8th World Congress on Structural and Multidisciplinary Optimization
June 1 - 5, 2009, Lisbon, Portugal

An Optimized Approach to Reduced Fuel Costs in the Operational Procedures of an Airline

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Abstract
Recent environmental and economical constraints have been forcing the airlines to take important measures in their operational procedures in order to reduce fuel costs. In this paper a study of the centre of gravity (CG) position effect on the total fuel consumption was carried out with an airline. The loading procedure is a critical operational issue since it has a direct influence on the weight and balance of the aircraft with a strong impact on its performance.

The correct selection of the CG position within the operational limits of the aircraft leads to an important aerodynamic drag reduction and hence a significant reduction in the fuel burnt. The results from this study were based on a statistical analysis of both load sheets and flight plans aiming at finding a relationship between the CG position and the trip fuel consumption resulting from the current loading practice of the airline which follows the procedures suggested by the aircraft manufacturer. The information gathered from these data enabled the formulation of an optimization problem that was designed towards finding the best ideal trim line for improved performance which, in turn, was validated with simulation results obtained by using a computer program provided by the aircraft manufacturer.
Keywords: aircraft operation, weight and balance, ideal trim, optimization problem

1. Introduction

With the surge of fuel prices in the early 1970s both airlines and aircraft manufacturers started developing systems and procedures to reduce fuel consumption. In some airlines, fuel cost at one point in time represented no less than 45% of their cash operating costs [1]. From the airlines side several operational procedures were adopted to overcome this problem, like flap reduced deflections at takeoff and landing, single-engine taxi, idle reverse at landing, reduced APU usage or optimized flight plan software. Beyond the operational procedures, also maintenance procedures like drag reduction programs and engine washing were put in place in the past years. With an impact in the fuel consumption, several ground operations procedures were introduced in the fuel conservation programs that proliferate within the airlines. On the ground operations procedures, potable water servicing and cargo loading for optimized centre of gravity are two of the most remarkable measures.

Technically, CG is the theoretical point where the total weight of the aircraft is assumed to be concentrated. On the aircraft design, specific structural and operational limits to the CG position are defined to ensure a safe operation [2,3]. Aircraft balance, both lateral and longitudinal, is important, but the prime concern is longitudinal balance, that is, the location of the CG along the longitudinal axis of the airplane

Airbus® has created a trim tank transfer system that controls the centre of gravity of the airplane. This system is installed in all Airbus A330®. When an airplane with a trim tank is in cruise, the system optimizes the centre of gravity position to save fuel by reducing the drag on the airplane. The system transfers fuel to the trim tank (aft transfer) or from the trim tank (forward transfer) [4]. This movement of fuel changes the centre of gravity position. The Fuel Control and Management Computer (FCMC) calculates the centre of gravity of the airplane from various parameters, including input values such as Zero Fuel Weight and the associated CG, ZFW CG, and the fuel tank contents. It continuously calculates the CG position during flight.

As long as the CG is maintained within the allowable limits for its weight, the airplane will have adequate longitudinal stability and control. If the CG is too far aft, it will be too near the centre of lift and the airplane will be unstable, and difficult to recover from a stall condition. On the other hand, if the CG is too far forward, the downward tail load will have to be increased in order to maintain level flight. This increased tail load has the same effect as carrying additional weight; the aircraft will have to fly at a higher angle of attack, and drag will increase. The position of the aircraft CG during the flight has an impact on the fuel consumption. Indeed, an aft CG position reduces the drag and thus implies fuel savings. That is the reason why a CG control system has been introduced on some long-range family aircraft. However, there are some limits that must be respected when designing a CG envelope. In fact, CG position has to stay within certain certified limits, which are mainly due to: structural limitations, handling qualities and/or a compromise between performances and aircraft loading.

