



UNIVERSIDADE DA BEIRA INTERIOR  
Ciências Sociais e Humanas

# The role of the time-of-use electricity tariffs in the control of the rebound effect in the household energy consumption

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Dissertação para obtenção do Grau de Mestre em  
**Economia**  
(2º ciclo de estudos)

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**Covilhã, Junho de 2015**



# Acknowledgements

I would like to thank my supervisor, the Professor António Cardoso Marques for giving me guidance since the first moment. His orientation and incentive was determinant for the final result of this master thesis. A thank you is also in order to Professor Pedro Morais for his valuable assistance and advices on improving this investigation. I would like to express my deepest gratitude to my family, girlfriend and friends for all their understanding, affection and immeasurable support through the years of my studies. Without them I would not have come this far.



# Resumo

Uma das formas de conservação de energia e de redução dos Gases de Efeito Estufa (GEE) é a utilização mais racional no consumo. No entanto, nem sempre o aumento da eficiência conduz à diminuição do consumo e conseqüentemente à diminuição dos GEE. O efeito da não obtenção da totalidade dos benefícios decorrentes do aumento da eficiência da energia é conhecido como *rebound effect*. Assim, este trabalho tem como objetivo focar-se no *rebound effect*, controlando-o através da definição de preços diferenciados consoante o período de consumo. É desenvolvida uma função de produção familiar que incorpora o consumo de eletricidade, diferenciando-o nos períodos em pico e em fora de pico. Calcula-se a função para o *rebound effect* direto através da elasticidade do serviço em eletricidade em relação ao aumento da eficiência de energia. Obtém-se as variações de consumo, em cada um dos períodos, que estão associadas a determinado *rebound effect* direto. Nesta perspetiva, é definida uma nova tarifa *time-of-use (TOU)* (bi-horária) sendo que os preços variam com o objetivo de anular o *rebound effect* total. Desta forma, o aumento da eficiência em energia provoca uma maior diminuição do consumo no período mais crítico (período de pico), promovendo a suavização do diagrama de carga. Propõe-se corte parcial da prioridade de despacho da eletricidade de base renovável em regime especial libertando assim recursos para promover maior eficiência de energia. O aumento da eficiência em energia será acompanhado pela nova tarifa *TOU*, tornando esses ganhos de eficiência efetivamente eficazes, uma vez que a tarifa proposta assegura o controlo dos *RE*.

## Palavras-chave

Eficiência da energia; Tarifas *Time of Use*; Consumo de electricidade em pico e fora de pico; *Rebound effect*.

## Resumo alargado

O aumento da eficiência em energia é a forma crucial para um mercado elétrico mais eficaz, seja no lado da procura como no lado da produção, levando a menores perdas e a um menor consumo de eletricidade.

Os aumentos de eficiência podem revelar-se de variadas formas, sendo que em todas as formas em que se revelem aumentos de eficiência é possível produzir um mesmo serviço através de uma utilização de energia menor. Apenas com o aumento da eficiência, seja na produção de eletricidade ou na sua utilização por parte de equipamentos mais eficientes, é possível “com menos, fazer mais”. No entanto, nem sempre os aumentos da eficiência garantem uma efetiva redução do consumo de eletricidade. Ou seja, um aumento da eficiência pode de facto reduzir o consumo de eletricidade associado a um determinado serviço, mas a utilização desse serviço poderá aumentar sempre que a diminuição do consumo não corresponda na sua totalidade ao aumento da eficiência ocorrido.

O fenómeno que corresponde às diminuições do consumo de energia inferiores às “esperadas” quando há um aumento da eficiência em energia é conhecido na literatura como o *rebound effect* (*RE*). Desta forma os aumentos da eficiência são subaproveitados e os benefícios decorrentes destes serão menores. Com a problemática do *RE* presente, este trabalho propõe a definição de uma nova tarifa de acordo com o período de consumo (Time-of-use (TOU)) que acompanha os aumentos de eficiência. A nova tarifa TOU será definida com o intuito de manter o nível inicial de consumo em fora de pico devido à problemática dos consumos muito “baixos”. Este problema é agravado em países onde o mix energético tenha como fontes de produção energias renováveis com tarifas feed-in. Essas tarifas ao garantirem à eletricidade, proveniente de produção renovável, uma prioridade de despacho, torna-a dificilmente acomodável em períodos de menor consumo.

A manutenção do consumo em períodos em fora de pico, quando existe um aumento da eficiência, leva a um *RE* de 100% porque o aumento da eficiência não se traduz em redução do consumo o que levará a um aumento da utilização dos serviços em eletricidade. Nos momentos de pico a tarifa TOU será definida de maneira a que o *RE* nos momentos de fora de pico seja anulado, mas anulando também o próprio *RE* no período em pico. Esta definição do preço em pico levará a uma diminuição do consumo através do aumento do preço. Assim para além de o *RE* ser controlado através da nova tarifa TOU, esta também levará a uma maior suavização do diagrama de carga, uma vez que o “gap” entre o consumo de pico e fora de pico será menor.

Será também proposta uma prioridade de despacho condicionada às eólicas, que num primeiro momento não se traduz numa redução direta no preço da tarifa de eletricidade aos consumidores, mas o montante libertado do condicionamento da prioridade de despacho será utilizado para um incentivo à eficiência em energia. Essa terá agora a sua eficácia potenciada

devido à proposta das variações simultâneas dos preços de uma tarifa TOU. A prioridade de despacho será condicionada de acordo com a eficiência das turbinas que produzirem a eletricidade, promovendo-se também a eficiência em energia no lado da produção de eletricidade.

A eficiência das turbinas será definida através do Limite de Betz, que permite averiguar quanto da força mecânica do vento uma determinada turbina eólica consegue extrair da passagem do próprio recurso vento. O trabalho de Betz demonstra que nenhuma turbina consegue extrair mais de 59,26% da força mecânica do vento e dessa forma será feita uma proporção para a prioridade de despacho “oferecida” às eólicas, sendo que apenas os parques eólicos onde todas as turbinas atinjam o valor de aproveitamento da energia mecânica do vento de 59,26% beneficiarão da totalidade da prioridade de despacho para a sua eletricidade produzida.

Como forma de demonstrar as políticas propostas é desenvolvido um modelo teórico inspirado em Ghosh e Blackhurst (2014). Serão definidas as funções de consumo de eletricidade no período de pico e fora de pico para que posteriormente seja possível definir as funções para o RE e as elasticidades preço-procura necessárias para averiguar as alterações dos preços da tarifa TOU. Através de um exemplo, onde são utilizados dados dos países europeus onde pelo menos 20% do consumo final de eletricidade seja de origem renovável, demonstra-se as variações dos preços da nova tarifa TOU, para que seja possível o controlo dos RE alcançando uma maior suavização do diagrama de carga, e as implicações da prioridade de despacho condicionada.



# Abstract

One way of energy conservation and Greenhouse Gases (GHG) reduction is a more rational energy consumption. However, increased energy efficiency does not always leads to the desired outcomes regarding the reduction of consumption and consequently the reduction of GHG. The effect that makes it impossible to obtain the totality of benefits due to increasing energy efficiency is known as the rebound effect. This study aims to focus on rebound effect, controlling them by setting different prices according to the consumption period. A family production function is developed incorporating the electricity consumption, dividing them between peak and off-peak periods. The function for the direct rebound effect is calculated through the elasticity of electricity service comparing to the increased energy efficiency. The consumption variations are thus obtained in each of the periods that are associated with a particular direct rebound effect. In this perspective, a new Time-of-Use tariff (TOU) dual tariff is defined wherein the prices vary in order to cancel the full rebound effect. This way, the increased energy efficiency causes a further decrease of consumption in the critical period (peak period), promoting the smooth of the load diagram. It is proposed a partial sectioning of the dispatch priority of renewables based electricity in a special regime (SR). This procedure allows freeing resources to promote greater energy efficiency. Increased energy efficiency will be accompanied by the TOU tariff we are proposing, making these actually effective efficiency gains, since the proposed tariff ensures the control of RE.

# Keywords

Energy Efficiency; Time-of-Use tariff; Electricity peak and off-peak consumption; Demand side management; Rebound effect.



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# Acronyms list

RE	Rebound Effect
Cop	Coefficient of performance
kWh	Kilowatt hour
TOU	Time-of-use tariff
kVa	Quilovoltampere

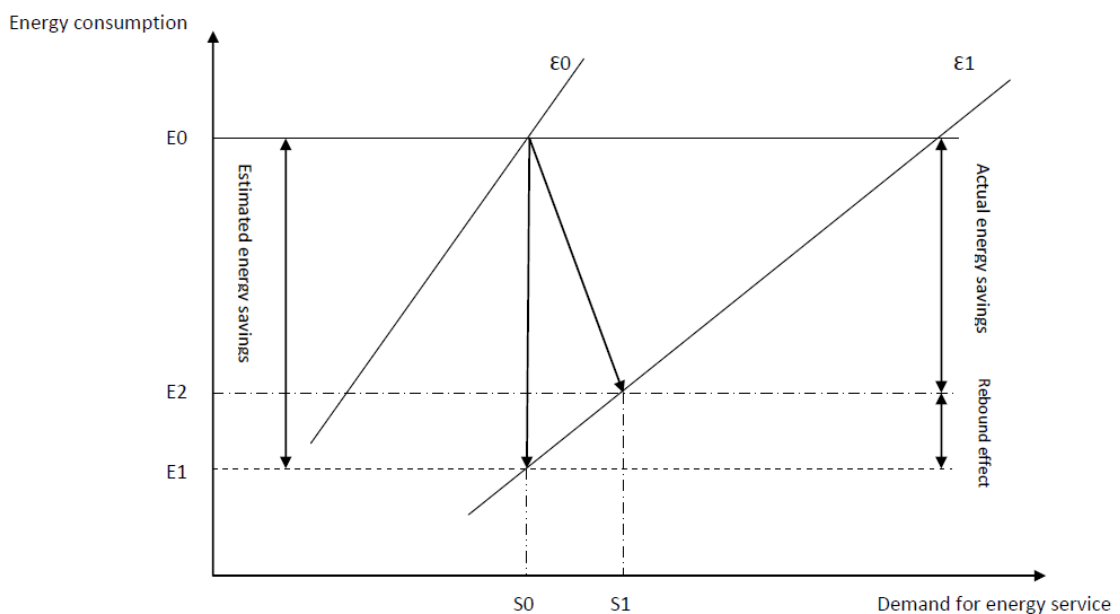


# 1. Introduction

One of the main goals for the development of an electricity market is the increase of energy efficiency. However, the increase in efficiency does not always have a positive effect on energy consumption, or at least not as good as would be expected. This occurs when the consumption does not reflect the increased efficiency in its entirety, hampering the management of energy demand. The phenomenon that constitutes a less than expected decrease in power consumption, when the energy efficiency increases, is known in literature as the "Rebound Effect". The energy efficiency gains do not always have the desired efficacy due to Rebound Effect (RE). The RE occurs when there is an unexpected result coming from the increased energy efficiency (Jin, 2007). Indeed, the RE is evident when the energy savings is lesser than the expected due to an increase in the demand for energy. The RE can thus be defined as the tendency to higher power consumption due to higher economic benefits (or energy savings) provided by energy efficiency (Berkhout et al., 2000).

