Multiquadric Approximation for Geodetic Location by Airborne Camera

André Silva[1], José Novais[2], K. Bousson[3]

Abstract – *This thesis addresses the development of a method for surface reconstruction and a vision based system for position estimation and camera control, for an unmanned aerial vehicle. A theory for response surface approximation was developed to interpolate a given set of data for application in surface reconstruction and geodetic location by focus of a camera for aerial surveillance. To lock the camera in the estimated location is necessary to control the orientation of the focus of the camera during the flight. The system can guide the camera to any point on the surface reconstructed using quaternion methods or any point location estimated, regardless of the direction and dynamics of the disturbances that the aircraft may have.*

Keywords: *Multiquadrics, surface reconstruction, robust camera control, quaternions*

I. Introduction

The development in the area of aerial surveillance technologies have been growing in the past years. Unmanned aerial vehicles are being used more and more in civilian applications such as monitoring of traffic, recognition and surveillance vehicles, search and rescue operations, homeland security, natural resource management and military operations.

For an effective detection and localization of a target or position is very important to know the geographical terrain. But the real world numerical data is usually difficult to analyze. Any function which would effectively correlate the data would be difficult to obtain and highly unwieldy.

There are many methods to reconstruct the continuous 3D surface from discrete scattered data. The scattered data approximation problem is easily described and occurs frequently in many branches of science.

To this end, the idea of the basic theory of Hardy multiquadric response surface approximation as in [1], [2] was developed to interpolate a given set of data for application in surface reconstruction and geodetic localization by focus of a camera for aerial surveillance.

Here we deal with functions of two independent variables, but the methods are easily extendible to arbitrary dimensions. The method is one of a class of methods know as radial basis functions methods, that includes other attractive schemes. The basic idea of such methods is quite simple.

Suppose that the data $\left(x_j, y_j, z_j\right)$, $j = 1,\ldots,N$ are given, assumed to be measurements of an underlying function $z = f\left(x, y\right)$.

The function f is to be approximated by a function $s(x, y)$ from the given data. For purposes of being definite it is pertinent to note that for the multiquadric method the radial function is:

$$\varphi\left(x, y\right) = \sqrt{\left|x - x_j\right|^2 + \left|y - y_j\right|^2 + c^2}$$

where $|\bullet|$ is the Euclidean norm.

The parameter c is known as the shift or smooth parameter in multiquadric approximation. For interpolation, that is, exact matching of the given data, a multiquadric function $\varphi\left(x, y\right)$ is associated with each data point. The approximation is a linear combination of the multiquadric functions, along with some polynomial terms that may be necessary in some cases, or may be used to assure that the approximation method has polynomial precision.

$$s\left(x, y\right) = \sum_{j=1}^{N} \alpha_j \varphi_j\left(x, y\right) + \sum_{l=1}^{L} \beta_l p_l\left(x, y\right) \quad (1.1.1)$$

The unknown coefficients $\left[\alpha \quad \beta\right]^T$, are computed by requiring the function $s\left(x, y\right)$ to pass through a given set of data $\left(x_j, y_j, f\left(x_j, y_j\right)\right)$, $j = 1,\ldots,N$, usually known as centers. Thus, given the smooth parameter c, the training of the multiquadric approximation model involves a solution of linear equations with which the coefficients $\left[\alpha \quad \beta\right]^T$ are associated as in [3].

The smooth parameter has great effect on the performance of multiquadric approximation and then, one of the objectives of this paper is to present an efficient method of computing the number of centers not necessarily equal to number of data points and the optimal smooth parameter.

After the surface reconstruction, there are many ways to compute the geodetic location of an object relative to its position within a focus of a camera. We propose an algorithm for the object location based on geodetic elevation of a reconstructed surface and the altitude and attitude of the focus of the camera [4].

The attitude of an aircraft, in a conventional mode, is defined by the three Euler angles ϕ, θ and ψ, being the roll, the pitch, and the yaw angle respectively. In the case of the camera, the rotational motion is also defined by three Euler angles.

The kinematic relations that determine the derivative of the Euler angles are given by:

$$\begin{aligned} \dot{\phi} &= w_x + \left(w_y \sin\phi + w_z \cos \right) \tan\theta \\ \dot{\theta} &= w_y \cos\phi - w_z \sin\phi \\ \dot{\psi} &= \left(w_y \sin\phi + w_z \cos \right) \left(1/\cos\theta \right) \end{aligned} \qquad (1.1.2)$$

where the angular velocities w_x, w_y, w_z, are obtained by the derivatives of Euler angles.