Most commercial airplanes have a Trimmable Horizontal Stabilizer (THS) that can be actuated as an aiding trimming device that guarantees an extra balancing moment [3]. THS creates a downward force which in turn creates a pitch-up moment that counters the pitch-down moment caused by the aircraft weight. However, THS actuation implies a drag increase whose magnitude depends on the aircraft CG position. The further forward the CG position is, the greater the counter moment to maintain flight level is required. This is due to the increased balance arm between lift and weight. In the case of a forward CG position, the THS is set to an aircraft nose-up position that creates important lift degradation therefore creating important drag. This drag will lead to an increase in fuel consumption. As a rule of thumb, one can say that the further aft the CG, the lower the fuel consumption.

In the case of the airplane considered in this paper (Airbus A330-200®), the CG position during cruise is controlled by transferring fuel to or from the trim tank. This operation is managed by the FCMC that is continuously seeking for an aft CG position, thus giving better in-flight performance.

2. Determination of operational limits

The aircraft CG position determination using a paper or computerized load sheet is influenced by the weight inaccuracy of the item that will be loaded [3]. Furthermore, the aircraft CG position changes during flight due to the movement of passengers and fuel transfer. Before flight, the aircraft CG position must be checked against certified envelopes. However, due to the CG position uncertainties some margins must be determined between the certified envelopes and the ones used on the trim sheet: *the operational limits*.

These operational limits are related to ZFW and Take-Off Weight (TOW). The first parameter is the total weight of the aircraft without any fuel whilst the latter considers the fuel weight added to ZFW. To determine the aircraft ZFW, TOW and CG position during flight one needs to know:

i. aircraft CG position and weight before loading any item;
ii. weight and position of each item loaded on the aircraft (cargo, passengers, fuel, any additional item);
iii. possible CG movements due to moving items during flight.

An inaccuracy on the final CG value can be introduced because of lack of precision either on item weight or on its location and/or CG position. The total possible inaccuracy incurred in the CG position estimation will be the result of a combination of all the individual errors due to initial conditions, cargo loading, passenger boarding or fuel loading.

In addition to these inaccuracy sources, the method used to determine the aircraft CG may also add an inaccuracy source if it needs index rounding or index interpolation.

The aircraft ZFW, TOW and CG position computation is based on the initial aircraft configuration corresponding to Dry Operating Weight (*DOW*) and Dry Operating Centre of Gravity (*DOCG*). Passenger and fuel movements during flight will cause changes on the aircraft configuration, and the subsequent CG movement needs to be taken into account in the operational limits determination. Also, during refuelling operations of long range aircraft a Fuel Control and Monitoring System (FMCS) follows a fuelling sequence which maintains the initial and final CG positions within specified limits. This sequence is illustrated in Figure 1 for the Airbus A330-200® aircraft. The parameter delta index, in the axis of abscises indicates the variation of the CG index of the aircraft due to fuel weight. The index parameter is a non-dimensional quantity that relates the CG position of any component in the aircraft with an arbitrarily chosen reference. Hence the final index parameter, after adding the fuel load to the ZFW, varies only by a small amount.

As it is shown in Figure 1 the refuelling procedure is divided in several segments where each segment corresponds to a different fuelling sequence: 1 - refuelling inner tanks up to 4500 kg; 2 - outer tanks full; 3- inner tanks up to total fuel of 36500 kg; 4 - trim tank up to 2400 kg; 5 - inner tank full; 6 - trim tank full and centre tank full (simultaneously). Segments 7 and 8 are not applicable for the A330-200® version considered in the present work. From Figure 1 it is also possible to conclude that the refuelling vector always positions the TOW CG forward of the ZFW CG. The first segment, the segment corresponding to the ZFW CG, has a -10 Delta Index in this refuelling example, whilst the last segment, has a -5 Delta Index. This point corresponds to the actual TOW CG.

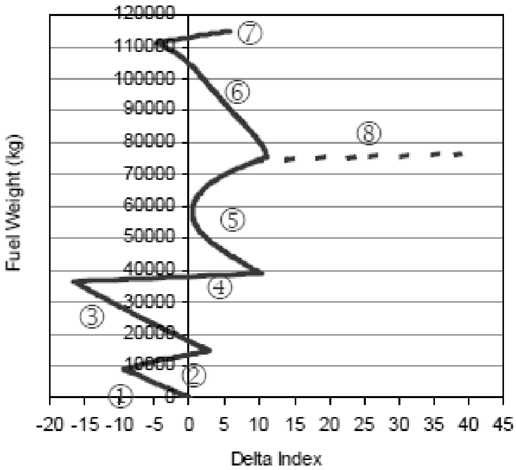

Figure 1: Refuelling sequence for the A330-200 aircraft family.