This phenomenon can be observed in the following figure:

Figure 1: Example of the RE



Source: Adapted from, Liu e Lin (2013)

This way, it is understood that the RE can assume different dimensions. Through the graphic analysis, it is observed the initial consumption situation "E0" and the predicted consumption after an increase of the energy efficiency (from  $\epsilon_0$  to  $\epsilon_1$ ), "E1". The RE represented in the graphic ranges from 0% to 100%. After the efficiency increase, if the consumption decreases from E0 to E1 and maintains that constant level, the demand (utility) of a certain energy service (S) will stay constant (S0). Therefore the RE will be null because an increase in the efficiency only caused what was "expected", a consumption decrease (from E0 to E1), which

is proportional to the efficiency gains. If the RE is 100% the consumption stays constant at  $E_0$ , resulting in a electricity service coinciding with the intersection between  $E_0$  and  $\epsilon_1$ . However, what happens in most of the cases in real world is what can be observed in the graphic of the shift from  $E_1$  to  $E_2$  energy consumptions level. In other words, the difference between  $E_1$  and  $E_2$  represents the RE in the way that that gap leads to a consumption decrease that is less than proportional to the efficiency gains, causing an increase in the service utility "S" ( $S_0$  to  $S_1$ ).

The RE can assume extreme values, or within its range. In other words, there is a super-conservation of energy every time the value of the RE is lower than 0 %, therefore the consumption would be less than  $E_1$ . Furthermore it is observed the phenomenon known in the literature as "backfire effects" every time the value of the RE is greater than 100 %, that way the consumption will be greater than  $E_0$ . The variations described above will never be observed with the impact of efficiency variation only.

Therefore, the RE could represent a major problem to an efficient management of electricity demand, in a way that they remove the efficacy of the energy efficiency aim. Fortunately, some ways to control the RE could be applied, considering that these vary inversely with the price of energy. One way to reduce them is by increasing the price of energy in order for the efficiency increase to be boosted to the maximum. At the same time that the income effect on other energy services is reduced, thus promoting a more efficient energy market as a whole.

As well known, since a long time the energy markets are helped by financial aids, both on the production and on consumption side, as a way to boost investment in certain sources of power generation in order to ensure the safety of the power grid. One of the financial aids widely used throughout Europe are the feed-in tariffs because they are a way that allows a massive development of renewable energy, particularly in wind and solar energy, such as noted by Gallego-castillo and Victoria, (2015) and Kwon, (2015). An important aspect in contracts concluded with these rates, is the dispatch priority in the grid. The problem with this priority comes when we realize that a forecast of wind generation is very unreliable because although it can be predicted with a day in advance, it is commonly subject to 20-50% forecast errors (Piwko et al., 2005), which is related with the high volatility of wind energy. There are then two possible scenarios that come with several difficulties. In off-peak hours, the consumption is "low" and it occurs usually during the periods coincident with night. In general, during these periods the wind resource is more available and as such, an increased productivity is expected. In this way, if there is suddenly a great wind production, that satisfies the entire consumption, with the wind energy benefiting from dispatch priority, as a result there will be problems at the hydro plants (greater impact on small hydro plants that also benefit from the feed-in tariff), that will not be able to continue to produce and may even have to waste water as not to cause an overload on the grid.

The opposite scenario occurs in peak periods, when the consumption is very high. Due to the volatility of the resource wind, in these situations all the possible means of energy production

must be used, in order to avoid the shortages and even the collapse of the grid. This is detrimental because some of the production plants that were needed only work as a way to complement the wind farms, and many of them were designed prior to the wind farms to work as a baseload power plant and with guaranteed pay contracts making their fixed costs highly disproportional on the market. The negative effects from the two scenarios presented can be minimized through a better disciplining of the demand, but also with improvements on the electricity production side. So as to obtain a better demand management whilst keeping the RE controlled, this work focuses on the creation of a TOU rate that will provide in a first instance the necessary reduction of the price in off-peak hours so that the power consumption does not decrease after an increase in efficiency which occurs when the RE is 1. Thus, the off-peak consumption becomes appropriate to the energy mix in each country. Later the price is set considering the peak in order to maintain the control on the RE (reach a global RE 0, taking into account the changes resulting from the decrease in price in off-peak at baseline). In reality, the rebound will never be 0 % since it will be annulled through the manipulation of the consumption with the change in prices, therefore, the price changes will always have to be calculated together with the efficiency variations. Thus a new TOU rate will be defined which allows the smooth of the load diagram, keeping the initial consumption levels in off-peak periods constant and decreasing the peak consumption by obtaining an RE of 0, which by itself potentiates the energy efficiency.

Accordingly, this study contributes to the literature by developing an analytical model that allows controlling the RE through a TOU rate that establishes an increase of the peak price that is higher than the price decrease in off-peak periods. With this strategy, there is a decisive contribution to discipline the demand, promoting thus the mooth of the load diagram.

With the control of the RE guaranteed by the TOU rate proposal it will be ascertained which feed-in tariff (for wind) is suitable for the existing energy mix in the target country. It is then expected a decrease of the feed-in tariff, making it possible to quantify which percentage of the price is being directed to the feed-in tariff and then conduct that amount to a policy that encourages the increase of energy efficiency.

This transferred amount, from the financial aids to production, will correspond to the amount saved by a restriction on the order of priority, i.e., imposing a limit to this priority. This way, the demand management is strengthened through a TOU rate to control the negative impacts of energy efficiency, and later this efficiency will be encouraged knowing that their negative impacts are controlled.

With the smooth of the achieved load diagram, an energy mix with a lower installed production capacity can be obtained. It is then expected a decrease of the feed-in tariff, making it possible to quantify which percentage of the price is being directed to the feed-in tariff and then conduct that amount to a policy that encourages the increase of energy efficiency. On the other hand, the increase of wind capacity will require an increase in the production capacity because at a constant level, the increase of wind power requires

increased pumping capacity and additional baseload capacity, therefore requiring an investment in two types of electricity production other than the investment in wind energy (if these are added in an environment of decreasing consumption of electricity, particularly in peak periods, the need for this tripling of costs would be lower). Savings will be achieved through lower grid losses and greater predictability of consumption that would translate into greater energy security. However, many thermal power plants with guaranteed cost coverage contracts could close, as well as wind farms where there are contracts that guarantee much higher electricity prices than in other types of electricity production. It would also lower the emissions of carbon dioxide (CO<sub>2</sub>) by increasing the efficiency of electricity combined with the TOU proposed. This reduction would be higher due to an increase of efficiency that translates into higher electricity output reductions in peak periods, which means less production of electricity from fossil fuels, leading to greater reductions in CO<sub>2</sub> emissions.

The rest of this dissertation is organized as follows. In Section 2 the review literature is presented. In Section 3 it is developed an analytical model that defines the RE and the necessary changes in the TOU tariff rates to control the RE. It is also stated the idea for the conditional dispatch priority. Section 4 presents an example where it is calculated the RE and the necessary changes in the TOU tariff proposal that control the RE, considering two different times (time 1: off-peak and time 2: peak). It also calculated the percentage by which the order of priority will be conditioned, assuming a specific model of a wind turbine. Section 5 concludes.

## **2. Literature, motivation and theoretical option**

One of the best forms of energy conservation, and consequently to cut off GHG emissions, is the energy efficiency, both in consumption and production. However in the presence of a RE it is not possible to obtain full effectiveness of increased energy efficiency. The RE is reported in the literature (e.g. Hong et al. (2013)) as direct or indirect. The direct RE reports to a decrease in a particular product energy use (reduction in the price per unit of use), which leads to an increased use of that product. The indirect RE relates to a decrease in energy prices resulting in a greater disposable income among households, thus leading to more spending on goods and energy services. Consequently, there is higher power consumption, since the household has now a bigger income availability for buying an electric product ("Gadget" that provides a service through the use of electricity, e.g. Air-conditioning) that previously would not be covered by the budget constraint. However, the RE may reveal itself through several ways, for example, with the increase of the number of end-uses, with the average size of the product /service, the average performance (higher ratio of output energy utility relatively to the input power), and also the occupancy rate (for example, greater washing machines) (Yu et al., 2013). A clear example of the RE is related to the car exchange by a family. For example, if a family trades its old car, which ran 100 km per

month, for a new car with lower fuel consumption per 100km but now going to go 200 km per month, there has been a RE of 100% because the energy service (go 100 km) rose to double.

From what has been said, when financial aids are given for consumption or production, by providing a lower price of electricity, it may cause a greater proliferation of RE. Thus, also the CO<sub>2</sub> emissions would increase and damage the environment (Liu and Li, 2011). Thus, a transfer of costs related to renewable support (aiding with lowering the price of electricity) would be interesting from the perspective of the reduction of electricity consumption, leading to a lower the RE. This way, we would be in a situation where the achieved energy efficiency would be enhanced.

The literature is no consensual regarding the scope of the RE. Authors like Bentzen (2004), Berkhout et al. (2000), Haas and Biermayr (2000), Hertwich (2005), B. Howarth et al. (2000), Laitner (2000) state that the RE is insignificant and therefore can be ignored. On the other hand, authors such as Brännlund et al. (2007), Grepperud and Rasmussen (2004), Roy (2000) argue that the impact of the RE is too important to be left out. Through these two perspectives Greening et al. (2000) conclude that the RE does not obey any rule so its size will have to be ascertained individually, either between countries or within the same country in different sectors.

There are several ways to reduce the RE identified in the literature. Among them is the increase in energy prices (Ouyang et al. (2010)). Within this thought, Lin and Liu (2013) and Jiang and Tan (2013) state that a reduction in subsidies to support a lower electricity prices, causes an increase in energy prices and thus it will be interesting for the control of the RE. Note that this increased energy prices, as well as contributing to the control of the RE may also cause other effects, as shown by Popp (2002). In fact, this increase may lead to more investment in R&D as well as to the improvement of energy efficiency. The increase in energy prices can still be used as a disciplinary tool of the demand, while controlling the RE. That is the focus of this study. In fact, with relative increases of differentiated prices, depending on the consumption period (Time-of-use) at peak and off-peak, a relative decrease in the RE at the same time it achieves a smooth of the load diagram. This discipline of demand contributes to lower generation costs for producers while allowing to advance towards a reduction of energy dependence, improving economic efficiency of the system and, of course, increasing social welfare. For example, Faruqui and George (2005) showed through the "Statewide Pricing Pilot" (SPP) experiment (experiment conducted in California applying different electricity tariffs with the aim of measuring the behavior of consumers towards the different prices) that consumers reduce their use of energy between 7.6% and 27% at peak consumption.

In short, two distinct policies could be carried out. The first one, differentiation of electricity prices between peak and off-peak hours. The second one, restrictions on the absolute dispatch priority of renewable energy which are associated with feed-in tariffs. Thus, electricity would become more predictable during the peak periods, which would provide a greater security in the grid and also a reduced need of capacity. With the movement of

renewable costs for this purpose it would also boost the new energy efficiency enhancement measures, since the price of electricity would rise and the RE could fail to have a very high impact.

## 2.1 Motivation

One way to reduce RE is through the development of renewable energy (Ouyang et al., 2010). This could be seen after the first stated order (increasing price), since Marques et al. (2014) concluded that there is in Greece a one-way relationship between economic growth and renewable electricity, and the economic growth provides the further development of renewable power. So, what is expected through the two political proposals is to encourage a more efficient energy economy so that economic growth is facilitated, and later an improvement of demand management policies keeping the RE controlled. The simultaneous control of the ER with a smooth of the load diagram (due to reduction of the peak consumption) enables, together with the economic growth, a healthier proliferation of renewable energies, since the need for backup and baseload for intermittent renewable energies will be lower.

