu} \equiv \nabla_{[\mu} A_{\nu]} - ig[A_\mu, A_\nu]$. Here we assume the Higgs field H_5 is not very relevant during inflation. We consider that the singlet field Φ is weakly coupled to the adjoint field Σ through the following tree level potential

$$V(\Phi, \bar{X}, \Sigma) = \frac{1}{4} a (\text{Tr}\Sigma^2)^2 + \frac{1}{2} b \text{Tr}\Sigma^4 - \frac{\lambda_2}{2} \Phi^2 \text{Tr}\Sigma^2 f\left(\frac{\Phi}{\bar{X}}\right) + \frac{\lambda_1}{4} \Phi^4 f^2\left(\frac{\Phi}{\bar{X}}\right), \quad (6.23)$$

where the coefficients $a \sim b \sim g^2$ (gauge couplings $g^2 \sim 0.3$). Following the discussion in section 6.1 we assume the coupling constants are field dependent, i.e., in (6.23) the coupling constants can be read as $\tilde{\lambda}_2 = \lambda_2 f\left(\frac{\Phi}{\bar{X}}\right)$, $\tilde{\lambda}_1 = \lambda_1 f^2\left(\frac{\Phi}{\bar{X}}\right)$ which depend on the ratio of fields (Φ, \bar{X}) . We consider

$$f\left(\frac{\Phi}{\bar{X}}\right) = \left(1 - \frac{|\Phi|^2}{|\bar{X}|^2}\right). \quad (6.24)$$

With the tree level potential in (6.23) the action (6.22) is conformally invariant under the following transformations

$$g_{\mu\nu} \rightarrow \Omega(x)^2 g_{\mu\nu}, \quad \bar{X} \rightarrow \Omega^{-1}(x) \bar{X}, \quad \Phi \rightarrow \Omega^{-1}(x) \Phi, \quad \Sigma \rightarrow \Omega^{-1}(x) \Sigma. \quad (6.25)$$

The SBCS occurs with gauge fixing $\bar{X} = \bar{X}^* = \sqrt{3}M$, where $M \sim \mathcal{O}(m_P)$. We assume inflation to happen in a direction $\text{Im}\Phi = 0$. Therefore, for the inflaton trajectory to be stable we require the mass of $\text{Im}\Phi$ to be¹⁰ $m_{\text{Im}\Phi}^2 \gg H_{inf}^2$. To arrange this, we can add a new term to the potential (6.23) as

$$V_S = V(\Phi, \bar{X}, \Sigma) + \frac{\lambda_{im}}{4} (\Phi - \Phi^\dagger)^2 (\Phi + \Phi^\dagger)^2, \quad (6.26)$$

such that the mass of the $\text{Im}\Phi$ in the inflationary direction $\text{Im}\Phi = 0$ is $m_{\text{Im}\Phi}^2 = \frac{\partial^2 V_S}{\partial (\text{Im}\Phi)^2} = \lambda_{im} (\Phi + \Phi^*)^2$. Therefore, If $\lambda_{im} \gg \lambda_{1,2}$ we can have $m_{\text{Im}\Phi}^2 \Big|_{\text{Im}\Phi=0} \gg H_{inf}^2$ during inflation. In this way, we can successfully obtain the stability of the inflaton trajectory during inflation [251]. Similarly to the SV model, here also we consider $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ by

¹⁰Where H_{inf} is the Hubble parameter during inflation.

$$\langle \Sigma \rangle = \sqrt{\frac{1}{15}} \sigma \cdot \text{diag} \left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2} \right), \quad (6.27)$$

Likewise to the SV model, we assume $\lambda_1 \ll \lambda_2 \ll a, b$ and due to the coupling $-\frac{\lambda_2}{2} \phi^2 \text{Tr} \Sigma^2 f \left(\frac{\phi}{\sqrt{6}M} \right)$, the GUT field σ reaches to its local field dependent minimum given by¹¹

$$\sigma^2 = \frac{2}{\lambda_c} \lambda_2 \phi^2 f \left(\frac{\phi}{\sqrt{3}M} \right). \quad (6.28)$$

Note that the above local minimum of the GUT field remains the same even though there is non-minimal coupling with the Ricci scalar. We can easily understand this by conformally transforming the action (6.22) into the Einstein frame.

After SU(5) symmetry breaking, the X gauge Bosons become superheavy whereas the field σ continues to follow the behavior of the field ϕ . The tree level potential for (ϕ, σ) sector is given by

$$V = \left[\frac{\lambda_c}{16} \sigma^4 - \frac{\lambda_2}{4} \sigma^2 \phi^2 f \left(\frac{\phi}{\sqrt{3}M} \right) + \frac{\lambda_1}{4} \phi^4 f^2 \left(\frac{\phi}{\sqrt{3}M} \right) \right]. \quad (6.29)$$

Substituting (6.28) in (6.22) and rescaling the field $\phi \rightarrow \sqrt{1 + \frac{\lambda_2}{\lambda_c}} \phi$ we obtain

$$S_G = \int d^4x \sqrt{-g} \left\{ (6M^2 - \phi^2) \frac{R}{12} - \frac{1}{2} (\partial\phi)^2 - \left[\frac{\lambda_c}{16} \sigma^4 - \frac{\bar{\lambda}_2}{4} \sigma^2 \phi^2 f \left(\frac{\phi}{\sqrt{3}M} \right) + \frac{\bar{\lambda}_1}{4} \phi^4 f^2 \left(\frac{\phi}{\sqrt{3}M} \right) \right] \right\}, \quad (6.30)$$

where $\bar{\lambda}_{1,2} = \lambda_{1,2} \sqrt{\frac{1}{1 + \frac{\lambda_2}{\lambda_c}}}$.

Since $\lambda_1 \ll \lambda_2$, the effective potential for the inflaton field ϕ due to the radiative corrections become

$$V_{eff}(\phi) = V + \delta V + m_\sigma^4 \ln \left(\frac{m_\sigma^2}{\mu^2} \right) + V_0, \quad (6.31)$$

where δV is the counter term, μ is the VEV of the field ϕ and V_0 is a constant. Using (6.28), choosing an appropriate $\delta V = \frac{\delta \bar{\lambda}_2}{4} \sigma^2 \phi^2 f^2 \left(\frac{\phi}{\sqrt{6}M} \right)$, a normalization constant such that $V_{eff}(\phi = \mu) = 0$ and the vacuum energy density such that $V(\phi = 0) = V_0 = \frac{A\mu^4}{4}$, we obtain

$$V_{eff}(\phi) = \frac{36AM^4}{m_P^4} \phi^4 f^2 \left(\frac{\phi}{\sqrt{3}M} \right) \ln \left(\left(\frac{6\phi^2 M^2 f \left(\frac{\phi}{\sqrt{3}M} \right)}{\mu^2 m_P^2} \right) - \frac{1}{4} \right) + \frac{A\mu^4}{4}, \quad (6.32)$$

where $A \sim \frac{\bar{\lambda}_2^2}{16\pi^2}$.

¹¹The similar scenario happens in the context of Hybrid inflationary scenario discussed in [302].

We note here that the CW potential we considered is the standard one obtained from 1-loop correction in Minkowski spacetime. In the de Sitter background 1-loop corrections are in principle different and their significance was discussed in literature [303-305]. In a recent Ref. [306], it was argued that during slow-roll inflation we can neglect the contribution of 1-loop corrections in the gravity sector. In addition, the contributions from higher loops can also be neglected by the consideration of slow-rolling scalar field [307, 308].

In order to get Planck mass m_P dynamically generated by the end of inflation, we should take the corresponding VEV of the inflaton field as

$$\langle \phi \rangle = \mu = \sqrt{6M^2 - 6m_P^2}. \quad (6.33)$$

Taking the function $f\left(\frac{\phi}{\sqrt{6}M}\right)$ from (6.24) and by doing a conformal transformation of the action (6.30) into Einstein frame, we obtain (expressing in the units of $m_P = 1$)

$$S_G^E = \int d^4x \sqrt{-g_E} \left[\frac{1}{2} R_E - \frac{1}{2M^2 \left(1 - \frac{\phi^2}{6M^2}\right)^2} \partial^\mu \phi \partial_\mu \phi - \frac{V_{eff}(\phi)}{36M^4 f^2\left(\frac{\phi}{\sqrt{3}M}\right)} \right]. \quad (6.34)$$

Under the conformal transformation the mass scales in the Einstein frame must be redefined as $\mu^2 \rightarrow \mu^2 (6M^2 - \phi^2)^{-1}$. This is very much an equivalent procedure to the 1-loop analysis of Higgs inflation. See Refs. [309-312] for a detailed discussion on the equivalence between Jordan and Einstein frames, which exactly matches if we redefine the mass scales accordingly by conformal factor. Subsequently, substituting (6.32) in (6.34)

$$S_G^E = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} R_E - \frac{1}{2M^2 \left(1 - \frac{\phi^2}{6M^2}\right)^2} \partial^\mu \phi \partial_\mu \phi - A \phi^4 \left[\ln \left(\frac{\phi^2}{\mu^2} \right) - \frac{1}{4} \right] - \frac{A \mu^4}{4} \right\}. \quad (6.35)$$

The kinetic term of (6.35) is similar the no-scale models [81]. Canonically normalizing the scalar field as $\phi = \sqrt{6}M \tanh\left(\frac{\varphi}{\sqrt{6}}\right)$ yields the Einstein frame potential

$$V_E(\varphi) = A \tanh^4\left(\frac{\varphi}{\sqrt{6}}\right) \left(\log \left(\frac{\sqrt{6}M \tanh\left(\frac{\varphi}{\sqrt{6}}\right)}{\mu} \right) - \frac{1}{4} \right) + \frac{A \mu^4}{4}. \quad (6.36)$$

The corresponding VEV of the canonically normalized field is $\langle \varphi \rangle = \sqrt{6} \arctan\left(\frac{\mu}{\sqrt{6}M}\right)$. The potential in (6.36) is a flattened version of CW potential (6.19). Due to SBCS the shape of the potential above VEV $\phi > \mu$ significantly gets flattened. In Fig. 6.1 we compare the CW potential of the SV model with the modified form (6.36) we obtained in our case. The shape of the potential reaches a plateau like in Starobinsky model when $\varphi \gg \mu$ i.e., $\phi \rightarrow \sqrt{6}M$. Inflation always starts near the plateau and continues to evolve as $\phi \lesssim \sqrt{6}M$, therefore $f\left(\frac{\phi}{\sqrt{3}M}\right)$ defined in (6.24) is always positive and consequently that avoids an anti-gravity regime. Note that the flat potential (6.36) is significantly different from the one of CW inflation studied with positive non-minimal coupling in [268]. In the next subsection we show that the inflationary observables for the potential (6.36) exactly match that of Starobinsky and Higgs inflation.