On the balance charts used by aircraft operators it is mandatory to check the TOW CG position against the takeoff operational limits in order to ensure that the aircraft CG position is within the certified takeoff limit. Nevertheless, this check is not sufficient to guarantee that the aircraft CG remains within the in-flight and landing limits during the whole flight. This procedure cannot be easily made by checking the landing CG position and the different flight CG positions because of the difficulty to estimate these values. Therefore, an additional limit has been introduced to perform this check procedure: the ZFW limit. ZFW limit is determined to ensure that during the whole flight (including the landing stage) the aircraft CG remains within the required operational limits.

For each type of ZFW limit the calculation principle is the same, consisting of:

i. Determining the possible aircraft CG position at takeoff, landing and in-flight, provided that the ZFW CG position is known, by analyzing the fuel vectors applicable to each flight phase.
ii. Using the fuel vectors, determining all the allowed ZFW CG positions so that all possible takeoff (for takeoff protected limit), landing and in-flight CGs remain within their operational limits from no fuel to maximum fuel quantity.

The Ideal Trim Line (ITL) is a theoretical line that defines an optimal position for aircraft centre of gravity, as a function of its weight, when the aircraft is out of fuel, i.e., in a ZFW condition. As mentioned before the CG position influences the THS angle of attack. Different angles of attack mean different fuel consumptions. The aircraft manufacturer recommends the aircraft loading process to be done in such a way that the CG position is as close as possible to the ITL.

3. Methodology

The aircraft CG position effect on the total flight fuel consumption varies significantly with the flight length, being more relevant on long-haul flights than on short/medium haul flights where such influence is not perceivable. The present study was applied only to long-haul flights using Airbus A330-200®, since this aircraft is equipped with a trim tank on the horizontal stabilizer that optimizes in-flight CG position.

Under this study, CG position was optimized using two distinct methods: (1) a statistical approach used to implement an ITL method was accomplished through the analysis of load sheets information for several flights, and (2) an optimization approach to determine the TOW CG position based on a computational numerical optimization procedure that utilizes Hardy Multiquadric Functions. In this latter approach information from actual load sheets and flight plans was used. The results obtained from the numerical optimization method were validated by comparing them against results obtained from a flight performance software provided by the aircraft manufacturer

3.1 The ITL implementation

One of the goals of the present work is to answer the question: *how much an aft ITL location is possible?*

Each flight has a specific load sheet. The load sheet is a standardized document where some parameters which affect aircraft load distribution and flight performance such as payload, DOW, TOW, ZFW, trip fuel, flight CG, among others are recorded. This information panoply of a total of 250 Airbus A330-200® flights was used to characterize the CG statistical distribution. From these data the statistical distribution of the ZFW CG position – in terms of mean aerodynamic chord (MAC) – is shown in Figure 2, which follows a typical Gauss distribution with an average value of about 29.76% and a corresponding standard deviation of 2.18%.

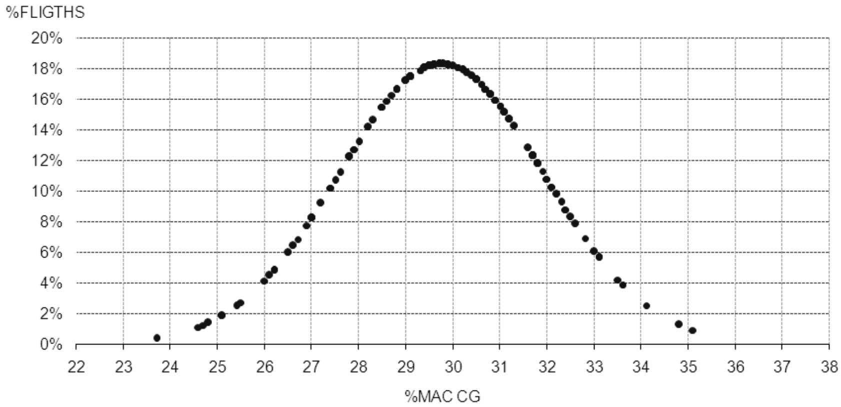

Figure 2: ZFW CG distribution as obtained from the analysis of the load sheets.