Knowing that the electricity service ( $C$ ) represents the electricity consumption according to the energy efficiency ( $C = E \cdot \epsilon$ ), and calculating the rebound effect through variation of the electric service when the energy efficiency varies (direct effect of the Energy efficiency variation, elasticity of electricity service in terms of efficiency), we can deduce what happens to consumption when we get a certain RE. For example, when we obtain the elasticity of 1, we can conclude that the electricity consumption is kept constant. Thereby we can infer that ER is 100%, meaning that the increase of the energy efficiency was absorbed by maintaining the initial level of consumption. A rebound of 100% may be associated with an energy service where the consumer cannot fully appreciate it and thus increased efficiency (a decrease in the cost of service) may allow these consumers to get more a more useful utilization out of it while keeping the cost electricity constant (increasing consumption) with the benefit of enjoying more of that service. On the opposite side, when the RE is 0%, it means that the electricity service does not change when the energy efficiency increases, thus there is a proportional reduction of electricity consumption. This case occurs in services that give no additional utility to the consumer for their increased use, and thus the consumer keeps a constant service that allows him to enjoy a lower cost of recurring use of lower power. Within this logic it is assumed, at first, that the rebound we want to achieve in times when off-peak are 100%, as well the level of consumption is kept constant. This maintenance of the level of consumption during off peak is beneficial because countries have divided an energy production capacity mix appropriate to the consumption levels, and causing these same levels to remain constant will be beneficial for the smooth of the load diagram, since the high and low peaks are not "apart".

After the appropriate setting of the price of off-peak consumption corresponding to the desired, the overall RE is calculated, i.e. the RE in each intake period is weighted by its size

(time elapsed during peak is higher than the time elapsed in off-peak), and so it will be defined the desired value of the RE at peak times to ensure the "cancellation" of the rebound in off-peak hours so that the global rebound is 0.

### 3. Theoretical model of price differentiation in the Rebound Effect

#### 3.1 Rebound effect and differentiated prices:

Consider a household production function, like the used by Ghosh and Blackhurst (2014). Assume now that the electricity service is differentiated between peak and off-peak hours<sup>1</sup>.

Nomeclature:

$Y$ - Composite good (represents the household production fuction)	$p_c$ - flat-rate price
$C$ - electricity service (work in electricity)	$p_f$ - off-peak price
$C_p$ - electricity service (work electricity) during peak periods	$E_c$ - total consumption of electricity
$C_f$ - electricity service (work electricity) during off-peak periods	$\tau$ - percentage of time in peak period
$E_p^1$ - peak period consumption (initial)	$t$ - number of peak hours
$E_f^1$ - off-peak period consumption (initial)	$H$ - Total hour period
$X$ - remaining services with another types of energy	$M$ - Families' income
$\varepsilon_c$ - electricity service efficiency	$P$ - Mechanical Power in W
$p_p$ - peak price	$\rho \approx 1,225 \text{ kg/m}^3$ , air density, in normal temperature conditions (15 °C) and pressure at sea level.
	$S$ - Circle area formed by the movement of the blades.
	$V$ - Wind speed in m/s

Considering the energy consumers as a company, the total production (household production function),  $Y$ , depends on the combination of work in electricity (measured by the entry of electricity escalated from its efficiency ( $E_i, \varepsilon_c$ ), at peak and off-peak, and of the other factors of production (types of energy)  $X$ :

Household production function:

$$Y = [(1 - \alpha_f - \alpha_p).X^{\frac{\sigma-1}{\sigma}} + \alpha_f.C_f^{\frac{\sigma-1}{\sigma}} + \alpha_p.C_p^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \quad (1)$$

<sup>1</sup> Throughout this study, it is assumed that the peak and off-peak periods remain constant every day of the week, like the TOU rate

Considering  $C_f = E_f \cdot \varepsilon_c$  and  $C_p = E_p \cdot \varepsilon_c$ .  $(\alpha_p, \alpha_f)$  are the parameters of energy inputs.  $\sigma$  is the elasticity of substitution .

**Proposition 1<sup>2</sup>:** *If  $U$  is an utility function with a strictly positive derivative then*

$$V(M, p_p, p_f, \varepsilon_c) = \max_{E_p, E_f, X} \{U(Y)\}$$

$$\text{s.a, } M \geq X + p_f \cdot E_f + p_p \cdot E_p$$

*has an unique solution defined by:*

$$E_f^* = \frac{M}{p_f} \cdot \frac{Zf}{(1 + Zp + Zf)} \quad (12)$$

$$E_p^* = \frac{M}{p_p} \cdot \frac{Zp}{(1 + Zp + Zf)} \quad (13)$$

$$X^* = M \cdot \left( \frac{1}{(1 + Zp + Zf)} \right), \text{ where} \quad (14)$$

$$Zp = \left( \frac{\alpha p}{(1 - \alpha p - \alpha f)} \right)^\sigma \cdot \left( \frac{p_p}{\varepsilon_c} \right)^{1-\sigma}$$

$$Zf = \left( \frac{\alpha f}{(1 - \alpha p - \alpha f)} \right)^\sigma \cdot \left( \frac{p_f}{\varepsilon_c} \right)^{1-\sigma}$$

**Proof.**

Considering that  $U$  is strictly increasing, then:

$$V(M, p_p, p_f, \varepsilon_c) = \max_{E_p, E_f, X} \{U(Y)\} = \max_{E_p, E_f, X} \{Y\}, \quad M \geq X + p_f \cdot E_f + p_p \cdot E_p$$

Furthermore, observing that,

$$\frac{\partial Y}{\partial I} = Y^{\frac{1}{\sigma-1}} \cdot \alpha_l \cdot \varepsilon_l \cdot I^{-\frac{1}{\sigma}} > 0,$$

Where  $I$ =Inputs  $(X, E_p, E_f)$ , we can affirm that the value of  $Y$  increases for any increase in the inputs  $(X, E_p, E_f) > 0$ . Therefore, the maximum value of  $Y$  will be reached at  $M = X + p_f \cdot E_f + p_p \cdot E_p$ .

Considering that  $C_p = E_p \cdot \varepsilon_c$  (peak electricity work) and  $C_f = E_f \cdot \varepsilon_c$  (off-peak electricity work), then, Substituting in (1):

$$Y = [(1 - \alpha_f - \alpha_p) \cdot X^{\frac{\sigma-1}{\sigma}} + \alpha_f \cdot (\varepsilon_c \cdot E_f)^{\frac{\sigma-1}{\sigma}} + \alpha_p \cdot (\varepsilon_c \cdot E_p)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \quad (2)$$

<sup>2</sup>  $p_x=1$  is normalized, so that energy prices are relative.

The Lagrangian comes:

$$L(X, E_p, E_f, \lambda) = Y + \lambda.(M - X - p_p.E_p - p_f.E_f)$$

The correspondent first-order conditions are,

$$[(1 - \alpha_f - \alpha_p)X^{\frac{\sigma-1}{\sigma}} + \alpha_c.C_f^{\frac{\sigma-1}{\sigma}} + \alpha_p.C_p^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}.(1 - \alpha_f - \alpha_p)X^{-\frac{1}{\sigma}} = \lambda \quad (3)$$

$$[(1 - \alpha_f - \alpha_p)X^{\frac{\sigma-1}{\sigma}} + \alpha_c.C_f^{\frac{\sigma-1}{\sigma}} + \alpha_p.C_p^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}.\alpha_p.\varepsilon_c.C_p^{\frac{1}{\sigma}} = \lambda.p_p \quad (4)$$

$$[(1 - \alpha_f - \alpha_p)X^{\frac{\sigma-1}{\sigma}} + \alpha_c.C_f^{\frac{\sigma-1}{\sigma}} + \alpha_p.C_p^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}.\alpha_f.\varepsilon_c.C_f^{\frac{1}{\sigma}} = \lambda.p_f \quad (5)$$

$$M - X - p_p.E_p - p_f.E_f = 0 \quad (6)$$

Dividing (5) and (4) by (3) then:

$$C_p = X.\left(\frac{\alpha p}{(1 - \alpha p - \alpha f)}\right)^{\sigma}.\left(\frac{p_p}{\varepsilon_c}\right)^{-\sigma} \quad (7)$$

$$C_f = X.\left(\frac{\alpha f}{(1 - \alpha p - \alpha f)}\right)^{\sigma}.\left(\frac{p_f}{\varepsilon_c}\right)^{-\sigma} \quad (8)$$

Substituting the budget constraint (6) in to (7), then:

$$C_p = (M - p_p.E_p - p_f.E_f).\left(\frac{\alpha p}{(1 - \alpha p - \alpha f)}\right)^{\sigma}.\left(\frac{p_p}{\varepsilon_c}\right)^{-\sigma} \quad (9)$$

Let  $Zp = \left(\frac{\alpha p}{(1 - \alpha p - \alpha f)}\right)^{\sigma}.\left(\frac{p_p}{\varepsilon_c}\right)^{1-\sigma}$ , then:

$$E_p = \frac{(M - p_f.E_f)}{p_p}.\frac{Zp}{(1 + Zp)} \quad (10)$$

Substituting (6) and then (10), in (8) comes:

$$C_f = \left(M - (M - p_f.E_f).\frac{Zp}{(1 + Zp)} - p_f.E_f\right).\left(\frac{\alpha f}{(1 - \alpha p - \alpha f)}\right)^{\sigma}.\left(\frac{p_f}{\varepsilon_c}\right)^{-\sigma} \quad (11)$$

Similarly to  $C_p$ , let  $Zf = \left(\frac{\alpha f}{(1 - \alpha p - \alpha f)}\right)^{\sigma}.\left(\frac{p_f}{\varepsilon_c}\right)^{1-\sigma}$ , then:

$$E_f^* = \frac{M}{p_f}.\frac{Zf}{(1 + Zp + Zf)} \quad (12)$$

Substituting (12) in to (10):

$$E_p^* = \frac{M}{p_p}.\frac{Zp}{(1 + Zp + Zf)} \quad (13)$$

And then substituting (12) and (13) in (6), it is obtained:

$$X^* = M \cdot \left( \frac{1}{(1 + Z_p + Z_f)} \right) \quad (14)$$

Concluding the proof. ■

The expressions (12) and (13) represent the point where the family reaches maximum utility for the consumption of electricity during off-peak and peak respectively, and of all the other family's production factors (14), taking into account the available income  $M$ .