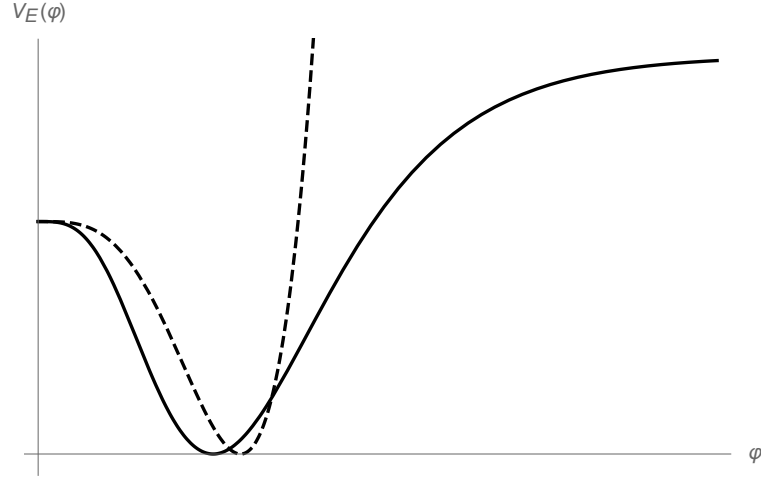


Figure 6.1: The dashed line denotes the CW potential in SV model. The full line indicates the shape of the potential obtained in (6.36) which comes from the insertion of conformal symmetry in SU(5). When $\varphi \gg \mu$ the above VEV branch of the potential approaches the plateau of Starobinsky model.

6.3.1 Inflationary predictions and proton lifetime

We assume the standard FLRW background. Let us define the general definitions of slow-roll parameters as

$$\varepsilon = \frac{H'}{H}, \quad \eta = -\frac{\varepsilon'}{\varepsilon}, \quad \delta_1 = -\frac{\eta'}{\eta}, \quad \delta_2 = -\frac{\delta_1'}{\delta_1}, \quad (6.37)$$

where H is the Hubble parameter and the prime $'$ denotes derivative with respect to e-folding number $N = \ln a(t)$ before the end of inflation.

The scalar power spectrum is given by

$$\mathcal{P}_{\mathcal{R}} = \frac{\gamma_s H^2}{8\pi^2 \varepsilon} \Big|_{k=aH}, \quad \gamma_s \equiv 2^{2\nu_s-3} \frac{\Gamma(\nu_s)^2}{\Gamma(3/2)^2} (1-\varepsilon)^2. \quad (6.38)$$

The scalar spectral index up to the first orders in slow-roll parameters is given by

$$n_s - 1 = 3 - 2\nu_s, \quad (6.39)$$

where $\nu_s = -\varepsilon - \eta/2$.

The running the spectral index can be expressed as [313]

$$\alpha_s \equiv \frac{dn_s}{d \ln k} \Big|_{k=aH} \simeq -2\varepsilon\eta - \delta_1\delta_2. \quad (6.40)$$

The ratio of tensor to scalar power spectrum is

$$r = 16\varepsilon \Big|_{k=aH}. \quad (6.41)$$

The potential (6.36) when $\varphi \gg \mu$ can be approximated as

$$\begin{aligned} V_E(\varphi) &\simeq A \left(1 - e^{-\sqrt{2/3}\varphi}\right)^4 \ln \left(\frac{\sqrt{6}M \left(1 - e^{-\sqrt{2/3}\varphi}\right)}{\mu} \right) \\ &\approx A \left(1 - e^{-\sqrt{2/3}\varphi}\right)^4 \ln \left(\frac{\sqrt{6}M}{\mu} \right). \end{aligned} \quad (6.42)$$

The equation of motion of the canonically normalized field is

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{E,\varphi} = 0,$$

which during the slow-roll regime reduces to

$$\frac{\partial \varphi}{\partial N} \approx \frac{V_{E,\varphi}}{V_E} = 4\sqrt{\frac{2}{3}}e^{-\sqrt{\frac{2}{3}}\varphi}, \quad (6.43)$$

where we use the fact that $H_{inf} \approx \frac{V_E(\varphi)}{3}$. Integrating (6.43) and expressing the slow-roll parameter $\varepsilon(N)$, $\eta(N)$ when $N \gg 1$ we get

$$\varepsilon = \frac{\partial \ln H}{\partial N} \approx \frac{1}{2} \left(\frac{V_{E,\varphi}}{V_E} \right)^2 \approx \frac{3}{4N^2}, \quad \eta = -\frac{\partial \varepsilon}{\partial N} \approx \frac{2}{N}. \quad (6.44)$$

Using (6.44) we can write the predictions for the scalar tilt (6.39) and tensor to scalar ratio (6.41) as

$$n_s \approx 1 - \frac{2}{N}, \quad r = \frac{12}{N^2}, \quad (6.45)$$

which exactly match with the predictions of Starobinsky and Higgs inflation [6, 44]. We emphasize that the predictions of our model in (6.45) are independent of the VEV of the inflaton field $\langle \phi \rangle = \mu$. In Table. 6.1 we support this result by numerically solving equation of motion of the field φ and the Friedmann equations.

In Table. 6.1 we present the inflationary predictions of the model together with the corresponding X bosons mass and proton life time using (6.20) and (6.21). In Table. 6.1 we present results for the case when the inflaton field rolls from above VEV (AV) i.e., when $\phi > \mu$. The predictions of below VEV (BV) branch i.e., when $\phi < \mu$ are not very interesting as those are nearly same in the original CW inflation without any conformal symmetry [128]. This is evident from Fig. 6.1 where we can see only the AV branch of the potential significantly different in our case, whereas the BV branch is nearly same as in the SV model. Therefore, our interest in this chapter is restricted to AV branch. For this case, from Table. 6.1 we can see that the inflationary predictions of the model are extremely stable with respect to the choice of VEV

and any value of M . In Fig. 6.2 we depict the evolution of field ϕ (also for the canonically normalized field φ) and slow-roll parameter ε for particular parameter values.

M (m_P)	A (10^{-12})	H_{inf} (10^{13} GeV)	N	φ_0 (m_P)	φ_e (m_P)	n_s	r	$-\alpha_s$ (10^{-4})	M_X ($\sim 10^{16}$ GeV)	τ_P (years)
1.1	4.79	1.74	50	7.24	2.10	0.960	0.0048	8.07	0.57	5.0×10^{34}
	3.95	1.59	55	7.35	2.10	0.963	0.0039	6.67	0.54	4.2×10^{34}
	3.32	1.46	60	7.46	2.10	0.966	0.0033	5.61	0.52	3.6×10^{34}
1.5	6.87	1.71	50	7.95	3.093	0.960	0.0046	7.88	1.53	2.6×10^{36}
	5.69	1.56	55	8.07	3.093	0.964	0.0038	6.52	1.46	2.1×10^{36}
	4.79	1.43	60	8.17	3.093	0.967	0.0032	5.48	1.39	1.8×10^{36}
2	7.59	1.70	50	8.63	3.897	0.960	0.0045	7.79	2.47	1.6×10^{37}
	6.29	1.55	55	8.75	3.897	0.964	0.0037	6.45	2.30	1.3×10^{37}
	5.29	1.52	60	8.85	3.897	0.967	0.0032	5.42	2.21	1.1×10^{37}
3	7.92	1.68	50	9.61	5.956	0.960	0.0044	7.73	3.99	1.2×10^{38}
	6.57	1.53	55	9.72	5.956	0.964	0.0037	6.40	3.81	1×10^{38}
	5.54	1.41	60	9.82	5.956	0.967	0.0031	5.39	3.65	8.5×10^{37}
5	8.07	1.68	50	12.5	7.95	0.960	0.0044	7.69	6.95	7.8×10^{38}
	6.70	1.53	55	12.7	7.95	0.964	0.0037	6.37	6.63	9.2×10^{38}
	5.65	1.41	60	12.8	7.95	0.967	0.0031	5.35	6.35	1.3×10^{40}
10	8.13	1.68	50	12.5	7.95	0.960	0.0044	7.68	14.1	1.9×10^{40}
	6.75	1.53	55	12.7	7.95	0.964	0.0037	6.35	13.5	1.6×10^{40}
	5.69	1.41	60	12.8	7.95	0.967	0.0031	5.33	12.9	1.3×10^{40}

Table 6.1: Inflationary predictions of the AV branch solutions for different parameter values.