The integration of the equations above lets us know the attitude of the camera. However, for $\tan\theta$ and $1/\cos\theta$ their integration becomes not possible for values of $\theta = \pm 90^o$ since these angles have infinite values. Thus, it is necessary to avoid this problem by another system of equations. This is possible thanks to the quaternions, this is, the four symmetric Euler parameters [4], [5].

If σ_1, σ_2 and σ_3 are the angles defined by the axes of the camera with respect to terrestrial axes and if μ is the only rotation that moves the axes of the camera to terrestrial axis, then we define the quaternion associated with this rotation for the following four parameters:

$$\begin{aligned} \eta_o &= \cos\left(\mu/2 \right) \\ \eta_1 &= \sigma_1 \sin\left(\mu/2 \right) \\ \eta_2 &= \sigma_2 \sin\left(\mu/2 \right) \\ \eta_3 &= \sigma_3 \sin\left(\mu/2 \right) \end{aligned} \qquad (1.1.3)$$

The equations that relate with the Euler angles are:

$$\begin{aligned} \tan\phi &= \frac{2\left(\eta_o\eta_1 + \eta_2\eta_3 \right)}{\eta_o^2 - \eta_1^2 - \eta_2^2 + \eta_3^2} \\ \sin\theta &= 2\left(\eta_o\eta_2 - \eta_1\eta_3 \right) \\ \tan\psi &= \frac{2\left(\eta_o\eta_3 - \eta_1\eta_2 \right)}{\eta_o^2 + \eta_1^2 - \eta_2^2 - \eta_3^2} \end{aligned} \qquad (1.1.4)$$

and the system can be shown by:

$$\begin{bmatrix} \dot{\eta}_o \\ \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -w_x & -w_y & -w_z \\ w_x & 0 & w_z & -w_y \\ w_y & -w_z & 0 & w_x \\ w_z & w_y & -w_x & 0 \end{bmatrix} \begin{bmatrix} \eta_o \\ \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} \qquad (1.1.5)$$

The equation of the rotational motion control of the camera is given by:

$$J\dot{W} + W \times \left(JW \right) = u \qquad (1.1.6)$$

where J is the inertia matrix, W is the angular velocity vector and u is the actuators torque vector. For the objective of lock the focus of the camera in a particular point from any position the camera may be, a method for trajectory control of the camera is provided.

II. Surface Reconstruction

II.1. Densification: centers and smooth parameter

Applying a densification procedure enables to approximate any multidimensional function by a mono-dimensional function. This method reduces the number of variables of the original problem and requires less function evaluations as in [6].

Let $B = \prod_{i=1}^{r} \left[a_i, b_i \right] \subset \Re^n$ be a box (or hyper-rectangle) and $J = \left[a, b \right]$ be a real interval. Then a function $h: J \to B$ is said to be an α - dense curve in B, if $\forall x \in B$, $\exists t \in J$: $\left\| h\left(t \right) - x \right\| \le \alpha$, where $\left\| \cdot \right\|$ denotes the Euclidean norm in $\Re^n$.

Now consider the h: $J = \left[0,1 \right] \to B = \prod_{i=1}^{n} \left[a_i, b_i \right]$ defined as $\forall \theta \in J$, $h\left(\theta \right) = \left(h_1\left(\theta \right), h_2\left(\theta \right), \ldots, h_n\left(\theta \right) \right)$, where h_n is $\alpha_n = \mu^{n-1} 2^n \pi$ dense.

The curve obtained by this method has a cosine representation. This means that all axes are filled with periodical functions, having different periods. The parameter μ is controlling the distance between all points in space.

With this densification of the domain, the shift or smooth parameter can be found by measuring the neighbor of each center ξ, as $\forall \xi_k \in B$, $c_k^2 = \max\left\{ c_j^2 = \left\| \xi_j - \xi_k \right\| \right\}$, $j \ne k$, where ξ_j is a center near ξ_k (see Fig. 1).

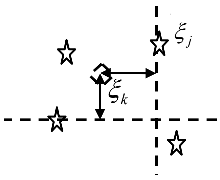

Fig. 1. Smooth parameter measure as the maximum shortest-path length of each center

II.2. Randomly sample of data points

The data points are divided into two data subsets I and II. A random sample algorithm is utilized so that all data points have an equal chance of being selected from the all set to the subset I or II. To all such data is given an equal probability.