**Definition 1.** Considering  $F$  as a function that depends on  $\varepsilon$ , it is called elasticity of  $F$  in relation to the variable  $\varepsilon$  to:

$$\frac{\partial F}{\partial \varepsilon} \cdot \frac{\varepsilon}{F}$$

The elasticity of the electricity service function ( $C$ ) in relation to efficiency ( $\varepsilon$ ) is called RE, i.e.:

$$r_\varepsilon(E) = \eta_\varepsilon(C) = \frac{\partial C}{\partial \varepsilon} \cdot \frac{\varepsilon}{C}$$

In other words, it represents the sensitivity between two variables, i.e., by defining the elasticity, the RE may be defined as the change in the electricity service when energy efficiency varies 1 %:

**Proposition 2.** *The peak RE is:*

$$r_{\varepsilon}(Ep) = \eta_{\varepsilon}(C_p) = \eta_{\varepsilon}(Ep) + 1 = (\sigma - 1) \cdot \left(1 - \frac{P_{p,Ep}}{M}\right) + 1 \quad (15)$$

**Proof.**

Calculating the elasticity price for peak consumption, it is obtained:

$$\eta_{p_p}(E_p) = \frac{\partial E_p}{\partial p_p} \cdot \frac{p_p}{E_p} = -1 + (1 - \sigma) \cdot \left(1 - \frac{P_{p,Ep}}{M}\right) \quad (16)$$

To find the elasticity of consumption with respect to the efficiency, then:

$$\eta_{\varepsilon}(Ep) = \frac{\partial Ep}{\partial \varepsilon} \cdot \frac{\varepsilon}{Ep} = \frac{\partial \frac{C_p}{\varepsilon}}{\partial \varepsilon} \cdot \frac{\varepsilon}{\frac{C_p}{\varepsilon}} = \left( \frac{\partial C_p}{\partial \varepsilon} \cdot \frac{\varepsilon}{C_p} - 1 \right) = \eta_{\varepsilon}(C_p) - 1$$

Therefore it is know that,

$$\eta_{\varepsilon}(C_p) = \frac{\partial C_p}{\partial \varepsilon} \cdot \frac{\varepsilon}{C_p} = \frac{\partial Ep \cdot \varepsilon}{\partial \varepsilon} \cdot \frac{\varepsilon}{C_p} = \left( \frac{\partial Ep}{\partial \varepsilon} \cdot \varepsilon + Ep \right) \cdot \frac{1}{Ep} = \eta_{\varepsilon}(Ep) + 1$$

In the literature, assuming that the price of energy is exogenous and efficiency changes are constant, some researchers affirm that the direct RE,  $\eta_{\varepsilon}(C_p)$ , is approximately the inverse of the elasticity price of demand (Greene, 2012; Sorrell et al., 2009; Binswanger, 2001):

$$\eta_{\varepsilon}(C_p) = -\eta_{pp}(E_p) \quad (17)$$

So,  $\eta_{\varepsilon}(Ep) = -\eta_{pp}(Ep) - 1$ , confirmed by,

$$\eta_{\varepsilon}(Ep) = \frac{\partial Ep}{\partial \varepsilon} \cdot \frac{\varepsilon}{Ep} = \frac{M}{p_p} \cdot \frac{1 + Zf}{(1 + Zf + Zp)^2} \cdot \frac{(\sigma - 1)}{\varepsilon} \cdot Zp \cdot \frac{\varepsilon}{E_p} = (\sigma - 1) \cdot \left(1 - \frac{P_{p.E_p}}{M}\right) \quad (18)$$

RE is calculated through the elasticity of the electricity ork in relation to its efficiency:

$$r_{\varepsilon}(E_p) = \eta_{\varepsilon}(C_p) = \eta_{\varepsilon}(E_p) + 1 = (\sigma - 1) \cdot \left(1 - \frac{P_{p.E_p}}{M}\right) + 1$$

Concluding the proof. ■

**Proposition 3.** *The off-peak RE is:*

$$r_{\varepsilon}(E_f) = \eta_{\varepsilon}(C_f) = \eta_{\varepsilon}(E_f) + 1 = (\sigma - 1) \cdot \left(1 - \frac{P_{f.E_f}}{M}\right) + 1 \quad (19)$$

Proof.

All calculations are performed in the same way that was used to define the peak RE (15), then the elasticity price in off-peak:

$$\eta_{p_f}(E_f) = \frac{\partial E_f}{\partial p_f} \cdot \frac{p_f}{E_f} = -1 + (1 - \sigma) \cdot \left(1 - \frac{P_{f.E_f}}{M}\right) \quad (20)$$

Concluding the proof. ■

Through the observation of (15) and (19) it can be concluded that having an higher  $p_p / p_f$  while keeping constant the consumption will result in a larger RE. This may lead to the idea that the rebound decreases by reducing the price and not through its increase. However, with a higher price, to get a larger rebound is expected considering that under that scenario, households are constrained with greater pressure on the budget. As such, an increase of energy efficiency allows a "relief" of that pressure. With a lower price, the pressure on the budget will be smaller and therefore the consumption of electricity is closest to providing maximum utility of its use<sup>3</sup>.

Through the RE equations (15) and (19) knowing that  $C = E \cdot \varepsilon$ , it is possible to calculate how much does the consumption of electricity vary when it is observed a particular value for RE. It is then possible to calculate the necessary price variations to accompany the efficiency increases so that the desirable RE is reached.

<sup>3</sup> This only happens when comparing an efficiency variation with different price levels without considering effects of a price variation. The RE corresponds to a variation in the consumption of energy through the variation of its efficiency, with the possibility of assuming different dimensions depending on the real price variations that occur simultaneously. Therefore, a positive price variation together with the increase in efficiency will lead indeed to a lesser impact of the rebound, since the positive price variation contributes to a decrease in consumption.

**Proposition 4.** *By knowing the initial RE in both consumption periods it is possible to design a new TOU rate so that the total RE is 0 %:*

The aim and scope of this proposition is deep enough to slice it on two different propositions, in order to accommodate the dynamics of the actions. The proof will be carried out separately.

**Proposition 4.1. Stage 1:** *The off-peak price change in order to keep the consumption in off-peak constant is:*

$$\left[ \frac{1 - \left( \frac{1 + \left( \frac{\eta_{\varepsilon_c}(C_f)}{100} \right)}{1,01} \right)}{\left( \eta_{p_f}(E_f) \right)} \right] \left[ \frac{\left( 1 + \left( \frac{\eta_{\varepsilon_c}(C_f)}{100} \right) \right)}{1,01} \right] \cdot 100 \quad (21)$$

Proof.

Consider the elasticity settings from (19) and (20):

Knowing that the off-peak rebound is  $(\eta_{\varepsilon_c}(C_f))$  (an increase of 1 % in efficiency causes a  $(\eta_{\varepsilon_c}(C_f))\%$  increase in the energy service), and as the variation of the energy service only ranges between 0 and 1, when there is a variation in the efficiency and everything else remains constant, we can conclude that the off-peak consumption  $E_f$  decreases when the increase in efficiency ( $\varepsilon_c$ ) is observed.

As a way of knowing how much does the off-peak consumption need to increase after the increase of the efficiency, the decrease of  $E_f$  can be computed through the following way

(knowing that  $C_f = E_f \cdot \varepsilon_c$ ):

$$\left( 1 + \left( \frac{\eta_{\varepsilon_c}(C_f)}{100} \right) \right) C_f = E'_f \cdot 1,01 \cdot \varepsilon_c \Leftrightarrow$$

$$\Leftrightarrow E'_f = \frac{\left( 1 + \left( \frac{\eta_{\varepsilon_c}(C_f)}{100} \right) \right)}{1,01} \cdot E_f \quad (22)$$

Therefore,  $\left[ \frac{\left( 1 + \left( \frac{\eta_{\varepsilon_c}(C_f)}{100} \right) \right)}{1,01} \right] - 1$  is the percentage change in consumption during the off-

peak period when the rebound is  $(\eta_{\varepsilon_c}(C_f))$ .

The objective in the off-peak period will be the definition of a off-peak price ( $p_f$ ) that ensures an RE of 1, i.e., that nullifies the effect of the efficiency over consumption, thereby

maintaining a constant initial level of consumption, so energy losses will be minimized because with this measure it is possible to avoid the problem supply excess.

Thus it is estimated that the percentage of off-peak consumption will have to increase:

$$\left[ \left( \frac{1 - \left( \frac{1 + \left( \frac{(\eta_{\varepsilon_c}(C_f))}{100} \right)}{1,01} \right)}{1,01} \right) / \left( \frac{1 + \left( \frac{(\eta_{\varepsilon_c}(C_f))}{100} \right)}{1,01} \right) \right] \cdot 100\% \quad (23)$$

Expression (22) represents the percentage by which the consumption will have to increase when there is an increase in efficiency.

Through the elasticity demand price (20), one could compute the necessary price that ensures the variation of the desired consumption. That is, for the variation of 1 % of the off-peak price, the off-peak consumption varies  $(\eta_{p_f}(E_f))\%$ . Then for an off peak consumption

variation of  $\left[ \left( \frac{1 - \left( \frac{1 + \left( \frac{(\eta_{\varepsilon_c}(C_f))}{100} \right)}{1,01} \right)}{1,01} \right) / \left( \frac{1 + \left( \frac{(\eta_{\varepsilon_c}(C_f))}{100} \right)}{1,01} \right) \right] \cdot 100\%$  the offpeak price will

have to vary:

$$\frac{\left[ \left( \frac{1 - \left( \frac{1 + \left( \frac{(\eta_{\varepsilon_c}(C_f))}{100} \right)}{1,01} \right)}{1,01} \right) \right]}{\left[ \left( \frac{1 + \left( \frac{(\eta_{\varepsilon_c}(C_f))}{100} \right)}{1,01} \right) \right]} \cdot 100$$

$(\eta_{p_f}(E_f))$

Concluding the proof. ■

A new RE is obtained for the off-peak period due to the new price and respective consumption. It is worthwhile to note that existing rebound is offset by the simultaneous variation of efficiency and price, but its new calculation will not be zero, because the rebound continues to exist, i.e., there is always the rebound, it may or may not be offset by other factors that influence the consumption other than efficiency. However, a calculation of cross-elasticity to verify the new peak consumption due to off-peak price changes will be needed:

$$\begin{aligned}
(\eta_{p_f}(E_p)) &= \frac{\partial E_p}{\partial p_f} \cdot \frac{p_f}{E_p} = \frac{M}{p_p} \cdot z_p \cdot \left[ - \frac{\left( \frac{\alpha_f}{1-\alpha_f-\alpha_p} \right)^\sigma \cdot \left( \frac{1-\sigma}{\varepsilon_c} \cdot \left( \frac{p_f}{\varepsilon_c} \right)^{-\sigma} \right)}{\left( 1+z_p + \left( \frac{\alpha_f}{1-\alpha_f-\alpha_p} \right)^\sigma \cdot \left( \frac{p_f}{\varepsilon_c} \right)^{1-\sigma} \right)^2} \right] \cdot \frac{p_f}{E_p} \Leftrightarrow \\
\Leftrightarrow (\eta_{p_f}(E_p)) &= \frac{M}{p_p} \cdot z_p \cdot \left[ - \frac{\frac{1-\sigma}{\varepsilon_c} \cdot \left( \frac{\alpha_f \cdot \varepsilon_c}{(1-\alpha_f-\alpha_p) \cdot p_f} \right)^\sigma}{\left( 1+z_p + \left( \frac{\alpha_f}{1-\alpha_f-\alpha_p} \right)^\sigma \cdot \left( \frac{p_f}{\varepsilon_c} \right)^{1-\sigma} \right)^2} \right] \cdot \frac{p_f}{E_p} \quad (24)
\end{aligned}$$

By observing the calculation of cross-elasticity price (24) the effects of off-peak price decrease in stage 1 over the electricity consumption during peaks in stage 2, are not considered because we observe a negative effect among them. This is not expected to occur, because when the price decreases in off-peak there is a shift of peak electricity consumption to off-peak period, and thus we would have a positive effect on cross-elasticity. This positive effect is not observed because the household production function is optimized by maximizing the utility, having Y a positive derivative for all inputs ( $X$ ,  $E_f$  and  $E_p$ ), and therefore each function of consumption, represents the maximum of utility that the families withdraw from the consumption considering the available income ( $M$ ). Therefore, the effect obtained from the elasticity is negative because:

- I. In the event of a decrease in  $p_f$ : The decrease in the off-peak price will cause a greater financial availability for households. Knowing that the families already retreated the maximum utility in electricity consumption, the privileged consumption will be the consumption in peak periods, because the price is higher and therefore an increase in consumption could provide greater utility. This respects the diminishing marginal utility, but also because the peak hours are those in which families will have a greater propensity to benefit from electricity.
- II. In the case of an increase in  $p_f$ : The off-peak price increase will cause a reduction of the total electricity consumption. As families consumed the maximum at the highest point of utility, they are deprived of part of the consumption in off-peak periods. This way, families favor a shift between consumption in peak towards off-peak periods, because consumption in off-peak periods has a lower price and therefore the abdicated consumption in peak periods is lower than the consumption "gain" on off-peak periods.