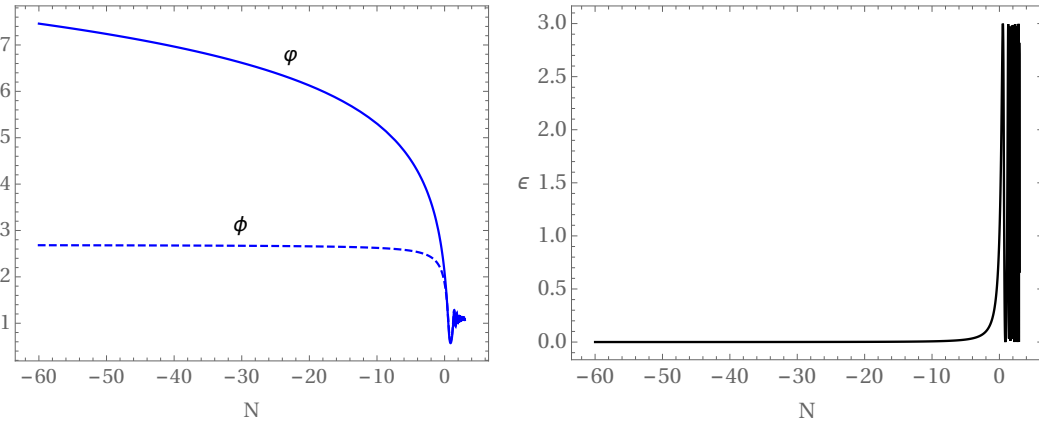


Figure 6.2: In the left panel we depict the evolution of scalar field during inflation verses the e-folding number. The solid blue line indicates the evolution of canonically normalized field φ , whereas the dotted blue line is for the original field ϕ . In the right panel we plot the corresponding slow-roll parameter ε verses N . Inflation ends when $\varepsilon = 1$. For both plots we have taken $\mu = 1.12m_P$.

6.4 Type I seesaw mechanism and neutrino masses

In this section, we further extend our model through type I seesaw mechanism with global lepton number symmetry, whose spontaneous breaking leads to the generation of neutrino masses. In this framework, we suppose the singlet field Φ carries two units of lepton number and is coupled to the three generation of singlet right handed Majorana neutrinos (RHNs), from [271]

$$V_N = V(\Phi, \bar{X}, \Sigma) + Y_D^{ij} \bar{l}_L^j i\tau_2 H^\star \nu_R^i + \frac{1}{2} Y_N^i \Phi f \left(\frac{\Phi}{\bar{X}} \right) \overline{\nu_R^i} \nu_R^i + h.c., \quad (6.46)$$

where l is the lepton doublet, τ_2 is the second Pauli matrix. Here Y_D is the Yukawa coupling matrix of the SM Higgs coupling to the left handed neutrinos and Y_N is the coupling matrix of the singlet field to the three generations of Majorana right handed neutrinos (ν_R^i). In principle, we can also weakly couple the inflaton with the SM Higgs boson as

$$V_h = V_N + \lambda_h f \left(\frac{\Phi}{\bar{X}} \right) \Phi^\dagger \Phi H^\dagger H. \quad (6.47)$$

We note that even with the new potential in (6.47), conformal symmetry in (6.22) can be preserved by the following additional transformations¹²

$$l_L^i \rightarrow \Omega^{3/2} l_L^i, \quad \nu_R^i \rightarrow \Omega^{3/2} \nu_R^i, \quad H \rightarrow \Omega H. \quad (6.48)$$

Applying SBCS via $\bar{X} = \bar{X}^\star = \sqrt{3}M$ and computing 1-loop corrections due to the additional couplings to neutrinos (6.46) and SM Higgs, the effective potential of the field ϕ becomes

$$V_f^{eff} = \frac{36A_f M^4}{m_P^4} f^2 \left(\frac{\phi}{\sqrt{3}M} \right) \phi^4 \ln \left(\frac{\phi^2 f \left(\frac{\phi}{\sqrt{3}M} \right)}{\mu_f^2} - \frac{1}{4} \right) + \frac{A_f \mu_f^4}{4}, \quad (6.49)$$

where $A_f = \frac{\beta_f}{32\pi^2}$ and

$$\beta_f = 20\bar{\lambda}_2^2 + 2\lambda_h^2 + 2\bar{\lambda}_2 \sum_i (Y_N^i)^2 - \sum_i (Y_N^i)^4. \quad (6.50)$$

In (6.50) we assume the coupling constant Y_N^i to be at least $\mathcal{O}(10)$ smaller than $\bar{\lambda}_2$ and $\lambda_h \ll Y_N^i$, such that $\beta_f \sim 20\bar{\lambda}_2^2$ and $\mu_f \sim \mu$. Therefore during inflation the coupling of a singlet field to the adjoint scalar Σ dominates, consequently the inflationary predictions in Table. 6.1 are unaffected by this additional couplings to Higgs and singlet neutrinos. However, since we impose $\lambda_h \ll Y_N^i$, the inflaton field dominantly decays to RHNs rather than to SM Higgs.

Let us consider that the lepton number violation happens at a scale when $\langle \phi \rangle = \mu$. Computing the mass matrix of singlet and doublet neutrinos in the basis of ν_L, ν_R using the Einstein frame potential of (6.46), we have

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & Y_D v_2 \\ Y_D^T v_2 & \frac{m_P^2 \langle \phi \rangle Y_N}{M^2 \sqrt{2}} \end{bmatrix}, \quad (6.51)$$

where $v_2 = 246 \text{ GeV}$ is the Electroweak vacuum. The light neutrino mass can be obtained from perturbative diagonalization of (6.51) as

¹²The kinetic terms and couplings of SM Higgs and RHNs to the Ricci scalar are irrelevant here and can be neglected in comparison with the inflaton dynamics.

$$m_{\nu_L} \simeq \sqrt{2} Y_D Y_N^{-1} Y_D^T \frac{v_2^2}{\mu} \frac{M^2}{m_P^2}. \quad (6.52)$$

The mass of heavy RHNs is given by

$$m_{\nu_R} = \frac{Y_N \langle \phi \rangle}{\sqrt{2}} \frac{m_P^2}{M^2}. \quad (6.53)$$

The essence of seesaw mechanism is the generation of neutrino masses resulting light left handed neutrinos and heavy right handed neutrinos. Both here are related to the VEV of the inflaton field.

The current *Planck* data indicates the sum of light neutrino masses constrained as $\sum m_{\nu_i} < 0.23 \text{ eV}$ [27]. Therefore considering the light neutrino mass to be $m_{\nu_L} \sim \mathcal{O}(0.1) \text{ eV}$, (6.52) gives a relation

$$Y_N \simeq 6\sqrt{2} Y_D^2 \frac{10^{14} \text{ GeV}}{\mu} \frac{M^2}{m_P^2}. \quad (6.54)$$

Taking $Y_D \sim \mathcal{O}(10^{-1})$ and from Table. 6.1 imposing $\mu \sim 1.2 m_P - 24.37 m_P$, we get $2.5 \times 10^{-6} \lesssim Y_N^i \lesssim 1.0 \times 10^{-5}$. This supports our previous assumptions after (6.50) that the couplings to the RHNs have negligible effect for inflation. Our generalization of the SV model successfully fits into explaining the origin of neutrino masses. We can also take $Y_D < \mathcal{O}(10^{-1})$ which results in smaller values for $Y_N < \mathcal{O}(10^{-6})$. Taking $Y_N \sim 10^{-6}$, the heavy RHN mass will be around $m_{\nu_R} \sim 4 \times 10^{12} \text{ GeV}$. For $Y_N < \mathcal{O}(10^{-6})$ we can lower the masses of RHNs. In the next section we aim to study reheating in our inflationary scenario, taking into account the constraints we have derived so far.

6.5 Reheating and non-thermal leptogenesis

We consider reheating through a dominant decay of the inflaton into heavy RHNs which requires $m_\varphi \gtrsim 2m_{\nu_R}$. The mass of the canonically normalized field φ at the minimum of the potential is given by the second derivative of the potential (6.36)

$$m_\varphi = \sqrt{V_{\varphi, \varphi}^E} \Big|_{\varphi=\langle \varphi \rangle} = 2 \times 10^{-6} \mu,$$

where we have taken a value for $A \sim 5 \times 10^{-12}$ from Table 6.1.

We implement the scheme of non-thermal leptogenesis proposed in [95, 314] which can give rise to baryogenesis through CP violating decays of RH Majorana neutrinos. In this section we closely follow in [273-275]. We consider:

- Hierarchical masses for RHNs $m_{\nu_R^1} \ll m_{\nu_R^2} \sim m_{\nu_R^3}$. To arrange this we require the coupling constants to be $Y_{N_1} \ll Y_{N_2} \sim Y_{N_3}$. We assume that the inflaton decays equally into the two heavy RHNs $\nu_R^{2,3}$ and the corresponding reheating temperature can be computed using [272, 273]

$$T_R = \left(\frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma_\varphi (\varphi \rightarrow \nu_R^i \nu_R^i) m_P}, \quad (6.55)$$

where $g_* = 105.6$ is the number of relativistic degrees of freedom and the decay rate is given by

$$\Gamma_\varphi (\varphi \rightarrow \nu_R^i \nu_R^i) \simeq \frac{m_\varphi}{4\pi} \sum_{i=1}^3 c_i^2 \left(\frac{m_{\nu_R^i}}{m_P} \right)^2 \left(1 - \frac{4m_{\nu_R^i}^2}{m_\varphi^2} \right)^{3/2}.$$

The masses of heavy RHNs are $m_{\nu_R^{2,3}} \sim \frac{Y_N^{2,3}}{\sqrt{2}}$, which for $Y_N^{2,3} \sim 10^{-8} - 10^{-6}$ we have $m_{\nu_R^{2,3}} \sim 10^{10} - 10^{12}$ GeV. In Fig. 6.3 we plot the possible reheating temperatures of our case taking $c_1 \approx 0$ and $c_2 = c_3 = 1$.