The first data set I objective is to train and find the approximation function. The second data set purpose is to validate the method.

The algorithm consists in selecting the data by raffling a number $\tau \in [0,1]$ for each data point x_j and if τ is for example odd (could be even), x_j belongs to the subset I for function training. The number of data points for function training is $T > N_i/2$ and all the others in subset II are for function evaluation.

II.3. Function Learning

The learning is equivalent to finding a surface in a multidimensional space that provides a best fit to the training data. We use for this training T data points in equation below:

$$s(x,y) = \sum_{k=1}^{N_c} \alpha_k \varphi_k(x,y) + \sum_{l=1}^{L} \beta_l p_l(x,y) \quad (2.3.1)$$

where $\varphi_k = \sqrt{c_k^2 + \|(x,y) - \xi_k\|^2}$ and $p_l = [1 \quad x \quad y]$.

To solve this function training is necessary to solve a minimization problem trough all the subset data I as follows:

$$J(\alpha,\beta) = \sum_{t=1}^{T} \left(\left\| \sum_{k=1}^{N_c} \alpha_k \varphi_k(x_t,y_t) + \sum_{l=1}^{L} \beta_l p_l(x_t,y_t) - F(x_t,y_t) \right\|^2 \right) \quad (2.3.2)$$

And then we can use the least squares pseudo inverse method for solving the over determined system:

$$A \cdot \eta = F \quad (2.3.3)$$

where $A = a_{t,k} = \sqrt{c_k^2 + \|(x_t,y_t) - \xi_k\|^2} + p_l(x_t,y_t)$. and $\eta = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$. Finally we get the coefficients $[\alpha \quad \beta]^T$ by:

$$\eta = (A^T A)^{-1} A^T F. \quad (2.3.4)$$

The algorithm for function training $s(x)$, is schematized in Fig. 2 where N_i, N_c are the number of total data points and total of centers respectively.

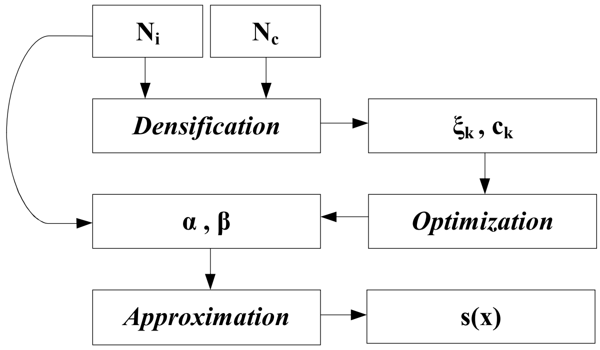

Fig. 2. Algorithm for function approximation

III. Position Estimation

For position estimation we suggest a traditional geodetic location method that use position data, a digital elevation model (reconstructed surface), and pose angles of the airborne camera to estimate the 3D coordinates of ground target as in [4]. The objective of geodetic location is to estimate the position of the focus of the camera from the aircraft.

The position estimation is obtained using a terrain model, and a nonlinear filter is used to estimate the position and velocity of moving ground based targets.

The estimator would require a very high load of computation to estimate the focus of the camera, but with our proposed method we can estimate the same position by pointing directly the reconstructed surface, thus saving computation and memory.

IV. Camera Control

In this stage the system can guide the camera to any point on the surface reconstructed using quaternion methods or any point location estimated regardless of the direction and dynamics of the disturbances that the aircraft may have [4].

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= x_{ref}(t) \end{aligned} \quad (4.1.1)$$

Being $x_{ref}(t)$ a given reference, possibly variable in time, the problem is to find $u^* = u^*(t)$ such that $\exists \tau > t_o$ and check $\forall t \geq \tau \left\| s(x, x_0, t_0) - x_{ref} \right\| < \varepsilon$. Where $s(x, x_0, t_0)$ is the solution of the Equation (4.1.1) for $u^* = u^*(t)$ and the initial conditions $x_o = x(t_o)$. For nonlinear systems there exist matrices A and B such that Equation (4.1.1) is equivalent to:

$$\dot{x} = Ax + Bu + \varphi(x, u) \qquad (4.1.2)$$

and the error corresponding to $x - x_{ref}$.