From this ZFW CG distribution one can conclude that the dispersion of CG positions is around 11% MAC CG. This figure is extremely important to identify the ITL maximum aft position which ensures that all CG positions are within the operational limits indicated in the operational manual of the aircraft. However, there are some drawbacks associated with the implementation of a corrected ITL, namely, the associated uncertainty level of the loading process and several difficulties related with the standardization of the process at the operational staff level. Thus, an alternative method has been analyzed in order to find an optimized CG location using a more solid approach.

3.2. Approximation using Hardy Multiquadric Functions

The Hardy multiquadric approximation methods described in [5,6] deal with the problem of approximating or interpolating a multivariate function $y = f(x)$ by a function $\hat{f}(x,\theta)$ having a fixed number of parameters θ. Variable x and parameter θ may be vectors of any finite dimension, that is, $x = (x_1 \ldots x_n)^T \in \Re^n$, $\theta = (\theta_1 \ldots \theta_m)^T \in \Re^m$, $n \geq 1, m \geq 1$. In the sequel we assume variable y to take scalar values $(y \in \Re)$.

The four dimensions in the variable input list, referred as vector *x*, represents cruise altitude (h), flight range (r), wind component (w) and take-off weight (TOW). Variable *y* that stands for the two dimensions output variable represents the optimal TOW CG and the corresponding minimal fuel consumption. Therefore, it may be seen as a sample of input-output pairs of a function *f* such that $y = f(x)$.

Approximating function *f* by a parametric function $\hat{f}$ having the following generic expression:

$$\hat{f}(x,\theta) = \sum_{k=1}^{m} \theta_k \psi_k(x) \tag{1}$$

where each ψ_k is a Hardy multiquadric function [6] defined by:

$$\psi_k(x) = \sqrt{\sigma^2 + \left\| x - x^k \right\|^2} \tag{2}$$

with $\sigma > 0$, called shape factor, and $x^k \in \Re^n$, the centre of function ψ_k.

Approximation of large scale samples, or dealing with online processes, requires classification-based [5,6] or densification-based methods to find appropriate centres and shape factors. However, in case of small scale data samples, the centres and the shape factor are determined as follows:

1. The set of the centres $\{x^k\}_{k=1,\ldots,m}$ is chosen to be equal to the set of the input data of the sample $\{x^i\}_{i=1,\ldots,p}$. Consequently, $m = p$ (in that case).
2. The shape factor is computed as ([7,8]):

$$\sigma = \max_{i=1,\ldots,p}(\min_{j:j<i}\left\|x^i - x^j\right\|) \tag{3}$$

Thus, having determined the shape factor and the centres of the functions, we can now search for the elements of the parameter vector $\theta = (\theta_1 \ldots \theta_m)^T$ in the expression of the approximating function – Eq. (1). Indeed, the parameter vector is sought as the minimiser of the following cost function:

$$J(\theta) = \frac{1}{2}\sum_{i=1}^{p}\left\|y^i - \hat{f}(x^i,\theta)\right\|^2 \tag{4}$$

However, the expression of the approximating function in Eq. (1) is equivalent to:

$$\hat{f}(x,\theta) = \theta^T\psi(x) \tag{5}$$

with $\psi(x) = (\psi_1(x) \;\; . \;\; . \;\; . \;\; \psi_m(x))^T$. Therefore, the cost function in Eq. (4) may be written as:

$$J(\theta) = \frac{1}{2}\sum_{i=1}^{p}\left\|y^i - \theta^T\psi(x^i)\right\|^2 \tag{6}$$

It is clear that the cost function is quadratic with respect to the parameter vector. Hence, the minimiser θ_{opti} is found analytically [8] as:

$$\theta_{opti}^T = \left(\sum_{i=1}^{p} y^i\psi^T(x^i)\right)\left(\sum_{i=1}^{p}\psi(x^i)\psi^T(x^i)\right)^{-1} \tag{7}$$

The previous equation is the output of the optimization problem representing the optimal TOW CG position that minimizes the fuel consumption for all inputs *x* considered.