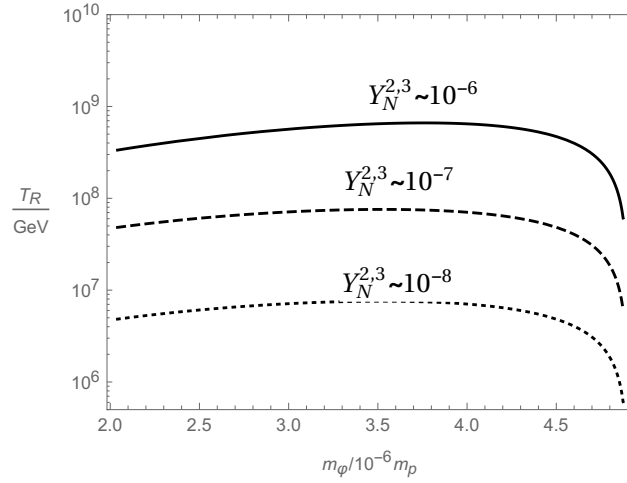


Figure 6.3: In this plot we depict the reheating temperatures T_R Vs. m_φ for the values of couplings $Y_N^{2,3} \sim 10^{-8} - 10^{-6}$.

- The decays of RH Majorana neutrinos ν_R^i break the lepton number conservation and leads to CP violation. There are two decay channels

$$\Gamma_i : \nu_R^i \rightarrow H + l_i, \quad \bar{\Gamma}_i : \nu_R^i \rightarrow H^\dagger + \bar{l}_i, \quad (6.56)$$

where H and l denote the Higgs field and the lepton doublets of the SM. The lepton asymmetry generated by the CP violation decays of ν_R^i is measured by the following quantity

$$\varepsilon_i \equiv \frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i} \ll 1. \quad (6.57)$$

CP asymmetry ε_i can be computed for the dominant decays of $\nu_R^{2,3}$ using [274, 315-317]

$$\varepsilon_i = -\frac{1}{8\pi} \frac{1}{(Y_D Y_D^\dagger)_{11}} \sum_{i=2,3} \text{Im} \left[\left\{ (Y_D Y_D^\dagger)_{1i} \right\}^2 \right] \left[f \left(\frac{m_{\nu_R^i}^2}{m_{\nu_R^1}^2} \right) + g \left(\frac{m_{\nu_R^i}^2}{m_{\nu_R^1}^2} \right) \right], \quad (6.58)$$

where

$$f(y) = \sqrt{y} \left[-1 + (y+1) \ln \left(1 + \frac{1}{y} \right) \right], \quad g(y) = \frac{\sqrt{y}}{y-1}.$$

Here we only aim to constrain the range of values for ε_i leaving for future the explicit computation of constraining Yukawa matrix Y_D^{ij} [273].

The lepton asymmetry is given by

$$\frac{n_L}{s} = \sum_{i=1}^3 \varepsilon_i \text{Br}_i \frac{3T_R}{2m_\varphi}, \quad (6.59)$$

where n_L is the difference between number of leptons and anti-leptons and s indicates the entropy density, Br_i denotes the branching ratio

- The production of RH Majorana neutrinos happens non-thermally and sufficiently late so that the produced lepton asymmetry sources the baryon asymmetry at a later stage. This essentially requires $m_{\nu_R^1} \gtrsim T_R$ so that the later decay of lightest RH Majorana neutrino ν_R^1 does not wash away the produced lepton asymmetry by the heavy ones. We assume there is an accidental $B - L$ conservation¹³ such that sphaleron process is active which brings a part of the above lepton asymmetry into the baryon asymmetry (see Ref. [318-320] for details). As the reheating temperature in our case is $T_R \sim 10^6 - 10^9 \text{ GeV}$ (see Fig. 6.3) we take $Y_N^1 \sim 10^{-10} - 10^{-9}$ such that $m_{\nu_R^1} \sim 10^8 - 10^9 \text{ GeV}$. Therefore, with values $m_{\nu_R^{2,3}} \sim 10^{10} - 10^{12} \text{ GeV}$, $m_{\nu_R^1} \sim 10^8 - 10^9 \text{ GeV}$ and $T_R \sim 10^6 - 10^9 \text{ GeV}$ we have met the conditions for successful leptogenesis, $m_{\nu_R^2} \sim m_{\nu_R^3} \gg m_{\nu_R^1}$ and $m_{\nu_R^1} \gtrsim T_R$.

Baryon asymmetry is proportional to the lepton asymmetry as

$$\begin{aligned} \frac{n_B}{s} &\simeq \frac{28}{79} \frac{n_L}{s} \\ &\simeq \frac{42}{79} \sum_{i=1}^3 \varepsilon_i \text{Br}_i \frac{T_R}{m_\varphi}. \end{aligned} \quad (6.60)$$

The baryon asymmetry which is measured by the ratio of the difference between the number of baryons minus the anti-baryons n_B to the entropy density in the present Universe, is constrained [27] in the following form

$$\frac{n_B}{s} = (6.05 \pm 0.06) \times 10^{-10}. \quad (6.61)$$

Considering branching ratios $\text{Br}_1 = 0$ and $\text{Br}_2 = \text{Br}_3 = \frac{1}{2}$ with $\varepsilon_1 \ll \varepsilon_2 \sim \varepsilon_3$ we have

$$\frac{n_B}{s} \approx \varepsilon_2 \frac{T_R}{m_\varphi}. \quad (6.62)$$

From Fig. 6.3 we can read that $\frac{T_R}{m_\varphi} \sim 10^{-7} - 10^{-4}$, which indicates the CP violation in the decay of RH Majorana neutrinos (ε_i) must be in the range $6 \times 10^{-6} \lesssim \varepsilon_{2,3} \lesssim 6 \times 10^{-3}$ to have the observed baryon asymmetry.

¹³ B, L refers to baryon number and lepton number respectively.

6.6 Summary

Coleman-Weinberg inflation [89] has been a successful and realistic model based on GUT and is consistent with current *Planck* data with $r \gtrsim 0.02$ [266]. In this chapter, we have further generalized the framework of CW inflation with an additional conformal symmetry. Spontaneous breaking of conformal symmetry is useful to create a hierarchy of mass scales, therefore it is natural to realize this symmetry in GUT models. In this respect, two complex singlet fields of $SU(5)$ or $SO(10)$ are considered and are coupled to the GUT fields in a suitable manner. We have showed that this setup, upon spontaneous breaking of GUT and conformal symmetry, leads to an interesting inflationary scenario driven by the real part of the singlet field. In our model, the above VEV branch of CW potential gets flattened to a Starobinsky plateau allowing for $n_s \sim 0.96 - 0.967$ and $r \sim 0.003 - 0.005$ for $50 - 60$ number of e -foldings. We found that these predictions are independent of the VEV of the inflaton field. However, values of inflaton VEV affect the masses of the superheavy gauge bosons that mediate the proton decay. We calculated the corresponding estimates for proton life time above the current lower bound from Super-K data $\tau_p(p \rightarrow \pi^0 + e^+) > 1.6 \times 10^{34}$. In the next step, we introduced a coupling between the complex singlet field with the generation of three singlet RHNs, where the singlet field is assumed to carry two units of lepton number. We implemented type I seesaw mechanism where spontaneous symmetry breaking of global lepton number results in generating neutrino masses. We put an upper bound to the inflaton couplings to RHNs assuming inflation is dominated by inflaton couplings to GUT field. For the non-thermal leptogenesis to happen, we have considered dominant decay of inflaton into some of the RHNs and obtained the corresponding reheating temperatures as $10^6 \text{ GeV} \lesssim T_R < 10^9 \text{ GeV}$. In summary, our new development of CW inflation can be tested within future CMB data [29].

7

Conclusions and outlook

Never theorize before you have data. Invariably, you end up twisting facts to suit theories, instead of theories to suit facts

- Sir Arthur Conan Doyle, *Sherlock Holmes*

Conclusions

In this thesis, we have studied inflationary scenarios in string theory, SUGRA and particle physics. We have covered aspects of inflationary models following a top-down or bottom-up motivations as we described in the introduction. It is important to understand the physics of inflation from the point of view of UV completeness as well as from the point of view of physics beyond SM. Both of these motivations are naturally appealing on their own. In the scope of the latest CMB data from *Planck* 2015 and the upcoming ground based and space based CMB probes, it is a greater necessity than before that we not only construct interesting models of inflation but test them observationally. Moreover, we need to concentrate on developing theoretical frameworks of inflation towards generality/naturality rather than simplicity. In this respect, this thesis uncovers models beyond the conventionality, towards realistic features within the fundamental theories. This perspective is made concrete within our model by model brief appraisal in what follows.

3-form fields are viable alternative to conventional scalar fields. 3-forms were known to have different dynamics than scalar fields, which allow inflation as an attractor phenomenon [106]. In chapter 2, bearing the fact that multifields are more natural in string theory settings, we have studied the multiple 3-form inflation. We have explored possible dynamics of two 3-form fields with suitable choice of potentials. We have put the model for test with (n_s, r) , running of n_s and detailed study of non-Gaussianities. Moreover, we must notice that even though *Planck* 2015 data favours single field inflation, there is a wide scope of interesting dynamics and parameter space for multifield inflation [154]. Therefore, in the future it would be interesting to consider a wider study of inflationary scenarios with 3-form fields with more complicated choice of potentials than we have studied herein.