The decision making dynamics allows the consideration of a balance on stage 2 that was obtained on stage 1. Let's see stage 2.

**Proposition 4.2. Stage 2:** The price variation during the peak period so that the final RE consumption in null is:

$$\beta = 1 - \left[ 1 - \frac{\left( -(\eta_{\varepsilon_c}(C_f)) \cdot \frac{(1-\tau)}{\tau} - \left( (\sigma-1) \cdot \left( 1 - \frac{P_{p.E_p}}{M} \right) + 1 \right) \right)}{100} \right] / (\eta_{p_p}(E_p)) \quad (25)$$

Proof.

In the calculation of the overall RE (while considering 2 distinct periods), it must be taken into account the periods of time elapsed during each period. Therefore, the calculation may be carried out as follows:

$$\tau \cdot (\eta_{\varepsilon_c}(C_p)) + (1-\tau) \cdot (\eta_{\varepsilon_c}(C_f)) = \tau \cdot \left( (\sigma-1) \cdot \left( 1 - \frac{P_{p.E_p}}{M} \right) + 1 \right) + (1-\tau) \cdot \left( (\sigma-1) \cdot \left( 1 - \frac{P_{f.E_f}}{M} \right) + 1 \right) \quad (26)$$

To cancel the RE in off-peak periods by reducing the peak consumption, it is defined by how much the peak RE should be reduced:

$$(\eta_{\varepsilon_c}(C_p)) = -(\eta_{\varepsilon_c}(C_f)) \cdot \frac{(1-\tau)}{\tau} \quad (27)$$

It is also calculated, the RE only for the peak period so that the RE on the same period is cancelled:

$$(\eta_{\varepsilon_c}(C_p)) = \left( (\sigma-1) \cdot \left( 1 - \frac{P_{p.E_p}}{M} \right) + 1 \right) \quad (15)$$

In order to obtain a null final RE, the calculations (15) and (27) represent the necessary electricity services decreases. Therefore the electricity service during the peak period will vary:

$$-(\eta_{\varepsilon_c}(C_f)) \cdot \frac{(1-\tau)}{\tau} - \left( (\sigma-1) \cdot \left( 1 - \frac{P_{p.E_p}}{M} \right) + 1 \right) \% \quad (28)$$

As it was observed,  $(\eta_{\varepsilon_c}(C_p)) > 0$ , there is a need to define an increase in price as to achieve the desired reduction of  $C_p$ .

Knowing that  $C_p = \varepsilon_c \cdot E_p$ , it is possible to check by how much the consumption in peak periods will have to vary when efficiency is increased by 1% and the electricity service varies in the percentage calculated on the equation (28).

$$\text{Then, } \left[ 1 + \frac{\left( -(\eta_{\varepsilon_c}(C_f)) \cdot \frac{(1-\tau)}{\tau} - \left( (\sigma-1) \cdot \left( 1 - \frac{P_{p.E_p}}{M} \right) + 1 \right) \right)}{100} \right] \cdot C_p = E'_p \cdot (1+0,01) \cdot \varepsilon_c \Leftrightarrow$$

$$\Leftrightarrow E'_p = \frac{\left[ 1 + \frac{\left( -(\eta_{\varepsilon_c}(C_f)) \cdot \frac{(1-\tau)}{\tau} \right) - \left( (\sigma-1) \cdot \left( 1 - \frac{P_{p.E_p}}{M} \right) + 1 \right)}{100} \right]}{1,01} \cdot E_p$$

So, for the final RE to be null,  $E_p$  will have to vary:

$$\left[ \frac{\left( -(\eta_{\varepsilon_c}(C_f)) \cdot \frac{(1-\tau)}{\tau} \right) - \left( (\sigma-1) \cdot \left( 1 - \frac{P_{p.E_p}}{M} \right) + 1 \right)}{100} \right]_{-1\%} \cdot 1,01 \quad (29)$$

Through price elasticity (16) it will then be possible to check the variation in the peak price  $p_p$  that causes a reduction in consumption  $E_p$  ensuring a reduction in the electricity service ( $C_p$ ) in order to achieve a rebound of 0. That is, when knowing that the peak price varies 1% than the consumption at peak periods varies  $(\eta_{p_p}(E_p))\%$ , in order for the peak consumption to vary in the percentage calculated in (26), the price varies

$$\beta = 1 - \left[ 1 - \frac{\left[ \frac{\left( -(\eta_{\varepsilon_c}(C_f)) \cdot \frac{(1-\tau)}{\tau} \right) - \left( (\sigma-1) \cdot \left( 1 - \frac{P_{p.E_p}}{M} \right) + 1 \right)}{100} \right]}{1,01} \right] / (\eta_{p_p}(E_p))$$

Concluding the proof. ■

Through propositions 4.1 and 4.2 it is defined how much the TOU rate must vary so that the RE remains controlled while the load diagram is smoothed. However, the proposed pricing policy will only be triggered when the occurrence of a RE is observed, i.e., when there is an increase of energy efficiency. From this perspective it will be proposed a reassessment of the dispatch priority that the wind farms benefit from, being that this dispatch priority will be conditioned by the efficiency of the wind turbines.

### 3.2 Conditional dispatch priority:

The conditional dispatch priority will follow a line of thought that is based on the work of Betz in 1919. According to Betz's limit, no wind turbine can extract more than 59.26% of wind energy.

Demonstrated by coefficient of performance:

$$C_{op} = \frac{P}{\frac{1}{2} \cdot \rho \cdot S \cdot V^3} \quad (30)$$

It is then proposed that the dispatch priority becomes subject to the capacity of wind turbines to turn wind energy into mechanical energy. In this way there will be a clear incentive to increase wind turbine efficiency. This new way of indexing the dispatch priority to wind energy has the opposite effect to the existing effect of the dispatch priority given to the RES-E. With the new indexing of dispatch priority to the efficiency of wind turbines, the wind energy companies would benefit if they operated with the most efficient turbines possible.

## 4. Example

In this section data for some European countries is used, those in which at least 20% of consumed electricity comes from renewable sources. This renewable electricity weight is important due to the subsequent release of funds by conditional dispatch priority. This is the reason why the criterion of selection of the countries is the penetration of renewables. Note that these countries are particularly faced with the need not only to an increase in efficiency of electrical equipment but also encourages an increasingly efficient renewable production. The sources of the data are Eurostat, Nationmaster, Datamarket, Shrinkthatfootprint, ERSE e REN.

### 4.1 Rebound effect and differentiated prices:

For all computations one considers the average values for the following countries (see the Table A.1 in annex):

- Germany
- Austria
- Croatia
- Norway
- Denmark
- Slovenia
- Slovakia
- Portugal
- Spain
- Finland
- Greece
- Romania
- Italy
- Ireland
- Latvia
- Sweden

Let's then see the differentiation in consumption according to the periods of that consumption. It is assumed:

- i) It is calculated the average income through its median for the countries considered, then  $M = 16025.6875$  (see the Table A.2);

- ii) The total electricity consumption is  $E_c = 4724.3283$  which comes from the average of per capita per household consumption, weighted by the average number of people per family (see the Table A.3);
- iii) A flat-rate with electricity price ( $p_c$ ) = 0.1963, which comes from the average of electricity prices (including all fees and taxes) (see the Table A.4);
- iv) The price on peak ( $p_p$ )=0,2333 and the price off-peak ( $p_f$ )=0,1224 are calculated, considering the differences between the flat tariffs and the TOU tariffs (see the Tables A.5 and A.6);

**Example 1: Consumption differentiation attending to the cost of electricity (considering the same utility between these periods).**

Knowing that  $\alpha_c$  is the relative electricity consumption burden on the family income (M):

$$\alpha_c = \frac{p_c \cdot E_c}{M} \Leftrightarrow \alpha_c = \frac{0,1963 \cdot 4724,3283}{16025,6875}, \text{ then families use } 5,787\% \text{ of their income on the}$$

total electricity consumption. This means that (on average) they dedicate € 927.4065 per year to the consumption of electricity.

This ways, we can divide the consumption at peak times and off-peak times:

$cE_p$  (at peak cost) and  $cE_f = (927,4065 - cE_p)$  (off-peak cost):

$$\frac{cE_p}{0,2333} + \frac{(927,4065 - cE_p)}{0,1224} = 4724,3283 \Leftrightarrow cE_p = 734,5031\text{€}, \text{ then } cE_f = 192,9034\text{€}$$

Considering the prices calculated through the tables 5 and 6, then:

$$E_p = \frac{cE_p}{p_p} = 3148,3202 \text{ kWh} \text{ and } E_f = \frac{cE_f}{p_f} = 1576,008 \text{ kWh}$$

**Data:**

$\sigma = 0,15$  (elasticity substitution collected from the literature, Calculated through the elasticity price (16) in Ghosh & Blackhurst (2014));  $p_p^1 = 0,2333$  and  $p_f^1 = 0,1224$ ;

$$M = 16025,6875; \text{ With } \alpha_p = \frac{p_p \cdot E_p}{M} \text{ and } \alpha_f = \frac{p_f \cdot E_f}{M}: \alpha_p = 0,0458 \text{ and } \alpha_f = 0,0120.$$

The first year of this policy is regarded as the starting year, with a varying annual efficiency,

then:  $\varepsilon_c = 1$ ; with  $\tau = \frac{t}{H}$ :  $\tau = 0,5833$  ( peak period: 8h-22h and off-peak period: 22h-8h);

$$E_p^1 = 3148,3202 \text{ and } E_f^1 = 1576,0081.$$

Consider now two stages. On the first one the desired off -peak rebound is 1. This ensures that the off-peak consumption remains constant and it will be easier to maintain a proper energy production level that does not run the risk of getting an even more disproportionate installed capacity in relation to consumption in off-peak periods. In the second stage, the objective is to set the price at peak. The starting point is the assumption that the total RE

must be 0, i.e., the efficiency increase corresponds to a decrease in consumption that ensures the same level energy service during peak periods and a further decrease of consumption that annuls the observed RE in off-peak periods. This way, the energy efficiency will be increased to the maximum because this will contribute to lower consumption, only in peak hours, with the guarantee of smoothing the load diagram.

**Stage 1:**

Off-Peak Rebound (19):  $(\eta_{\varepsilon_c}(C_f)) = ((\sigma - 1) \cdot (1 - \frac{P_f \cdot E_f}{M}) + 1) = 0,1602$ , knowing that at

first a 1% efficiency increase will cause an increase of 0.1602% in the electricity service during off-peak hours, corresponding to a RE of 16.02% .

With the elasticity demand price being (19):

$$\eta_{p_f}(E_f) = \frac{\partial E_f}{\partial p_f} \cdot \frac{p_f}{E_f} = -1 + (1 - \sigma) \cdot (1 - \frac{P_f \cdot E_f}{M}) = -1 + (0,85) \cdot (1 - 0,0120) = -0,1602\%$$

The found value means that when the off-peak price varies 1%, the off-peak consumption varies -0.1602%.