4. Results and Discussion

As previous explained, an aft CG position minimizes the aircraft fuel consumption. A procedure to place the ITL in a more aft position is derived. To achieve this, one needs to take into consideration that all CG translations to a more aft position must be done without compromising aft CG limits as represented in Figure 3. In addition, the most critical CG position, the one that corresponds to the maximum aft CG point of the ZFW flight envelope, must be identified. Therefore, the corrected ITL must be placed as close as possible to the operational aft limit assuring that all CG locations fall inside the envelope. Due to safety and operational requirements, a small safe margin should be kept between ITL and ZFW limit (of about 3% Index). As previously explained, the refuelling vector will be an aiding factor to accomplish this requirement, since when adding fuel, all ZFW CG locations will be forced to move up and left (forward) inside the flight envelope of Figure 3.

Load sheets from about 250 Airbus A330-200® long-haul flights were analyzed under this work. Figure 3 shows the dispersion of ZFW and TOW CG positions throughout the flight envelope for the considered flights. These CG positions were found by following the default ITL ("ITL" shown in Figure 3) as a guide. The results obtained from this analysis show an average ZFW CG of 29.76% and an average TOW CG of 28.53%. The ZFW CG distribution follows the default ITL with a dispersion of approximately 11% MAC. However, it is clear that there is still a significant margin to place the ITL in an aft position (when compared with its actual position) without compromising the operational limits.

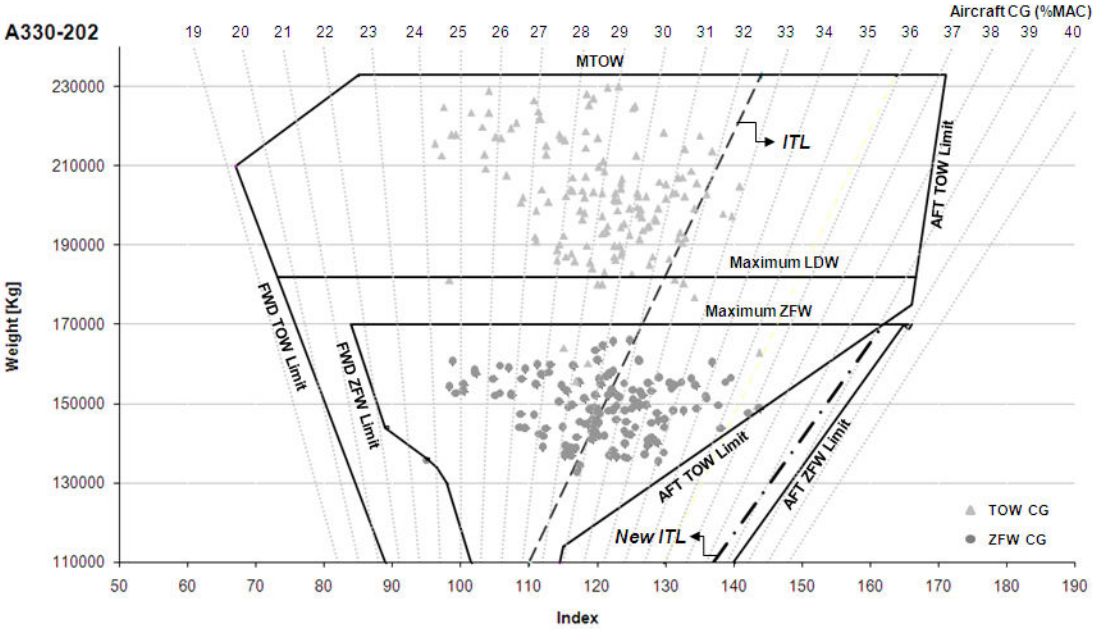

Figure 3: Flight Envelope of an Airbus A330-200® aircraft with the ZFW and TOW CGs distributions.