Our study of DBI Galileon model in chapter 3, is also beyond the conventional DBI model. We studied DBIG model as it is indeed a natural framework in string theory, where the motion of the D-brane in the bulk space imparts effects of induced gravity [189]. We significantly scanned the parameter space of the model with respect to n_s and r . We also contemplated our study of parameter space with available results on “equilateral” shape of non-Gaussianities. Overall, we have shown that the model is observationally improved over DBI inflation. However, we must note that it is important to understand the role of the geometry of the bulk space, in which the motion of the D-brane induces the inflationary expansion. We mostly relied on numerical analysis but theoretical studies regarding warped geometries and inflaton potential remain to be

done. Moreover, a detailed study of non-Gaussianities, especially of orthogonal shape [193], is very much required for this model, to allow it to be rigorously tested in the future observations. Identifying SFT as a crucial part of string theory to be UV complete, in the framework of SFT, we have proposed in chapter 4, a class of effective models based on the phenomenon of TC and non-locality. This study introduces a very new framework of inflation driven by closed string dilaton in SFT, where we consider the TC to happen above the inflationary energy scale, which is very different from popular models of inflation driven by tachyon [214]. Within our considerations, we obtained single field inflation with the potential (1.8), where the parameter B is obtained within our SFT setup. The interesting part of this study is that we have demonstrated how conformal symmetry can emerge in our SFT framework, given that conformal inflationary models are one of the best fit with current data. A further study of non-Gaussianity in this setup is essential to distinguish these models from the other competitive scenarios in the literature. The models in this chapter are, though viable with respect to observational data, quite speculative. More theoretical progress has to be done within SFT, to strengthen this framework. In the SUGRA framework, we have explored the quite well known α -attractor model in chapter 5. Our study, that follows a non-slow-roll approach, highlighted the importance of its inflaton's non-canonical kinetic term and we also found a new class of potentials. With our efforts, the connection between Kähler manifold and the inflaton dynamics is more clear. However, as far as inflationary predictions are concerned our approach is indistinguishable from the original model with the respective potentials. Perhaps, only a study of the subsequent reheating might help in this regard, to falsify our approach.

Realizing conformal symmetry as the fundamental symmetry of nature [279], in the GUT model studied in chapter 6, we generalized CW inflation [89]. We have shown that the inflationary predictions (n_s, r) in this model are the same as with the Starobinsky and Higgs inflation. Moreover, we obtain several predictions in particle physics context such as proton life time, neutrino masses and leptogenesis. Therefore, this model can be tested outside of the CMB observations. It would be interesting to extend this model in SUGRA with superconformal symmetries to attain UV completion which we defer to future studies.

In summary, the thesis presents many facets of inflationary cosmology in fundamental theories and all of them fit successfully with *Planck* 2015 data. Moreover, we developed several theoretical aspects which opens new routes for interesting research in future. In the next section, we present an outlook focusing on recent trends in inflationary cosmology and point out to an outlook regarding the successful models we have studied so far.

Outlook

Although the inflationary paradigm is successful and observationally consistent with CMB data so far, the physical origin of inflation is still uncertain. Even after *Planck* 2015 data, by means of which simple inflationary models were ruled out, there are still many models being viable [33]. Moreover, studying inflation is so far the only way to probe observationally the physics of very high energy scales (e.g., GUT scale). Hence, it is quite natural to hope and look in inflationary cosmology for the signatures of string theory, SUGRA and beyond SM. Being more precise, inflationary cosmology is playing a vital role in the broad area of high energy physics, strengthening our efforts to ultimately build a UV complete theory as well as its connection to

the current understanding of Universe.

We summarize here some lines of future research which could be useful to further distinguish not only the models we have studied in thesis but also among the other still viable models in literature, by means of upcoming CMB probes [33]:

- Reheating/preheating after inflation and connecting to SM is crucial, especially in string/SUGRA based models. Although it is hard to observationally probe the reheating epoch, it might be crucial for the completeness of a model [46, 321]. In this aspect, we must study the dynamics of inflaton field at the reheating epoch and consider possible decays of inflaton field into SM or beyond SM degrees of freedom [14].
- *Planck* 2015 data has strongly indicated the presence of certain anomalies in the CMB such as power suppression at low multipoles, hemispherical asymmetry and the regions of hot and cold spots [322], which has no consistent theoretical explanation so far. These could indicate new physics and perhaps it is time to intensify our theoretical efforts to explain these anomalies [323, 324].
- A more careful understanding is required for the case of single field inflation addressing the issue of the so called η -problem [70]. If inflation is believed to be originated in the UV complete theories such as string theory or SUGRA, ubiquitous presence of heavy fields might leave non-trivial imprints in the primordial bispectrum and power spectrum [325-327].
- Studying tensor scalar cross correlations have recently gained much of interest and indeed they are a powerful tool to classify several models. Although observation of cross correlation spectra is yet not viable, it is worth to invest on these studies [328].
- It is worthy to consider inflationary scenarios when addressing the origins of dark matter, leptogenesis, baryogenesis and neutrino masses. This would enable testing inflationary models outside CMB i.e., at collider and astrophysical observations [97].
- To expand the scope of testability for inflationary models, it is useful to build unified models of inflation and dark energy such as Higgs-dilaton model [329].

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Appendices

Appendix A

Inflationary observables

This appendix provides complementary information concerning chapter 1.

A.1 General definitions

Consider FLRW spacetime

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad (\text{A.1})$$

where $a(t)$ is the scale factor with t being the cosmic time, \mathbf{x} is three dimensional space vector. The curvature perturbation $\zeta(t, \mathbf{x})$ in the comoving gauge is defined by the perturbation of the spatial part of the metric

$$\delta g_{ij} = a^2 (h_{ij} + 2\zeta(t, \mathbf{x}) \delta_{ij}), \quad (\text{A.2})$$

where h_{ij} denotes the tensor fluctuation in the 3+1 ADM (Arnowitt-Deser-Misner) decomposition of the metric [15, 330].

The second and higher order (quantum) correlations function of ζ relates to the properties of temperature anisotropies $\frac{\Delta T}{T}$ at a point in the (CMB) sky \mathbf{x} . A two-point function correlates the density or temperature fluctuations at two points in space, measured by the power spectrum $P_\zeta(k)$ whose distribution is in general Gaussian

$$\langle \zeta(\mathbf{x})\zeta(\mathbf{x}') \rangle|_{t=t_f} \equiv \int \frac{d^3k}{(2\pi)^3} \mathcal{P}_\zeta(k) e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')}, \quad (\text{A.3})$$

In the Fourier space (A.3) can be written as

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2) \rangle|_{t=t_f} = (2\pi)^5 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) \frac{1}{2k_1^3} P_\zeta(k_1). \quad (\text{A.4})$$

where t_f is the time corresponds to superhorizon scales, the power spectrum $\mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2} |\zeta_k(k, t)|^2$ and $\zeta_k(k, t)$ is the mode function of curvature perturbation $\zeta_k = \int d^3x \zeta(\mathbf{x}, t) e^{i\mathbf{k} \cdot \mathbf{x}}$, which is computed usually from the second order perturbation of inflationary action (see Sec. A.3). In general, scalar and the tensor power spectrum scales in the power-law form as [15, 25]

$$\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta(k_*) \left(\frac{k}{k_*} \right)^{n_s - 1}, \quad \mathcal{P}_t(k) = \mathcal{P}_t(k_*) \left(\frac{k}{k_*} \right)^{n_t}. \quad (\text{A.5})$$

where $k_* = 0.002 \text{ Mpc}^{-1}$ is the pivot scale. From the Planck 2015 data [27], the pivot scale power spectrum is measured as $P_\zeta(k_*) \approx 2.2 \times 10^{-9}$. Here n_s and n_t are named scalar and

tensor spectral tilt (or spectral index) respectively. The ratio of tensor to scalar power spectra $r = \frac{P_t}{P_\zeta}$ is another important inflationary observable.

The 3-point, 4-point correlation functions of curvature perturbation are useful to further characterize the nature of inflaton and these are defined, respectively, by

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \mathcal{B}_\zeta(k_1, k_2, k_3), \quad (\text{A.6})$$

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \zeta(\mathbf{k}_4) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) \mathcal{T}_\zeta(k_1, k_2, k_3, k_4), \quad (\text{A.7})$$

where $\mathcal{B}_\zeta(k_1, k_2, k_3)$, $\mathcal{T}_\zeta(k_1, k_2, k_3, k_4)$ are called the bispectrum and the trispectrum¹ respectively. Often the bispectrum is normalized to form the reduced bispectrum $f_{\text{NL}}(k_1, k_2, k_3)$ as

$$\mathcal{B}_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\text{NL}}(k_1, k_2, k_3) \left[P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right]. \quad (\text{A.8})$$

And $f_{\text{NL}}(k_1, k_2, k_3)$ signifies the shape of the bispectrum, it is also called the non-linear or non-Gaussianity parameter.

The 3-point correlation function usually computed in *in-in* formalism which is the standard method of quantum field theory. For this we require the interaction Hamiltonian (\mathcal{H}_{int}) which can be derived from the 3rd order perturbation of the inflationary action,

$$\langle \zeta(t, \mathbf{k}_1) \zeta(t, \mathbf{k}_2) \zeta(t, \mathbf{k}_3) \rangle = -i \int_{t_0}^t dt' \langle [\zeta(t, \mathbf{k}_1) \zeta(t, \mathbf{k}_2) \zeta(t, \mathbf{k}_3), \mathcal{H}_{\text{int}}(t')] \rangle. \quad (\text{A.9})$$

The computation of 3-point function is well known for various models, can be read from [35, 54, 112, 170].

A.2 Single field consistency relations

The following two consistency relations can ultimately test (standard) scalar field inflation².

- Tensor consistency relation that corresponds to the relation between tensor to scalar ratio and the tensor tilt as $r = -8n_t$ [15].
- Maldacena consistency relation [35, 38] that relates the bispectrum in the squeezed limit to the scalar spectral index as $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^2 \delta^3(\sum \mathbf{k}_i) (1 - n_s) \mathcal{P}_{\zeta_{\mathbf{k}_1}} \mathcal{P}_{\zeta_{\mathbf{k}_2}}$. In other words, the "local" shape of non-Gaussianity is proportional to the scalar tilt as $f_{\text{NL}}^{\text{local}} = \frac{5}{12} (1 - n_s)$ and it was argued that this relation holds not only for standard single but for any general single scalar field inflation [38].

¹The discussion on trispectrum is beyond the scope of this thesis since the current constraints are far less stringent [25].

²These relations are also valid for Starobinsky and Higgs inflation as their action can be written in the form of the standard scalar action, with the assistance of a conformal transformation [43, 44].