Thus, let be:

$$\dot{e} = \dot{x} - \dot{x}_{ref} = Ax + Bu + \varphi(x, u) - \dot{x}_{ref} \qquad (4.1.3)$$

such that $\varphi(x(t), u(t)) \cong \varphi(x(t), u(t-h)),\ h \ll 0$.

$$u^*(t) = -B^T P \left(BB^T P + \sigma I \right)^{-1} \cdot \left(De + Ax + \varphi(x, u_h) - \dot{x}_{ref} \right) \qquad (4.1.4)$$

The matrix P is such that for each x, $xPx' > 0$, D is positive definite and $\sigma > 0$.

V. Some Results

We give three examples to demonstrate the accuracy and applicability of our proposed technique for surface reconstruction and robust camera control. The algorithms were implemented in Matlab.

For surface reconstruction the flexibility of the implementation allows an interactive experimentation. The algorithm was implemented to obtain the best c value, for fixed centers found by the densification method.

The use of a number of centers different from the number of total data points improves the complexity of simulation by decreasing the dimension of matrix A. This dimension reduction is improved too by randomly selecting a subset of data points.

Thus, we used for exemplification two different applications with two sets with 2500 points distributed in a uniform 50×50 grid. These data was randomly placed in subset I and II and for function training we use $T = 0.7 \times N_i$ and the total number of centers is $N_c = 500$.

To compare our proposed method for surface reconstruction we first consider the use of one Franke´s test function:

$$g(x,y) = 0.75\exp\left[-\left((9x-2)^2 + (9y-2)^2\right)/4\right] + 0.5\exp\left[-\left((9x-7)^2 + (9y-3)^2\right)/4\right] + 0.75\exp\left[-(9x-2)^2/49 - (9y-2)^2/10\right] - 0.2\exp\left[-(9x-4)^2 - (9y-2)^2\right] \qquad (4.1.5)$$

For the second example, we used real numeric data of a real geodetic area in Covilhã, Portugal.

To Franke´s test function as in Equation (4.1.5) that is a surface with low complexity the proposed method shows a very close to the best fits as we can see in Fig. 1 and Fig. 2. The values for root mean square (RMS) are very low for any subset (see Table I).

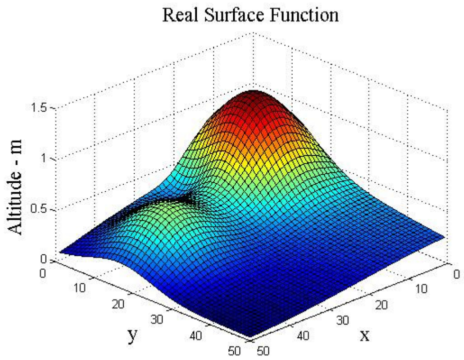

Fig. 3. Real surface function with 500 centers

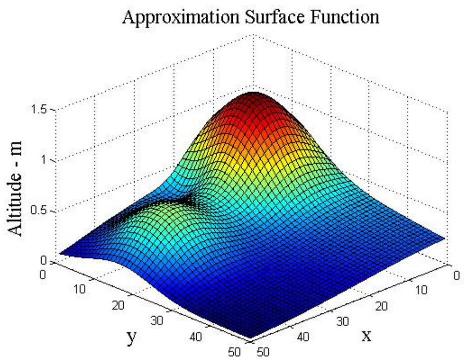

Fig. 4. Approximated surface function with 500 centers

TABLE I
FRANKE´S DATA SET WITH 500 CENTERS

Subset Evaluation	RMS	Error %
Function I	4.2666e-005	0.0145
Validation II	4.2442e-005	0.0399
Total	4.2599e-005	0.0107

Of course, most surfaces with other complexity cannot be fit so well as the example for the real geodetic data. The results of the algorithm for the same conditions as in Franke´s test function are showed next in Fig. 3 and Fig. 4.