Through the function (21) from **Proposition 4.1**, it is calculated the necessary variation in the off-peak price in order for the off-peak consumption to remain constant:

$$\frac{\left[ 1 - \frac{\left( 1 + \left( \frac{0,1602}{100} \right) \right)}{1,01} \right]}{-0,1602} \cdot \frac{\left[ \left( 1 + \left( \frac{0,1602}{100} \right) \right) \right]}{1,01} \cdot 100 = -5,234\%$$

In other words, with a 16,02% RE, the off-peak consumption is decreased. Assuming that the optimum point is to maintain the consumption in off-peak, the off-peak price will have to decrease 5,234%. As such, the new tariff comes:

$$p_f^2 = 0,9477 \cdot p_f^1 \Leftrightarrow p_f^2 = 0,1160$$

**Stage 2:**

Considering the same data, except the new price calculated in the previous stage, which guarantees the same initial level of off-peak consumption, we have:

$$\sigma = 0,15; \quad p_p^1 = 0,2333; \quad p_f^2 = 0,9478 \cdot p_f^1 \Leftrightarrow p_f^2 = 0,1160; \quad M = 16025,6875; \\ \alpha_p = 0,0458 \quad \text{and} \quad \alpha_f = 0,0120; \quad \varepsilon_c = 1; \quad \tau = 0,5083; \quad E_p^1 = 3148,3202 \quad \text{and} \\ E_f^1 = E_f^2 = 1576,0081$$

It is calculated the necessary reduction of service at peak times in order to cancel the rebound effect observed during off-peak periods (it is considered for this the dimension of time elapsed in each period):

$$\tau \cdot (\eta_{\varepsilon_c}(C_p)) + (1 - \tau) \cdot (\eta_{\varepsilon_c}(C_f)) = 0 \Leftrightarrow$$

$$\Leftrightarrow (\eta_{\varepsilon_c}(C_p)) = -(\eta_{\varepsilon_c}(C_f)) \cdot \frac{(1 - \tau)}{\tau} = -((\sigma - 1) \cdot (1 - \frac{P_{f.E_f}}{M}) + 1) \cdot \frac{(1 - \tau)}{\tau} = -0,1144$$

Then  $C_p$  should be reduced by 0,1144% in order to cancel the RE when off-peak.

It is also calculated the peak RE, to know by how much will the consumption in peak hours have to decrease in order to cancel the said rebound at peak times:

$$(\eta_{\varepsilon_c}(C_p)) = ((\sigma - 1) \cdot (1 - \frac{P_{p.E_p}}{M}) + 1) = 0,1889, \text{ i.e., the peak RE is 18,89\%}.$$

So in order to obtain a negative rebound during peak hours so that the final rebound is zero, we must have a reduction of peak consumption corresponding to a decrease in the electricity service of  $0,1144 + 0,1889 = 0,3033\%$ .

With the Price Elasticity being (15):

$$\eta_{p_p}(E_p) = \frac{\partial E_p}{\partial p_p} \cdot \frac{p_p}{E_p} = -1 + (1 - \sigma) \cdot (1 - \frac{P_{p.E_p}}{M}) = -0,1889$$

The found value means that when the off-peak price varies 1%, the off-peak consumption varies 0.1889% in reverse.

Then, through the function (25) from **Proposition 4.2**, it is calculated the necessary variation of the peak price in order for the peak consumption to remain constant:

$$1 - \left( 1 - \frac{\left[ 1 + \frac{(-0,1144) - (0,1889)}{100} \right]}{1,01} \right) / -0,1889 = 0,06831$$

In other words, with a 18,89% RE in peak and a 16,02% RE in off-peak, to assure a final RE that is null, the peak price will increase 6,831%, with a decrease in the peak consumption.

So in front of a variation of 1% in the energy efficiency, the new tariff will be:  $p_f^2 = 0,1160$  and  $p_p^2 = 0,2492$ .

The alteration of the TOU rate, shown above, will be proposed when the energy efficiency increases. The energy efficiency is also increased with the help of funds released by decreasing the renewable energies priority of dispatch.

**Example 2: Consumption differentiation considering the REN's data on hourly consumption**

Assuming a daily hourly cycle without distinction between week days or between Summer and Winter (peak: 08h-22h/ off-peak: 22h-08h), it is obtained the following average consumptions for hour in seven days (March 1st, 2015 to March 7th, 2015), in the two distinct periods (consumption data from REN):

Table 7 - Electricity consumption per hour during around the clock

	01/03/15	02/03/15	03/03/15	04/03/15	05/03/15	06/03/15	07/03/15
0h-1h	5180,125	5004,133	5432,075	5331,025	5269,875	5283,225	5228,375
1h-2h	4789,075	4726,15	5069,325	4969,1	4926,5	4949,05	4888,375
2h-3h	4531,125	4503,075	4854,025	4768,525	4739,45	4745,125	4634,125
3h-4h	4364,95	4418,075	4748,2	4677,075	4644,15	4634	4509,9
4h-5h	4298,275	4400,5	4677,4	4656,025	4634,025	4628,525	4448,125
5h-6h	4286,45	4512,95	4762,025	4716,45	4704,075	4693,975	4448,05
6h-7h	4233,4	4820,775	4971,6	4938,45	4946,125	4904,2	4389,275
7h-8h	4153,45	5419,175	5509,55	5498,5	5484,45	5463,35	4436,625
8h-9h	4466,35	6345,6	6418,05	6309,45	6306,225	6279,825	4890,625
9h-10h	4948,125	6831,25	6785,3	6613,325	6595,225	6560,375	5265,25
10h-11h	5344,225	6989,5	6865,1	6647,05	6667,125	6634,2	5409,85
11h-12h	5683,225	7139,9	6960,4	6729,325	6772,45	6693,95	5526,325
12h-13h	5768,5	6994,775	6776,15	6495,225	6563,775	6514,375	5538,75
13h-14h	5587,225	6806,25	6647,75	6336,325	6380,15	6383,875	5356,025
14h-15h	5384,25	6915,05	6787,75	6437,725	6473,25	6420,75	5256,575
15h-16h	5279,475	6829,95	6656,35	6352,075	6396,375	6314,325	5147,95
16h-17h	5187,6	6757,15	6529,425	6230,6	6317,05	6171,875	4996,2
17h-18h	5344,325	6542,875	6403,1	6043,525	6129,85	5934,1	4984,575
18h-19h	5878,975	6795,45	6675,425	6354,2	6419,5	6160,825	5412,25
19h-20h	6426,275	7364,45	7289,975	7017,875	7161,775	6830,05	6235,425
20h-21h	6434,9	7261	7168,275	6912,225	7051,625	6753,5	6189,375
21-22h	6227,75	6982,8	6902,15	6642,825	6774,575	6500,5	5861,575
22h-23h	5974,875	6653,95	6482,175	6344,625	6450,375	6253,1	5596,225
23h-0h	5438,075	5995,35	5908,275	5776,35	5847,075	5697,775	5202,775

**-Peak:**

Average per day: **day 1:** 5568,6571; **day 2:** 6896,8571; **day 3:** 6776,0857; **day 4:** 6508,6964; **day 5:** 6572,0679; **day 6:** 6439,5089; **day 7:** 5433,625.

**-Off-peak:**

Average per day: **day 1:** 4724,98; **day 2:** 5045,4133; **day 3:** 5241,465; **day 4:** 5167,6125; **day 5:** 5164,61; **day 6:** 5125,2325; **day 7:** 4778,245.

**Total average consumption at peak (per hour):**6313,6426 kWh

**Total average consumption in off-peak (per hour):** 5035,3655 kWh

The consumption at peak corresponds to 0,5563 of the total consumption, therefore, with an annual total consumption averaging on 4724,3283 kWh, the **peak consumption** will be: **2628,1438 kWh**. The consumption in off-peak corresponds to 0,4437 of the total consumption, therefore the **off-peak consumption** will be: **2096,1845 kWh**

Data:

$\sigma = 0,15$ ;  $p_p^{-1} = 0,2333$  and  $p_f^{-1} = 0,1224$ ;  $M = 16025,6875$ ; With  $\alpha_p = \frac{p_p \cdot E_p}{M}$  and

$\alpha_f = \frac{p_f \cdot E_f}{M}$ :  $\alpha_p = 0,0383$  and  $\alpha_f = 0,0160$

The first year of this policy is regarded as the starting year, with a varying annual efficiency,

then:  $\varepsilon_c = 1$ ; with  $\tau = \frac{t}{H}$ :  $\tau = 0,5833$ ( peak period: 8h-22h and off-peak period: 22h-8h);

$E_p^{-1} = 2628,1438$  and  $E_f^{-1} = 2096,1845$

Please note that the assumptions from example 1 remain, both for stage 1 and 2.

**Stage 1:**

Off-Peak Rebound (19):  $(\eta_{\varepsilon_c}(C_f)) = ((\sigma - 1) \cdot (1 - \frac{p_f \cdot E_f}{M}) + 1) = 0,1636$ , knowing that at

first a 1% efficiency increase will cause an increase of 0,1636% in the electricity service during off-peak hours, corresponding to a RE of 16,36%.

With the Price Elasticity being (19):

$$\eta_{p_f}(E_f) = \frac{\partial E_f}{\partial p_f} \cdot \frac{p_f}{E_f} = -1 + (1 - \sigma) \cdot (1 - \frac{p_f \cdot E_f}{M}) = -1 + (0,85) \cdot (1 - 0,0120) = -0,1636\%$$

The found value means that when the off-peak price varies 1%, the off-peak consumption varies 0.1636% in reverse.

Through the function (21) from **Proposition 4.1**, it is calculated the necessary variation in the off-peak price in order for the off-peak consumption to remain constant:

$$\frac{\left[1 - \frac{\left(1 + \frac{0,1636}{100}\right)}{1,01}\right]}{-0,1636} \cdot \frac{\left[\frac{\left(1 + \frac{0,1636}{100}\right)}{1,01}\right]}{1,01} \cdot 100 = -5,104\%$$

In other words, with a 16,36% RE, the off-peak consumption is decreased. Assuming that the optimum point is to maintain the consumption in off-peak, the off-peak price will have to decrease 5,104%. So the new tariff comes:

$$p_f^2 = 0,949 \cdot p_f^1 \Leftrightarrow p_f^2 = 0,1162$$

### Stage 2:

Considering the same data, except the new price calculated in the previous stage, which guarantees the same initial level of off-peak consumption, we have:

$$\sigma = 0,15; \quad p_p^1 = 0,2333; \quad p_f^2 = 0,9488 \cdot p_f^1 \Leftrightarrow p_f^2 = 0,1162; \quad M = 16025,6875; \\ \alpha_p = 0,0383 \quad \text{and} \quad \alpha_f = 0,0160; \quad \varepsilon_c = 1; \quad \tau = 0,5833; \quad E_p^1 = 2628,1438 \quad \text{and} \\ E_f^1 = E_f^2 = 2096,1845$$

It is calculated the necessary reduction of service at peak times in order to cancel the rebound effect observed during off-peak periods (it is considered for this the dimension of time elapsed in each period):

$$\tau \cdot (\eta_{\varepsilon_c}(C_p)) + (1 - \tau) \cdot (\eta_{\varepsilon_c}(C_f)) = 0 \Leftrightarrow$$

$$\Leftrightarrow (\eta_{\varepsilon_c}(C_p)) = -(\eta_{\varepsilon_c}(C_f)) \cdot \frac{(1 - \tau)}{\tau} = -((\sigma - 1) \cdot (1 - \frac{P_{f \cdot E_f}}{M}) + 1) \cdot \frac{(1 - \tau)}{\tau} = -0,1169$$

Then  $C_p$  should be reduced by 0,1169% in order to cancel the RE when off-peak.