The actual loading procedure in the airline is based on manual inputs to approximate the ZFW CG as much as possible to the ITL by manipulating the passenger and cargo weight distribution along the longitudinal axis of the aircraft. The software user tries to locate the ZFW CG as close as possible to the ITL, disregarding which side of the ITL is chosen. However, and from a theoretical point of view concerning a CG optimal location, the best ITL position clearly coincides with the right limit of ZFW flight envelope. Due to inherently manual procedure and the limitations on the software currently used in the loading distribution, a practical implementation of a corrected ITL method is not feasible.

The abovementioned limitations of the ITL method led to the development of an alternative optimization procedure based on Hardy multiquadratic functions, as explained in the previous section. The results obtained from this method were validated through a direct comparison against those obtained from the Performance Computational Program (PEP)® provided by the aircraft manufacturer. With this software one can obtain a set of operational output variables for a specific input flight configuration based on performance algorithms of the aircraft. Table 1 shows a set of input parameters that were considered in order to reproduce real flight conditions.

Table 1: An example of input parameters used in a performance analysis with PEP®.

Parameter	Condition
Air Conditioning	100%
Anti Icing	off
Temperature (K)	ISA+10,5
Drag factor	1.035
Fuel Reserves	5%
Cruise Mach	0.780
Taxi Fuel (Kg)	150
Taxi Time (min)	12

From the inputs described in Table 1, standard values were used for air conditioning (frequently used at 100%), anti-icing system (usually used in off position), outside air temperature, fuel reserves (which is a fixed parameter due to international standard requirements), Taxi (the movement of the aircraft at the airport since de ramp-off to the line-up runway position) Fuel and Taxi Time. The drag factor (that is a correction performance factor due to some aerodynamic degradation) considered was the actual average value for the Airbus A330-200s® used by the airline. The cruise Mach number (ratio between actual velocity and sound velocity propagation at the same place) considered was the average value of the flights analyzed.

An evaluation of the influence of the fuel consumption on the CG position using the applicable flight performance software is presented on Table 2. The input values considered correspond to typical long-range flight conditions, which roughly have a range around 3500 nm and a cruise altitude around 39000 ft. The payload is defined as the sum of passenger and cargo weights. Constant cruise wind intensity was considered for both tailwind (positive sign) and headwind (negative sign). Wind plays a relevant role in fuel consumption, and its effect can be predicted based on specific analytical expressions found in the literature [9].

The aircraft CG position effect on the total Fuel On Board (FOB), Trip Fuel and TOW is shown in Table 2.. It is clear that there is an optimal CG position that minimizes the total flight fuel consumption. This minimum value can be easily determined from a graphical representation of these results, as seen in Figure 4. As one can observe, for this specific example, the optimal TOW CG position is around 37%. An aft or forward CG movement from this position will result in an increase of total Trip Fuel, due to the reasons explained in sections 1 and 2.

Table: 2 Inputs and Outputs for a sample flight using the PEP® software.

Inputs									
Cruise Altitude (ft)	39000	39000	39000	39000	39000	39000	39000	39000	39000
CG (%)	30	31	32	33	34	35	36	37	38
Payload (kg)	30000	30000	30000	30000	300000	30000	30000	30000	30000
Distance (nm)	3500	3500	3500	3500	3500	3500	3500	3500	3500
Cruise Wind (kts)	-10	-10	-10	-10	-10	-10	-10	-10	-10
Outputs									
Total FOB (kg)	51175	51140	51068	51032	51002	50969	50983	50964	50974
Trip Fuel (kg)	44161	44126	44057	44021	43992	43959	43970	43950	43955
TOW (kg)	206989	206954	206882	206846	206817	206783	206797	206778	206788