A.3 Power spectra in generalized G-inflation

The most general scalar-tensor theory in 4D with second order field equations³ [48, 49] is given by the Lagrangian

$$\mathcal{S}_G = \int d^4x \sqrt{-g} \left[\frac{m_{\text{p}}^2}{2} R + P(\phi, X) - G_3(\phi, X) \square\phi + \mathcal{L}_4 + \mathcal{L}_5 \right], \quad (\text{A.10})$$

where

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4,X} [(\square\phi)^2 - (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi)], \quad (\text{A.11})$$

$$\begin{aligned} \mathcal{L}_5 = & G_5(\phi, X) G_{\mu\nu} (\nabla^\mu \nabla^\nu \phi) - \frac{1}{6} G_{5,X} [(\square\phi)^3 - 3(\square\phi) (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) \\ & + 2(\nabla^\mu \nabla_\alpha \phi) (\nabla^\alpha \nabla_\beta \phi) (\nabla^\beta \nabla_\mu \phi)]. \end{aligned} \quad (\text{A.12})$$

Here P and G_i 's ($i = 3, 4, 5$) are functions in terms of ϕ and $X = -\partial^\mu \phi \partial_\mu \phi / 2$ with the partial derivatives $G_{i,X} \equiv \partial G_i / \partial X$, and $G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} R / 2$.

Scalar power spectrum

The second order action of (A.10) for scalar perturbations is given by [49, 332],

$$S_s^{(2)} = \int dt d^3x (a^3 \mathcal{G}_s) \left[\dot{\zeta} - \frac{\mathcal{F}_s / \mathcal{G}_s}{a^2} (\nabla \zeta)^2 \right], \quad (\text{A.13})$$

where $\mathcal{F}_s, \mathcal{G}_s$ are arbitrary functions of time⁴ and $c_s \equiv (\mathcal{F}_s / \mathcal{G}_s)^{1/2}$ is the sound speed for scalar perturbations. In addition to the slow-roll conditions in (1.2), we introduce the following new parameters which has to be sufficiently small during inflation [49].

$$f_s \equiv \frac{d \ln \mathcal{F}_s}{d \ln a}, \quad f_s^{(2)} \equiv \frac{d \ln f_s}{d \ln a}, \quad g_s \equiv \frac{d \ln \mathcal{G}_s}{d \ln a}, \quad g_s^{(2)} \equiv \frac{d \ln g_s}{d \ln a}. \quad (\text{A.14})$$

Moreover, using the definition of c_s we have

$$\varepsilon_s \equiv \frac{d \ln c_s}{d \ln a} = \frac{1}{2} (f_s - g_s), \quad \eta_s \equiv \frac{d \ln \varepsilon_s}{d \ln a} = \frac{1}{2\varepsilon_s} (f_s f_s^{(2)} - g_s g_s^{(2)}). \quad (\text{A.15})$$

To quantify the amplitude and tilt of the spectrum we introduce the variables $dy_s \equiv \frac{c_s}{a} dt$, $z_s \equiv \sqrt{2} a (\mathcal{F}_s \mathcal{G}_s)^{1/4}$ and $u \equiv z_s \zeta$, using which the action (A.13) can be canonically normalized

$$S_s^{(2)} = \frac{1}{2} \int dy_s d^3x \left[(u')^2 - (\nabla u)^2 + \frac{z_s''}{z_s} u^2 \right]. \quad (\text{A.16})$$

Imposing the Bunch-Davies vacuum initial condition in the subhorizon limit $c_s k \gg aH$, the solution for perturbation mode u is given by

$$u_k = \frac{\sqrt{\pi}}{2} \sqrt{-y_s} H_{\nu_s}(-ky_s), \quad \nu_s^2 - \frac{1}{4} \equiv y_s^2 \frac{z_s''}{z_s}. \quad (\text{A.17})$$

Using now $\zeta_k = u_k / z_s$, we obtain the the scalar power spectrum as

³That is free from Ostrogradski instabilities [331].

⁴Whose definitions can be found in [49].

$$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2} |\zeta_k|^2 = \frac{\gamma_s}{2} \frac{\mathcal{G}_{s*}^{1/2}}{\mathcal{F}_{s*}^{3/2}} \frac{H_*^2}{4\pi^2} \quad , \quad \gamma_s \equiv 2^{2\nu_s-3} \frac{\Gamma(\nu_s)^2}{\Gamma(3/2)^2} \left(1 - \varepsilon_* + \frac{g_{s*}}{2} - \frac{f_{s*}}{2}\right)^2, \quad (\text{A.18})$$

where “*” labels the time of sound horizon crossing when $ky_s = -1$.

To compute the spectral index of the scalar perturbations

$$n_s - 1 \equiv 3 - 2\nu_s, \quad (\text{A.19})$$

first we need to compute ν_s . Using the definition of z_s we find

$$\frac{z_s''}{z_s} = \left(\frac{Ha}{c_s}\right)^2 \left[\left(1 + \frac{f_s + g_s}{4}\right)^2 + \left(1 - \varepsilon - \frac{f_s}{2} + \frac{g_s}{2}\right) \left(1 + \frac{f_s + g_s}{4}\right) - \frac{f_s f_s^{(2)} - g_s g_s^{(2)}}{4} \right]. \quad (\text{A.20})$$

The next step is to integrate $dy_s = (c_s/a) dt$. Assuming small and constant η and η_s to neglect second order terms and integrating by parts we obtain

$$y_s = -\frac{c_s}{(1 - \varepsilon - \varepsilon_s) aH} \left(1 + \frac{\varepsilon\eta + \varepsilon_s\eta_s}{(\varepsilon + \varepsilon_s - 1)^2}\right). \quad (\text{A.21})$$

If the slow-roll parameters are sufficiently small, in the linear approximation [156, 333], the scalar spectral index can be written as

$$n_s - 1 \simeq \frac{4\varepsilon_* + 3f_{s*} - g_{s*}}{-2 + 2\varepsilon_* + f_{s*} - g_{s*}}. \quad (\text{A.22})$$

Using (A.22) we can compute the running index $n'_s \equiv \frac{dn_s}{d \ln k}$. Since we assumed η, η_s approximately constant and small, the use of (A.21) allows us to write

$$n'_s = -\frac{y_s aH}{c_s} \frac{dn_s}{d \ln a} \simeq \frac{1}{(1 - \varepsilon - \varepsilon_s)} \left(1 + \frac{\varepsilon\eta + \varepsilon_s\eta_s}{(\varepsilon + \varepsilon_s - 1)^2}\right) \frac{dn_s}{d \ln a} \simeq \frac{1}{(1 - \varepsilon - \varepsilon_s)} \frac{dn_s}{d \ln a}. \quad (\text{A.23})$$

Provided η and η_s are small and approximately constant, we can expand (A.22) to first order in η, η_s . Using (A.15) and (A.23) the running index becomes

$$n'_s \simeq \frac{2\varepsilon_* f_{s*} (4 - f_{s*} + g_{s*}) - 2g_{s*} g_{s*}^{(2)} (1 + \varepsilon_*)}{(2 - 2\varepsilon_* - f_{s*} + g_{s*})^2}.$$

For the action with $P(X, \phi) = K(\phi)X - V(\phi)$ and $\mathcal{L}_4 = \mathcal{L}_5 = 0$, ν_s can be computed up to the third order in the parameters ε, η , by using the definition of z_s and (A.21), we obtain⁵ [334]

$$\begin{aligned} \nu_s = & \left(\frac{3}{2} + \varepsilon + \varepsilon^2 + \varepsilon^3\right) + \left(\frac{1}{2} + 2\varepsilon + \frac{29\varepsilon^2}{6} + \frac{82\varepsilon^3}{9}\right) \eta + \left(-\frac{1}{6} + \frac{23\varepsilon}{18} + \frac{1069\varepsilon^2}{108} + \frac{5807\varepsilon^3}{162}\right) \eta^2 \\ & + \left(\frac{1}{18} + \frac{23\varepsilon}{54} + \frac{707\varepsilon^2}{108} + \frac{19633\varepsilon^3}{486}\right) \eta^3 + \mathcal{O}(\varepsilon^4, \eta^4). \end{aligned} \quad (\text{A.24})$$

⁵The expression (A.24) we use it in chapter 5.2.

Tensor power spectrum

Similarly to the case of scalar perturbations, the second order action for tensor perturbations can be written as [49]

$$S_t^{(2)} = \frac{1}{8} \int dt d^3x (a^3 \mathcal{G}_t) \left[\dot{h}_{ij}^2 - \frac{\mathcal{F}_t/\mathcal{G}_t}{a^2} (\nabla h_{ij})^2 \right], \quad (\text{A.25})$$

where \mathcal{F}_t and \mathcal{G}_t are functions of time and $c_t \equiv (\mathcal{F}_t/\mathcal{G}_t)^{1/2}$ is the sound speed for tensor perturbations.