It is also calculated the peak RE, to know by how much will the consumption in peak hours have to decrease in order to cancel the said rebound at peak times:

$$(\eta_{\varepsilon_c}(C_p)) = ((\sigma - 1) \cdot (1 - \frac{P_{p \cdot E_p}}{M}) + 1) = 0,1826, \text{ i.e., the peak RE is 18,26\%.}$$

So in order to obtain a negative rebound during peak hours so that the final rebound is zero, we must have a reduction of peak consumption corresponding to a decrease in the electricity service of  $0,1169 + 0,1826 = 0,2995\%$ .

With the Price Elasticity being (15):

$$\eta_{p_p}(E_p) = \frac{\partial E_p}{\partial p_p} \cdot \frac{p_p}{E_p} = -1 + (1 - \sigma) \cdot (1 - \frac{P_{p \cdot E_p}}{M}) = -0,1826$$

The found value means that when the off-peak price varies 1%, the off-peak consumption varies 0.1826% in reverse.

Then, through the function (25) from **Proposition 4.2**, it is calculated the necessary variation of the peak price in order for the peak consumption to remain constant:

$$1. - \left( 1 - \frac{\left[ 1 + \frac{(-0,1169) - (0,1826)}{100} \right]}{1,01} \right) / -0,1826 = 0,07046$$

In other words, with a 18,26% RE in peak and a 16,36% RE in off-peak, to assure a final RE that is null, the peak price will increase 7,046%, with a decrease in the peak consumption.

So when we have a variation of 1% in energy efficiency, the new tariff will be:  $p_f^2 = 0,1162$  and  $p_p^2 = 0,2497$ .

The alteration of the TOU rate, shown above, will be proposed when the energy efficiency increases. The energy efficiency is also increased with the help of funds released by decreasing the renewable energies priority of dispatch.

### Example 3: Consumption Differentiation through an enquiry

Assuming a daily hourly cycle without distinction between week days or between Summer and Winter (peak: 08h-22h/ off-peak: 22h-08h).

Enquiry with 37 answers on the peak consumption and off-peak consumption of consumers with a TOU tariff (monthly average).

Table 8 - Peak and off-peak consumption sample

Off-peak	Peak	(continue) Off-peak	(continue) Peak
250✓	300✓	100✓	16✓
375✓	205✓	105✓	155✓
130✓	130✓	165✓	150✓
100✓	100✓	35✓	40✓
471✓	323✓	150*	450*
200✓	100✓	134✓	177✓
57✓	84✓	63✓	84✓
500✓	100✓	147✓	114✓
36✓	58✓	137✓	153✓
80✓	120✓	400✓	300✓
115✓	165✓	79✓	88✓
188✓	52✓	33✓	52✓
253✓	408✓	269✓	138✓
73✓	78✓	66✓	42✓
124✓	161✓	98✓	98✓
150*	300*	200*	400*
128✓	179✓	186✓	293✓
250✓	250✓	585✓	295✓
50✓	35✓		
		Total: 5982 kWh	Total: 5043 kWh

Only the consumptions where the TOU tariff is compensatory over the flat tariff are taken into consideration.

It is possible to calculate from which percentage the TOU tariff can be compensatory, by equalizing the two cost functions for the flat tariff and the TOU tariff:

$$\begin{cases} cb = p_f \cdot E_f + p_p \cdot E_p \\ cc = p_c \cdot (E_f + E_p) \end{cases}$$

$$p_f \cdot E_f + p_p \cdot E_p = p_c \cdot (E_f + E_p) \Leftrightarrow (p_f - p_c) \cdot E_f = (p_c - p_p) \cdot E_p$$

$$\Leftrightarrow E_p = \frac{p_f - p_c}{p_c - p_p} \cdot E_f$$

Assuming the prices:  $p_f=0,1224$ ,  $p_p=0,2333$  and  $p_s$  (flat tariff price)= $0,1963$  then:

$E_p = \frac{0,1224-0,1963}{0,1963-0,2333} \cdot E_f \Leftrightarrow E_f = 0,5007 \cdot E_p$ , i.e., in order for the TOU tariff not to be more expensive than the flat tariff, the off-peak consumption has to correspond to at least 50,07% of the peak consumption.

Therefore, 3 answers are excluded because the off-peak consumption is less than 50,07% of the peak consumption and it is not defined as a rational option by the families.

Peak consumption: 5043Kwh (monthly, 34 families)

Off-peak consumption: 5982Kwh (monthly, 34 families)

It is concluded that the off-peak consumption represents 54,26% of the total consumption.

Peak: The consumption corresponds to 0,4574, therefore with a total annual consumption, for family, on average of 4724,3283 kWh, the peak consumption will be: 2160,9078 kWh

Off-peak: The consumption corresponds to 0,5426, therefore the off-peak consumption will be: 2563,4205 kWh

Data:

$\sigma = 0,15$ ;  $p_p^1 = 0,2333$  and  $p_f^1 = 0,1224$ ;  $M = 16025,6875$ ; With  $\alpha_p = \frac{p_p \cdot E_p}{M}$  and

$\alpha_f = \frac{p_f \cdot E_f}{M}$ :  $\alpha_p = 0,0315$  and  $\alpha_f = 0,0196$

The first year of this policy is regarded as the starting year, with a varying annual efficiency,

then:  $\varepsilon_c = 1$ ; with  $\tau = \frac{t}{H}$ :  $\tau = 0,5833$  (peak period: 8h-22h and off-peak period: 22h-8h);

$E_p^1 = 2160,9078$  and  $E_f^1 = 2563,4205$

Please note that the assumptions from example 1 remain, both for stage 1 and 2.

### Stage 1:

Off-Peak Rebound:  $(\eta_{\varepsilon_c}(C_f)) = ((\sigma - 1) \cdot (1 - \frac{P_f \cdot E_f}{M}) + 1) = 0,1667$ , knowing that at first a 1% efficiency increase will cause an increase of 0,1667% in the electricity service during off-peak hours, corresponding to a RE of 16,67% .

With the Price Elasticity being (19):

$$\eta_{p_f}(E_f) = \frac{\partial E_f}{\partial p_f} \cdot \frac{p_f}{E_f} = -1 + (1 - \sigma) \cdot (1 - \frac{p_f \cdot E_f}{M}) = -1 + (0,85) \cdot (1 - 0,0120) = -0,1667\%$$

The found value means that when the off-peak price varies 1%, the off-peak consumption varies 0,1667% in reverse.

Through the function (21) from **Proposition 4.1**, it is calculated the necessary variation in the off-peak price in order for the off-peak consumption to remain constant:

$$\frac{\left[ 1 - \frac{\left( 1 + \frac{0,1667}{100} \right)}{1,01} \right]}{-0,1667} \cdot \frac{\left[ \frac{\left( 1 + \frac{0,1667}{100} \right)}{1,01} \right]}{1,01} \cdot 100 = -4,9905\%$$

In other words, with a 16,67% RE, the off-peak consumption is decreased. Assuming that the optimum point is to maintain the consumption in off-peak, the off-peak price will have to decrease 4,9905%. So the new tariff comes:

$$p_f^2 = 0,9501 \cdot p_f^1 \Leftrightarrow p_f^2 = 0,1163$$

### Stage 2:

Considering the same data, except the new price calculated in the previous stage, which guarantees the same initial level of off-peak consumption, we have:

$$\sigma = 0,15; \quad p_p^1 = 0,2333; \quad p_f^2 = 0,9501 \cdot p_f^1 \Leftrightarrow p_f^2 = 0,1163; \quad M = 16025,6875;$$

$$\alpha_p = 0,0315 \quad \text{and} \quad \alpha_f = 0,0196; \quad \varepsilon_c = 1; \quad \tau = 0,5833; \quad E_p^1 = 2160,9078 \quad \text{and}$$

$$E_f^1 = E_f^2 = 2563,4205$$

It is calculated the necessary reduction of service at peak times in order to cancel the rebound effect observed during off-peak periods (it is considered for this the dimension of time elapsed in each period):

$$\tau \cdot (\eta_{\varepsilon_c}(C_p)) + (1 - \tau) \cdot (\eta_{\varepsilon_c}(C_f)) = 0 \Leftrightarrow$$

$$\Leftrightarrow (\eta_{\varepsilon_c}(C_p)) = -(\eta_{\varepsilon_c}(C_f)) \cdot \frac{(1 - \tau)}{\tau} = -((\sigma - 1) \cdot (1 - \frac{P_f \cdot E_f}{M}) + 1) \cdot \frac{(1 - \tau)}{\tau} = -0,1191$$

Then  $C_p$  should be reduced by 0,1191% in order to cancel the RE when off-peak.

It is also calculated the peak RE, to know by how much will the consumption in peak hours have to decrease in order to cancel the said rebound at peak times :

$$(\eta_{\varepsilon_c}(C_p)) = ((\sigma - 1) \cdot (1 - \frac{P_p \cdot E_p}{M}) + 1) = 0,1768, \text{ i.e., the peak RE is 17,68\%}.$$

So in order to obtain a negative rebound during peak hours so that the final rebound is zero, we must have a reduction of peak consumption corresponding to a decrease in the electricity service of  $0,1191+0,1768=0,2959\%$ .

With the Price Elasticity being (15):

$$\eta_{p_p}(E_p) = \frac{\partial E_p}{\partial p_p} \cdot \frac{p_p}{E_p} = -1 + (1 - \sigma) \cdot \left(1 - \frac{p_{p,E_p}}{M}\right) = -0,1768$$

The found value means that when the off-peak price varies 1%, the off-peak consumption varies 0,1768% in reverse.

Then, through the function (25) from proposition 4.2, it is calculated the necessary variation of the peak price in order for the peak consumption to remain constant:

$$1 - \left(1 - \frac{\left[1 + \frac{(-0,1191) - (0,1768)}{100}\right]}{1,01}\right) / -0,1768 = 0,07257$$

In other words, with a 17,68% RE in peak and a 16,67% RE in off-peak, to assure a final RE that is null, the peak price will increase 7,257%, with a decrease in the peak consumption.

So when we have a variation of 1% in energy efficiency, the new tariff will be:  $p_f^2 = 0,1163$  and  $p_p^2 = 0,2502$ .

The alteration of the TOU rate, shown above, will be proposed when the energy efficiency increases. The energy efficiency is also increased with the help of funds released by decreasing the renewable energies with dispatch priority.

## 4.2 Conditional dispatch priority:

Considering the model of the wind turbine Gamesa G114-2.0MW, which reaches its rated power of 2.0MW for a wind speed of about 10 m/s and has a circle area formed by the movement of the blades of 10207 m<sup>2</sup>, its coefficient of performance (30) will be:

$$C_{op} = \frac{2 \cdot 10^6}{\frac{1}{2} \cdot 1,225 \cdot 10207 \cdot 10^3} = 0,32, \text{ i.e., only 32\% of wind energy is converted into}$$

mechanical energy by the wind turbine.

By proportion, where the maximum turbine efficiency (59.26%) is associated with 100% dispatch priority, for a 32% efficiency it will be given an order of priority for 54% of the energy produced.

Considering the following electricity generation costs<sup>4</sup>:

Wind (offshore): 11.0 cent/kWh

<sup>4</sup> Source: Pinto de Sá, 2010, accessed on 05/05/2015 (<http://a-ciencia-nao-e-neutra.blogspot.pt/2010/06/custos-e-precos-da-electricidade.html>)

Wind (onshore): 7.0 cent/kWh  
 Biogas: 5.5 cent/kWh  
 Combined cycle gas: 6.0 cent/kWh  
 Hydropower (average): 5.7 cent/kWh  
 Desulfurized coal: 5.6 cent/kWh  
 Nuclear: 4.9 cent/kWh

It is only considered the production with lower costs than wind farms. Solar energy is not considered because to determine the amount available with the dispatch priority, it is important to relate wind energy in competition with the energies that have a lower cost.