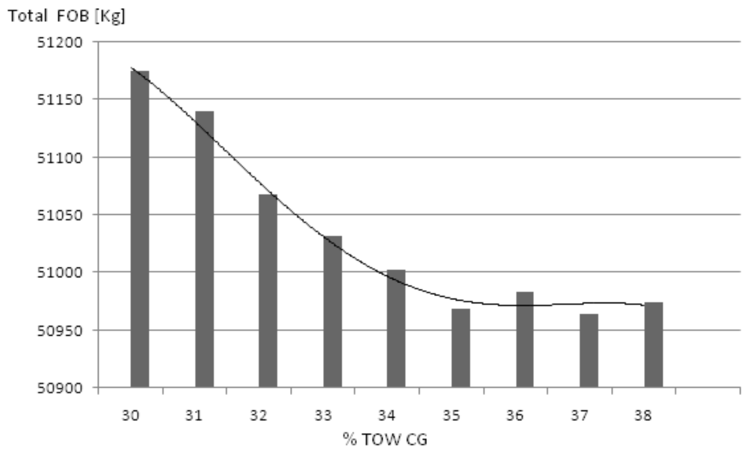

Figure: 4 - Effect of TOW CG position in total FOB.

The Hardy computational method was used to minimize a function that relates TOW CG with cruise altitude, TOW, flight distance and cruise wind using actual flight data. The outputs of this minimization problem are listed in Table 3. From the data considered, the best TOW CG is 37%, with a corresponding Trip Fuel of 43950 kg. Nevertheless, Hardy's method only works within bounds of the input parameters within which the function is valid. The associated error by using this approximation is 1.005752779×10^{-7}.

Table 3: Inputs and outputs using Hardy's computational method.

Inputs		**Interval {min ; max}**
Cruise Altitude (ft)	39000	{37000 ; 40000}
TOW (kg)	206797	{174592 ; 227951}
Distance (nm)	3500	{522 ; 4293}
Cruise Wind (kts)	-10	{-27 ; 4}
Outputs		
Optimal Trip Fuel (kg)	43950	
Optimal TOW CG (%)	37	

For the flight example seen in Table 3 the same result as that obtained with the Airbus PEP® program was computed. The Optimal Trip Fuel and the Optimal TOW CG obtained were 43950 kg and 37%, respectively, with both methods.

There is one specific CG location where $\partial C_{M,CG} / \partial \alpha = 0$, i.e., the change of aircraft pitching moment with angle of attack is zero. In this condition, the position of the CG is defined as the *neutral point* [10]. In Figure 4, the TOW CG of 37% corresponds to the minimum fuel consumption. For this aircraft type, the neutral point is located at 35%. Aft of this point, a slight increment in fuel consumption is present. Recalling that the definition of the aerodynamic centre for a wing is the point where moments are independent of the angle of attack, then the neutral point might be considered the aerodynamic centre of the complete airplane. Therefore the optimal TOW CG position can be defined at about 37% MAC. However, and by analyzing other flight data, the neutral point is not perceivable, i.e., in practice this point may not always be located at the theoretical location due to various parameters like downwash angle (the angle between the relative wind at the tail and the non-perturbed flow).

5. Conclusions

The objective of this paper was to understand how the aircraft centre of gravity position could influence the fuel consumption and hence to determine the position of the aircraft CG that minimizes the fuel consumption and consequently the airline costs and engine emissions per flight.

Two distinct methods were used: an ITL method and a Hardy multiquadric function approximation to actual flight data. A new ITL was suggested to reduce fuel requirements but an ITL optimal position implementation is not very easy to implement using the current airline load program due to its intrinsic constraints. The Hardy multiquadric function implementation which was based on actual flight data showed to be a good approximation with a much reduced associated error. This method can be easily implemented into a computer program that can be utilized by the airline as an alternative to its current program.

6. Acknowledgments

The authors would like to express their gratitude to Dr. K. Bousson (UBI) for his contribution to the Hardy multiquadratic function implementation, and also to Eng. Rogério Zacarias for all his technical support during the course of this work.

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