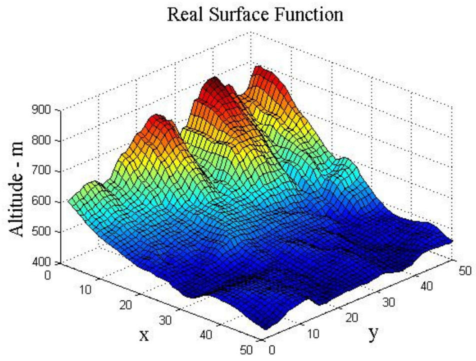

Fig. 5. Real geodetic surface function with 500 centers

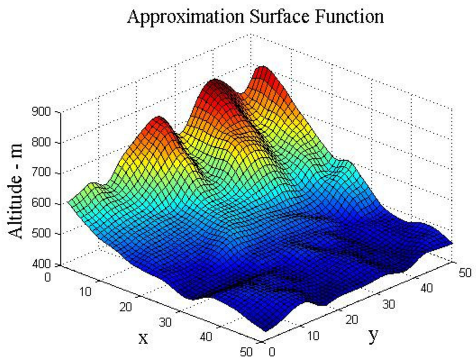

Fig. 6. Approximated surface function with 500 centers

TABLE II
REAL GEODETIC DATA SET WITH 500 CENTERS

Subset Evaluation	RMS [m]	Error %
Function I	4.0011	1.0698
Validation I	5.3006	3.2704
Total I+II	4.4311	0.8266

The RMS by evaluating each subset is showed in Table II. We can observe that we reach a good approximation function and very smooth surface.

Now we give an example to demonstrate the accuracy and applicability of our proposed technique for robust camera control. For the trajectory simulation of the airborne camera we use values for the principal inertia of 0.5 $kg.m^2$ for x, y, z axis.

One time it is found the equilibrium point for this system, we can see in Fig. 7 to Fig. 8 that initializing the system from any position the quaternions η_0 and η_1 (exactly the same procedure for η_2 and η_3) can follow the reference trajectory with a very near null error. These errors could be improved by adjusting the gains of the controller.

This reference trajectory is obtained by a simple sine and cosine function such that the attitude of the focus of the camera oscillate in the interval $[-90^o, 90^o]$.

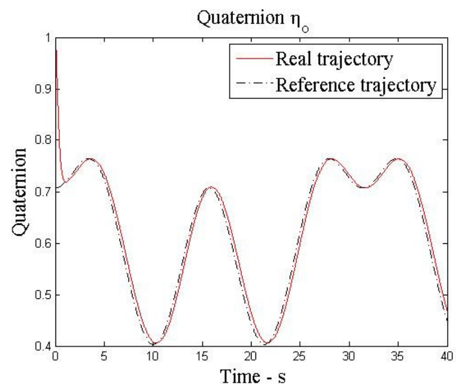

Fig. 7. Quaternion η_o in function of time

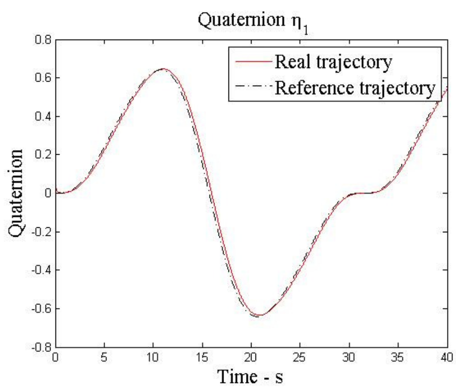

Fig. 8. Quaternion η_1 in function of time

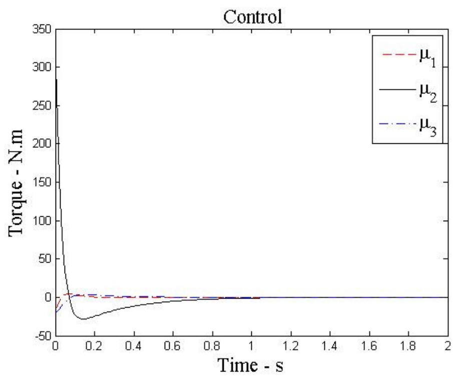

Fig. 9. Time response of the controller

We can see in Fig. 10 that the system can reach the reference trajectory very fast. There exists a good answer from the controller to the torque due to reference trajectory.

VI. Conclusion

Multiquadrics is a powerful method in multivariate interpolation. Some results state that the proposed algorithm is very simple and efficient with realistic accuracy for real geodetic data approximation. Even in an irregular terrain we get a smooth surface approximation associated with a small RMS.

The use of variable centers but in a fixed number and their variable smooth parameters can give a greatly improved approximation when using multiquadric functions.

The attitude of a camera set in quaternions is an alternative for practical applications as it can make the system follow any desired trajectory or orientation. Compared with the Euler angles, our proposed method has a better performance in control and has higher rotations amplitudes.

The control method applied to his nonlinear system presents goods results concerning the performance, stabilization in any reference and providing good margins of gain and phase.

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His thesis addresses the development of a method for response surface reconstruction, where Professor K. Bousson is his orientator.

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