Similarly to (A.14), we consider now the additional slow-roll parameters

$$f_t \equiv \frac{d \ln \mathcal{F}_t}{d \ln a}, \quad f_t^{(2)} \equiv \frac{d \ln f_t}{d \ln a}, \quad g_t \equiv \frac{d \ln \mathcal{G}_t}{d \ln a}, \quad g_t^{(2)} \equiv \frac{d \ln g_t}{d \ln a}, \quad (\text{A.26})$$

and using the definition of c_t we also have

$$\varepsilon_t \equiv \frac{d \ln c_t}{d \ln a} = \frac{1}{2} (f_t - g_t), \quad \eta_t \equiv \frac{d \ln \varepsilon_t}{d \ln a} = \frac{1}{2\varepsilon_t} (f_t f_t^{(2)} - g_t g_t^{(2)}). \quad (\text{A.27})$$

Similarly to the case of the scalar spectrum, we introduce the variables $dy_t \equiv \frac{c_t}{a} dt$, $z_t \equiv \frac{a}{2} (\mathcal{F}_t \mathcal{G}_t)^{1/4}$ and $u_{ij} \equiv z_t h_{ij}$ so that the action in (A.25) can be canonically normalized

$$S_t^{(2)} = \frac{1}{2} \int dy_t d^3x \left[(u'_{ij})^2 - (\nabla u_{ij})^2 + \frac{z_t''}{z_t} u_{ij}^2 \right]. \quad (\text{A.28})$$

Imposing the Bunch-Davies vacuum initial condition as in (A.17) we find

$$u_{ij} = \frac{\sqrt{\pi}}{2} \sqrt{-y_t} H_{\nu_t}^{(1)}(-ky_t) e_{ij}, \quad \nu_t^2 - \frac{1}{4} \equiv y_t^2 \frac{z_t''}{z_t}. \quad (\text{A.29})$$

where e_{ij} is the polarization tensor and

$$\begin{aligned} \frac{z_t''}{z_t} = & \left(\frac{aH}{c_s} \right)^2 \left[\left(\frac{f_t + g_t}{4} + 1 \right) \left(-\frac{f_t}{2} + \frac{g_t}{2} - \varepsilon + 1 \right) + \left(\frac{f_t + g_t}{2} + 1 \right)^2 \right. \\ & \left. - \frac{(f_t f_t^{(2)} - g_t g_t^{(2)})}{4} \right]. \end{aligned} \quad (\text{A.30})$$

Using that $h_{ij} = u_{ij}/z_t$ and taking into account the two polarization states, we arrive at the tensor power spectrum given by

$$\mathcal{P}_t = 8\gamma_t \frac{\mathcal{G}_{t_*}^{1/2}}{\mathcal{F}_{t_*}^{3/2}} \frac{H_*^2}{4\pi^2}, \quad \gamma_t \equiv 2^{2\nu_t-3} \frac{\Gamma(\nu_t)^2}{\Gamma(3/2)^2} \left(1 - \varepsilon_* + \frac{g_{t_*}}{2} - \frac{f_{t_*}}{2} \right)^2. \quad (\text{A.31})$$

The spectral index of the tensor spectrum is

$$n_t \equiv 3 - 2\nu_t, \quad (\text{A.32})$$

Similar to the scalar tilt, if slow-roll parameters are sufficiently small we can write the tensor

tilt as [156, 333]

$$n_t \simeq \frac{4\varepsilon_* + 3f_{t_*} - g_{t_*}}{-2 + 2\varepsilon_* + f_{t_*} - g_{t_*}}, \quad (\text{A.33})$$

where the subindex “*” indicates the time of sound horizon crossing, determined by the condition $ky_t = -1$.

In the case of $P(X, \phi) = K(\phi)X - V(\phi)$ and $\mathcal{L}_4 = \mathcal{L}_5 = 0$, calculating ν_t up to the third order in the parameters ε, η , by using the definition of z_t , we obtain [334]

$$\begin{aligned} \nu_t = & \left(\frac{3}{2} + \varepsilon + \varepsilon^2 + \varepsilon^3 \right) + \left(\frac{4\varepsilon}{3} + \frac{37\varepsilon^2}{9} + \frac{226\varepsilon^3}{27} \right) \eta + \left(\varepsilon + \frac{227\varepsilon^2}{27} + \frac{875\varepsilon^3}{27} \right) \eta^2 + \\ & \left(\frac{28\varepsilon^2}{9} + \frac{6491\varepsilon^3}{243} \right) \eta^3 + \mathcal{O}(\varepsilon^4, \eta^4). \end{aligned} \quad (\text{A.34})$$

Finally, the tensor to scalar ratio in generalized G-inflation is

$$r \equiv \frac{\mathcal{P}_{t_*}}{\mathcal{P}_{\zeta_*}} = 16 \frac{\gamma_t}{\gamma_s} \left(\frac{\mathcal{G}_{t_*}}{\mathcal{G}_{s_*}} \right)^{1/2} \left(\frac{\mathcal{F}_{s_*}}{\mathcal{F}_{t_*}} \right)^{3/2}. \quad (\text{A.35})$$

From (A.33) and (A.35) we observe that the standard single-field inflationary consistency relation, $r = -8n_t$, is in general violated. In Ref. [209] it has been shown that, in the case of power law G-inflation, we can have either $r > -8n_t$ or $r \leq -8n_t$ depending on the model parameters. However, the requirement of subluminal propagation speed of the scalar perturbations restricts $r \leq -\frac{32}{3}n_t$.

Appendix B

Stability of type I fixed points

This appendix provides a complementary feature for chapter 2.

Let us now discuss the stability of these fixed points for specific choice of potentials. The eigenvalues of \mathcal{M}_{ij} corresponding to the fixed point (χ_{1c}, w_{1c}) are $\zeta_1 = -3$, $\zeta_2 = 0$. Since the second eigenvalue is zero, we cannot decide on the stability of this fixed point. The eigenvector for the null eigenvalue is given by

$$v_0 = \begin{pmatrix} \sqrt{2/3} \\ 1 \end{pmatrix}. \quad (\text{B.1})$$

Let us consider the nonlinear order perturbation in the expansion

$$\delta r' = \mu^{(n)} \delta r^n \quad (\text{B.2})$$

where $\delta r = \sqrt{2/3} \delta \chi_1 + \delta w_1$ is the perturbation along the direction of the eigen vector (B.1). The general solution of (B.2) at order n is

$$\frac{\delta r^{(-n+1)}}{(-n+1)} = \mu^{(n)} N + \frac{\delta r_0^{(-n+1)}}{(-n+1)} \quad \text{with} \quad \delta r_0 = \delta r \quad (N=0). \quad (\text{B.3})$$

For $n > 1$, an initial negative perturbation $\delta r_0 < 0$ will decay if $\mu^{(n)}$ is positive, with n even, or $\mu^{(n)}$ is negative and n odd. If the initial perturbation is positive, then it will decay for $\mu^{(n)}$ is negative, for all $n > 1$. If we require that $\mu^{(1)} = 1$ in (B.2), we must have $\delta \chi_1 = \sqrt{3/2} \delta r / 2$ and $\delta w_1 = \delta r / 2$. The procedure consists in evaluating

$$\delta r' = \sqrt{2/3} \delta \chi_1' + \delta w_1' \quad (\text{B.4})$$

and collecting the second order terms of the expansion of (2.44) and (2.45) when the dynamical system is perturbed around the fixed point

$$\begin{aligned} \chi_1 &= \sqrt{\frac{2}{3}} \cos \theta + \delta \chi_1, \\ w_1 &= \cos \theta + \delta w_1. \end{aligned}$$

As the constraint (2.41) imposes that the dynamical system, near the fixed point, can only be subjected to a small negative perturbation, thus, we will consider an initial negative perturbation $\delta r_0 < 0$. Otherwise, a positive perturbation, that would slightly increase the value of the two fields above the fixed point, would imply that the Friedmann constraint (2.39) would blow up to infinity.

As it is seen from (2.44) and (2.45), the presence of the functions λ_1 and λ_2 , which, in turn, depend on the potentials and their derivatives, does not allow to study in general the stability of the type I solutions. Therefore, we illustrate this study for some simple and suitable choice of potentials.

B.1 Identical quadratic potentials

Let us consider the simple case when the two fields are under the influence of identical quadratic potentials, i.e., $V(\chi_1) = \chi_1^2$ and $V(\chi_2) = \chi_2^2$. In this situation, (2.45) and (2.47) exhibit type I solutions for any $0 < \theta < \pi/2$. The fixed points for these solutions are constrained by (2.50). Collecting the second order term in (B.4) we have

$$\mu^{(2)} = -\frac{9}{4} \left(3 \cos \theta + \cos 3\theta \right), \quad (\text{B.5})$$

which is always negative for $0 < \theta \leq \pi/4$. This means that all fixed points with $0 < \theta < \pi/4$ are unstable. If θ gets larger than $\pi/4$ then the fixed point coordinates $\chi_{2c} > \chi_{1c}$ and we can also collect the second order terms in $\delta r' = \sqrt{2/3} \delta \chi_2' + \delta w_2'$ for a negative perturbation $\chi_2 = \sqrt{\frac{2}{3}} \sin \theta + \delta \chi_2$. The coefficient yields

$$\mu^{(2)} = -\frac{9}{4} \left(3 \sin \theta - \sin 3\theta \right), \quad (\text{B.6})$$

which is always negative for $\pi/4 \leq \theta < \pi/2$. When the angle θ is close to $\pi/2$, then the 3-form field χ_1 approaches zero and (B.5) produces positive values for $\mu^{(2)}$. This means that in the asymmetric situation where $\chi_1 \approx 0$ and $\chi_2 \approx \sqrt{2/3}$, the solution $\chi_1(N)$ converges to zero, however $\chi_2(N)$ will be unstable. In fact, from (B.6), the second field will eventually diverge from $\sqrt{2/3}$, when subjected to a small negative perturbation. Furthermore, the decrease in the value of χ_2 implies that the variable w_2 will start to fall faster, as (2.46) suggests. The decrease of χ_2 will proceed until it reaches zero. At this point, we can show that both fields will start to oscillate around zero with a damping factor. The discussion for the situation where θ is near zero, is the same, in the sense that the roles of χ_1 and χ_2 , in the previous discussion, are interchanged. In Fig. 3.1 (left panel), the behavior of the two fields, at the end of the inflationary period, when the angle θ is close to $\pi/2$, are shown. Therein, we see that the two fields are going to a damped oscillatory regime, after the divergence of x_2 from its fixed point. The herein analytical description is numerically confirmed.