Average energy production cost (excluding wind):

$$\frac{5,5 + 6,0 + 5,7 + 5,6 + 4,9}{5} = 5,54 \text{cent} / kWh$$

Average cost of wind power production:  $\frac{11+7}{2} = 9 \text{cent} / kWh$

Taking into account the data of the countries considered in the work:

Assuming the average annual household electricity consumption ( $E_c$ ): 4724.3283 kWh

The average impact of wind power for electricity consumption in the 16 considered countries is 8.15% of the total electricity consumption:

Therefore, 385,0338 kWh shall be provided by wind, with an average annual cost per family of 9 cent / kWh.  $385,0338 kWh = 34,6530€$ .

If the dispatch priority was only of 54%:

Only 207,9177 kWh would be provided by wind farms at cost of 9 cent / kWh, being the total cost 9 cent / kWh.  $207,9177 kWh = 18,7126€$ .

The remaining consumption previously provided by the dispatch priority,  $385,0328 kWh - 207,9177 kWh = 177,1151 kWh$ , will now enter the market at the lowest price of 5,54 cent / kWh, with the cost of 5,54 cent / kWh.  $177,1157 kWh = 9,8122€$ .

It is possible to calculate how much is saved by a family in a year if the dispatch priority is only of 54%:

207,9177 kWh will enter the grid resulting in a cost of 18,7126€. The remaining 177,1157 kWh will enter the market in competition and so will the electricity at the lowest price, i.e., 5,54 cent / kWh, with the cost of 9,8122€. The average cost of the 385,0338 kWh will now be  $18,7126€ + 9,8122€ = 28,5248€$ , with the savings for each family, after conditioning the order of priority, being:  $9 \text{cent} / kWh \cdot 385,0338 kWh - (9 \text{cent} / kWh \cdot 207,9177 + 5,54 \text{cent} / kWh \cdot 177,1157 kWh) = 6,1282€$ .

Assuming the average of households per country,  $7 \cdot 318 \cdot 492$ , the annual savings will be of  $73184926,1282\text{€} = 448491826744\text{€}$ . This will be the amount that will be used for the incentive to increase energy efficiency.

## 5. Discussion

One concludes that a negative price change contributes in fact to an increase in the RE due to the increased consumption of electricity, and vice versa. However, when we start from a point where they comply lower prices, the RE tends to be lower. This outcome is consistent with the empirically observed. In fact the RE is associated with the willingness to pay of household using a certain electricity service, considering that in a market where a starting price is lower, the willingness to pay of households are closer to the desired level. Thus an increase in efficiency that is an indirect decrease in electricity prices will not lead to an "increase" so high in consumption. So it is consistent with the principle of diminishing marginal utility, since with the lowest starting price families obtain a higher utility of electricity service.

This may raise some problems. With the maintenance of a constant pricing strategy throughout the years, the RE at peak associated with peak successive price increases will be increasing in this most critical period leading to a "snowball" effect. The idea appears of a potential RE, i.e., the effect that can occur due to increased energy efficiency in view of the starting electricity prices. Thus with an higher starting price the associated RE will also be higher and the way of the controlling it will be through an increase of that price. This work conceptualizes a way to deal with this problem, making use of cutting the traditional dispatch priority that renewable energies benefit from. At first glance, this moderation of the dispatch priority will be reflected in the release of funds associated with higher rates, directing them to the direct incentive of increasing the efficiency of technology. In this first approach there will be no change in base prices. In a second approach, as peak prices increased such that the RE becomes too high, prices rate would decrease. This necessary reduction of the prices would be achieved through the reduction of incentives towards the increase of the efficiency of technology.

## 6. Conclusions and Implications

This study contributes to the literature with a theoretical model that is evocative of the control of the Rebound Effect, through the use of a differentiated pricing policy depending on the time of consumption. The control of the RE is promoted with the additional goal of promoting shifts in electricity consumption, from peak to off-peak periods. The work is as much focused on to establish itself as a useful tool not only for literature, but primarily for policy makers.

Three examples were taken into account and it was confirmed that with the TOU tariff we are proposing the final RE is null. This is effect is achieved by decreasing the consumption in the peak period, which ensures a greater smooth of the load diagram. It is also possible to observe through the analysis of the RE that the higher the RE, the smaller will be the necessary variations of the prices. This happens because, with a larger RE, the consumptions are more price sensitive. This perspective can give the idea that a high RE is beneficial because it leads to lower price increases. However, a high RE means that the sensitivity of consumers to price fluctuations is higher. This means that when the RE is very high, the price increase may be lower, but this increase will be associated, in relative terms, to a greater impact on consumption due to the higher price sensitivity among consumers.

The proposed policy will lead, over the years, to a higher peak price and a lower off-peak price. Through the observation of expressions (15) and (19) we can conclude that the incentive of off-peak consumption, and consequently the smooth of the load diagram, will be higher, since the sensitivity regarding the off-peak price will be increasingly lower and in relation to the peak price, increasingly bigger. We will then be facing an off-peak consumption increasingly more efficient.

This work does not consider the shifts, that are expected, from the consumption between peak periods to off peak periods. In that context there could be a possibility to develop a model in which consumption shifts between the periods would be considered. As such, the future research aims to go further, by leaving this hypothesis of constant consumption pattern between the consumptions period. That option will allows appraising how the household consumption pattern recomposition could be affected by the TOU tariffs. Even more, it would be possible to define differentiated tariffs which incentive the household to define a consumption pattern more appropriated to achieve the global efficiency on the electricity generation system as a whole. This is a future challenge.

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# Appendix

## Appendix 1:

It was chosen the countries where the renewable electricity productions corresponds to at least 20% of the total electricity consumption:

**Table A.1-** Countries and respective shares of renewable energy in the final consumption

Country	Share of renewable energy in gross final energy consumption
Alemanha	25,59%
Áustria	68,08%
Croácia	38,68%
Dinamarca	43,12%
Eslovénia	32,82%
Eslováquia	20,80%
Espanha	36,39%
Finlândia	31,11%
Grécia	21,24%
Itália	31,30%
Irlanda	20,89%
Letónia	48,75%
Noruega	105,47%
Portugal	49,16%
Roménia	37,52%
Suécia	61,81%

Source: Eurostat, 2013

## Appendix 2:

Monthly income (M):

**Table A.2:** Median income by household

Country	Median income by household
Germany	19545€
Áustria	22073€
Croatia	5078€
Denmark	26858€
Slovenia	11852€
Slovakia	6737€
Spain	13523€
Finland	23272€
Greece	8377€
Italy	15733€
Ireland	19065€
Latvia	4700€
Norway	42937€
Portugal	8177€
Romania	2071€
Sweden	26413€

Source: Eurostat, 2013

The average family income, in countries where the renewable electricity production represents at least 20% of the total electricity consumption:

$$M = \left( 19545 + 22073 + 5078 + 26858 + 11852 + 6737 + 13523 + 23272 + 8377 + 15733 + 19065 + 4700 + 42937 + 8177 + 2071 + 26413 \right) / 16 = 16025,6875€$$

### Appendix 3:

Average annual consumption of electricity, by the families, in kWh:

**Table A.3-** Average electricity consumption by household

Country	Consumption by households per capita	Average household size	Consumption by households
Germany	1719,43 kWh	2	3438,86 kWh
Austria	1654,14 kWh	2,3	3804,522 kWh
Croatia	1425,71 kWh	2,8	3991,988 kWh
Denmark	1928,06 kWh	1,9	3663,314 kWh
Slovenia	1475,15 kWh	2,5	3687,875 kWh
Slovakia	872,66 kWh	2,9	2530,714 kWh
Spain	1442,09 kWh	2,5	3605,225 kWh
Finland	3922,54 kWh	2,1	8237,334 kWh
Greece	1519,73 kWh	2,6	3951,298 kWh
Italy	1157 kWh	2,4	2776,8 kWh
Ireland	1805,81 kWh	2,7	4875,687 kWh
Latvia	683,33 kWh	2,4	1639,992 kWh
Norway	7246,79 kWh	2,1	15218,259 kWh
Portugal	1255,23 kWh	2,6	3263,598 kWh
Romania	426,82 kWh	2,3	981,686 kWh
Sweden	4724,81 kWh	2,1	9922,101 kWh

Note: Italy: consumption by household per capita in Shrinkthatfootprint, 2010. Sources: Consumption by households per capita: NationMaster, 2005. Average household size: Datamarket, 2013.

Therefore the annual average consumption of electricity per family is:

$$E = \frac{(3438,86 + 3804,522 + 3991,988 + 3663,314 + 3687,875 + 2530,714 + 3605,225 + 8237,334 + 3951,298 + 2776,8 + 4875,687 + 1639,992 + 15218,259 + 3263,598 + 981,686 + 9922,101)}{16}$$

$$= 4724,3283kWh$$

$$E_c = 4724,3283kWh$$

### Appendix 4:

Average prices (Pp and Pf):

**Table A.4-** Electricity prices (flat tariff)

Country	Electricity prices for domestic consumers
Germany	0,2921
Austria	0,2018
Croatia	0,1350
Denmark	0,2936
Slovenia	0,1657
Slovakia	0,1678
Spain	0,2273
Finland	0,1559
Greece	0,1697
Italy	0,2323
Ireland	0,2405
Latvia	0,1358
Norway	0,1778

Portugal	0,2131
Romania	0,1279
Sweden	0,2046

Note: All taxes and levies included. Source: Eurostat, 2° semester of 2013

Average price of the flat tariff:

$$p_c = \frac{(0,2921+0,2018+0,135+0,2936+0,1678+0,2273+0,1559+0,1697) + 0,2323+0,2405+0,1358+0,1778+0,2131+0,1279+0,2046}{16}$$

$$= 0,1963\text{€} / kWh$$

### Appendix 5:

According to the reference prices of several companies, in the liberalized portuguese market, the peak and off-peak prices vary, on average, in the following way:

**Table A.5-** Reference electricity prices (flat and TOU tariff) - Power  $\leq 6,9kVa$

	Flat tariff	TOU		$\Delta\%$ peak	$\Delta\%$ off-peak
Audax	0,1528	0,1785	0,0946	16,82	38,09
Edp-Casa	0,1555	0,1853	0,0978	19,16	37,11
Edp-Casa Verde	0,1587	0,1853	0,0978	16,76	38,37
Galp-Plano Base	0,1587	0,1853	0,0978	16,76	38,37

**Table A.6-** Preços Reference electricity prices (flat and TOU tariff) - Power  $>6,9 kVa \wedge \leq 20,7 kVa$

	Flat tariff	TOU		$\Delta\%$ peak	$\Delta\%$ off-peak
Audax	0,1464	0,1847	0,0959	26,16	34,49
Edp-Casa	0,1570	0,1871	0,0976	19,17	37,83
Edp-Casa Verde	0,1602	0,1890	0,0986	17,98	38,45
Galp-Plano Base	0,1602	0,1890	0,0986	17,98	38,45

The TOU prices vary on average, in relation to the flat tariff, 18,85% higher on peak periods and 37,65% lower on off-peak periods.

This way, the average TOU tariff can be defined through the average flat tariff:

$$p_p = 0,1963.(1+0,1885) = 0,2333 \text{ e } p_f = 0,1963.(1-0,3765) = 0,1224$$