B.2 Quadratic and quartic potentials

When the two fields are subjected to the potentials $V(\chi_1) = \chi_1^2$ and $V(\chi_2) = \chi_2^4$, the evolution is generally of the type II. However, (2.45) and (2.47) exhibit type I solutions when the condition (2.52) holds, which in this case becomes

$$\left(\frac{1}{\frac{3}{4} (\cot \theta)^2 \csc \theta + \sin \theta} \right)^2 + \left(\frac{6 \cos \theta}{6 - \cos 2\theta + \cos 4\theta} \right)^2 = 1. \quad (\text{B.7})$$

This last condition is satisfied for $\theta \rightarrow \pi/3$, $\theta \rightarrow \pi/2$ and at $\theta \rightarrow 0$. Collecting the second order term in (B.4) we have

$$\mu^{(2)} = -\frac{3}{5}, \quad (\text{B.8})$$

which is negative. This means that the fixed point with $\theta = \pi/3$ is unstable. At $\theta = 0$, i.e., the scenario with the quadratic term dominance, we must go to third order since, $\mu^{(2)} = 0$. In that

case, collecting the third order terms we have $\mu^{(3)} = 0.28$, which means that the fixed point is unstable. At $\theta = \pi/2$, scenario with the quartic term dominance, $\mu^{(2)} = -7.5$, which means that the fixed point is unstable.

Appendix C

Analytical approximations

This appendix constitutes a complement for chapter 3.

Parametrization 1

Using the definition of the hypergeometric function [207] we have

$${}_2F_1(1, 1 + \beta; 2 + \beta; z) = \frac{\Gamma(2 + \beta)}{\Gamma(1 + \beta)} \sum_{n=0}^{\infty} \frac{\Gamma(1 + n)\Gamma(1 + \beta + n)}{\Gamma(2 + \beta + n)} \frac{z^n}{n!} = (1 + \beta) \sum_{n=0}^{\infty} \frac{z^n}{1 + \beta + n}. \quad (\text{C.1})$$

For $\beta < 1$ we can approximate

$$\frac{1}{1 + \beta + n} = \left(\frac{1}{1 + n} \right) \frac{1}{1 + \frac{\beta}{1+n}} \simeq \frac{1}{1 + n} \left(1 - \frac{\beta}{1 + n} \right) = \frac{n + 1 - \beta}{(n + 1)^2}, \quad (\text{C.2})$$

and substituting in (C.1) we arrive at

$${}_2F_1(1, 1 + \beta; 2 + \beta; z) \simeq (1 + \beta) \sum_{n=0}^{\infty} \frac{n + 1 - \beta}{(n + 1)^2} z^n = -\frac{(1 + \beta)}{z} (\ln |1 - z| + \beta \text{Li}_2(z)), \quad (\text{C.3})$$

where $\text{Li}_n(z) = \sum_{k=1}^{\infty} k^{-n} z^k$ is the polylogarithm function [207]. Despite its being an excellent approximation for $\beta < 1$, substituting the above into (3.38) leads to a differential equation still too complicated (to solve for $a(t)$) due to the polylogarithmic function $\text{Li}_2(z)$. Our aim, therefore, is to find a simple analytical solution reproducing the qualitative behaviour of the scale factor. The simplest manner to achieve this is to neglect the term in the polylogarithm function in (C.3). This simplification can be justified after approximating

$$\sum_{n=0}^{\infty} \frac{z^n}{1 + \beta + n} \simeq \sum_{n=0}^{\infty} \frac{z^n}{1 + n} = -\frac{\ln |1 - z|}{z} \quad (\text{C.4})$$

in (C.1), which holds provided $\beta \ll 1$. In that case, after substituting $z \rightarrow \lambda_1 H^2 / \lambda_2$, the resulting background equation (3.39) has the advantage of being relatively simple.

Parametrization 2

Using the variables z and y defined in (3.43), our (3.11) becomes

$$\lambda_{1*} e^{y\alpha_1} e^{2z} - \lambda_{2*} e^{y\alpha_2} - e^{2z} z'(y) = 0. \quad (\text{C.5})$$

After multiplying by $\mu(y) = \exp[-(2\lambda_{1*}/\alpha_1)e^{y\alpha_1}]$, (C.5) becomes an exact differential equation

$$df = P(y, z) dy + Q(y, z) dz = 0, \quad (\text{C.6})$$

where

$$P(y, z) = \mu(y) [\lambda_{1*} e^{y\alpha_1} e^{2z} - \lambda_{2*} e^{y\alpha_2}] \quad \text{and} \quad Q(y, z) = -\mu(y) e^{2z}. \quad (\text{C.7})$$

Integral curves are of the form: $f(y, z) = \kappa$, where κ is a constant. Integrating f with respect to y in the first place we have

$$f(y, z) = \int P(y, z) dy + g(z), \quad (\text{C.8})$$

where $g(z)$ is to be computed by demanding $\partial_z f(y, z) = Q(y, z)$. After integrating and solving for $g(z)$ we find that the integral curves $f(y, z) = \kappa$ are determined by (3.44).

Appendix D

A review of SFT and Tachyon condensation

This appendix assists for chapter 4.

In generic words SFT is an off-shell description of interacting strings [122, 221, 229, 335-337]. It describes a string by means of a string field Ψ . This object is a shorthand for encoding all the string excitations in one instance. The corresponding action for open string field¹ can be written as

$$S = \frac{1}{g_o^2} \left(\frac{1}{2} \int \Psi \star Q\Psi + \frac{1}{3} \int \Psi \star \Psi \star \Psi \right), \quad (\text{D.1})$$

where \star and \int are Witten product and integral for string fields respectively. Q is the BRST charge. The first term clearly corresponds to the motion of free strings while the second term represents the interaction. The second term is the three-string vertex responsible for the non-perturbative physics. g_o is the open string coupling constant, it is dimensionless.

It has been understood [211, 340-343] that the tachyon of open strings is responsible for the decay of unstable D -branes or D -brane-anti- D -brane pairs. The corresponding process is the condensation of the tachyon (TC) to a non-perturbative minimum. Upon the TC the unstable brane (or pair) decays. It is the cornerstone of Sen's conjecture regarding TC that the depth of the tachyon potential minimum is exactly the tension of an unstable brane to which the string is attached to. The decay of a brane represents a configuration in which open strings must not exist, because the brane, to which they were attached, has decayed [344, 345]. This being said, let us assume Sen's conjecture, which prescribes the disappearance of open string excitations. The latter phenomenon of open strings extinction can be formalized as follows in the field-theoretical language. Given a field φ the following quadratic Lagrangians are non-dynamical

$$L = -m^2\varphi^2 \text{ or } L = \varphi e^{\gamma(\square)}\varphi. \quad (\text{D.2})$$

The left Lagrangian is clearly a mass term without any dynamics. In the right Lagrangian, \square is the space-time d'Alembertian and γ is an entire function. Although it may look like \square produces dynamics as it is a differential operator, as long as we require that the function in the exponent is an entire function, the whole exponent has no eigenvalues as an operator. This means that the inverse of such an exponent gives no poles in the propagator and effectively we have no dynamics at all.

We further notice that the right Lagrangian in (D.2) is an essentially non-local Lagrangian. It is obviously non-dynamical on the quadratic level and as long as the field φ is alone. However, novel and unusual effects can be generated upon coupling to other fields or in the non-linear physics [123, 212, 213, 216, 346].

The essence of SFT is that as long as a string interaction is involved then the non-locality of the above type emerges. Technically, we can understand this as follows. Strings are extended

¹An action for a closed SFT can be written only in a non-polynomial form, even for the bosonic strings [338, 339].

objects by construction. When a field-theoretic model describes strings, this property of an extended object is encoded in the non-locality of interactions. SFT straightforwardly creates vertex terms of the form

$$\sim \left(e^{\alpha' \square} \varphi_1\right) \left(e^{\alpha' \square} \varphi_2\right) \left(e^{\alpha' \square} \varphi_3\right) \quad (\text{D.3})$$

Here α' is the string length squared (which may be different from the inverse of the Planck mass squared). We aim to convey in the course of this paper² that non-locality indeed proves crucial in constructing (SFT inspired) cosmological models.

It is sufficient for the purposes of the present paper only to note that upon lengthy computations [122], the quadratic Lagrangian of the open string tachyon \mathcal{T} near the vacuum is non-dynamical of the form

$$L_{\mathcal{T}} = -\frac{T}{2} v(\square, \mathcal{T}). \quad (\text{D.4})$$

For zero momenta, i.e. when $\square = 0$ the resulting $v(0, \mathcal{T})$ is exactly the tachyon potential. The dependence on \square is analytic and being linearized near the vacuum value of field $\mathcal{T} = \mathcal{T}_0 + \tau$ it produces

$$L_{\tau} = -\frac{T}{2} \frac{v''(\mathcal{T} = \mathcal{T}_0)}{2} \tau e^{\gamma(\square) \tau}, \quad (\text{D.5})$$

with some entire function $\gamma(\square)$. The coupling T is nothing but the tension of the unstable D -brane given as

$$T = \frac{1}{2\pi^2 g_o^2 (\alpha')^{\frac{p+1}{2}}}, \quad (\text{D.6})$$

where α' is the string length squared, g_o is the open string coupling constant and p comes from the dimensionality of the Dp -brane. Thus, as expected for a 3-brane, T has a dimension $[\text{length}]^{-4}$ and the tachyon field τ is dimensionless.

²Computing any process in SFT leads to much more complicated results than presented above. TC is not an exclusion. Schematically, to describe the TC we should first compute an effective action in which all massive modes of a string with positive mass square are integrated out. Upon this computation a non-local interaction of several tachyons arise. The non-local operators are not just identical exponents but rather algebraic combinations of them. This effective action is enough to test Sen's conjecture for both, depth of the potential and absence of dynamics at the bottom of the potential. It is worth noting that actual computations in SFT are indeed difficult and technical performed by means of a level truncation scheme (i.e. including only fields up to a given mass m and the next iteration includes fields up to mass $m + 1$, etc. [347]). This scheme was proven to be convergent [347].