Optimal Output Control: Load Frequency Control of a Large Power System

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***Abstract*—This paper addresses the stabilization and performance of the load frequency regulator. The problem is solved by using the theory of the optimal control. An algorithm, based on the new technique, proposed by the authors, to overcome the difficulties of specifying the weighting matrices *Q* and *R*, is presented. The algorithm here proposed considers the multi-area electric energy system. The paper presents illustrative numerical simulation results for this model. The results indicate that the obtained controller exhibits better performance then those based on classic control.**

***Keywords*—Generation of electrical energy, Optimal control, Control methods for electrical systems.**

I. Introduction

The objective of the automatic generation control (AGC) of a power system is to maintain the frequency of each area and tie-line power flow (in interconnected system) within specified tolerance. Many investigations in the area of AGC problem of interconnected systems have been reported in the past. A number of control schemes have been employed in the design of load frequency controllers in order to achieve better dynamic performance.

The application of the optimal control theory to power systems has shown that an optimal load frequency controller can improve the dynamic stability of a power system [1], some difficulties to apply this technique still remain, mainly because (1) the optimal control is a function of all the states of the system, in practice all the states may not be available, the inaccessible states or missing states are required to be estimate, (2) it may not be economical to transfer all the information over long distances and (3) the specification of matrices *Q* and *R* [2].

The problem of selecting the weighting matrices *Q* and *R* has shown up in optimal control design [1,3]. Assigning appropriate values to the matrices *Q* and *R* in a systematic manner, together with the determination of the optimal output controller, is a major concern and is the main point addressed in this paper.

This paper develops a new practicable decentralized structures control algorithm that is based upon the authors previous results [4]. This new approach only requires the information available of the systems states in each area.

II. Background

To design a linear optimal control, a linearized model of the power system is sought. The linear model of the system is described in state space form, as

$$\begin{aligned} \dot{x} &= A\,x + B\,u \\ y &= C\,x \end{aligned} \qquad x \in \Re^{n_x},\; u \in \Re^{n_u},\; y \in \Re^{n_y}\,. \tag{1}$$

where A and B are system matrices, x the state vector, u the control signal vector, y the output vector and n_x, n_u and n_y are the number of state, control and output variables, respectively.

Consider first that the complete state x is available for feedback. Then it is possible to allocate the set of closed--loop system poles to an arbitrary position by means of a suitable linear state feedback. Thus the control law is

$$u = -K\,x\,, \tag{2}$$

where k is the feedback gain matrix.

When the number of output variables n_y is less than the order of the system n_x, then it is always possible to allocate n_y poles of the closed-loop system. Unfortunately, we can not assign the location of the remaining (n_x–n_y) poles of the closed-loop system [3].

As the location may be arbitrary, the first step in pole assignment is to decide on their location. To choose the location of the poles, one should bear in mind that the required physical effort is dependent on the distance to the imaginary axis. Particularly, zeros in open loop attract poles, being very difficult to remove the poles near zeros. Consequently, for an arbitrary location, the control vector could lead to physically or economically undesired action. These considerations lead to the linear optimal regulator problem.

For the linear system described in (1), the optimal control signal u that minimizes the performance index

$$J = \int_0^\infty \left[x^T Q\,x + u^T R\,u \right] dt \tag{3}$$

is a linear function in terms of the system state variables x as [1,5]

$$u = -K\,x = -R^{-1} B^T P\,x\,, \tag{4}$$

where Q and R are the weighting matrices and P is the solution of the matrix Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0\,. \tag{5}$$

Riccati equation is the key to obtain the optimal control. Once the matrices Q and R are known, the matrix P can be obtained by solving the Riccati equation. From equation (4) one gets the optimal control vector u.

So far it was assumed that the complete state x is available for feedback. That assumption is unrealistic for the most of the systems. Usually, only a reduced number of state variables, or a linear combination thereof, are available — the so-called output vector y. In that case, it is necessary to recur to output feedback.

Therefore, suppose that the control vector must be directly obtained from the system output [6]:

$$u = -F\,y\,, \tag{6}$$

where F is the feedback gain matrix. The linear model of system (1) can be rewritten as

$$\dot{x} = \left[A - BFC\right]x\,. \tag{7}$$

Let $A^* \triangleq \left[A - BFC\right]$. The F matrix, for which the control vector u is optimal, minimizes the performance index (3):

$$F = R^{-1}B^T K^* L^* C^T \left[CL^* C\right]^{-1}. \tag{8}$$

The matrix K^* is a positive semi-definite solution of

$$K^* A^* + A^{*T} K^* + Q + C^T F^T RFC = 0 \tag{9}$$

and L^* is a positive definite solution of

$$L^* A^{*T} + A^* L^* + I = 0\,. \tag{10}$$

In the special case that C^{-1} exists, the result is identical to that obtained with the optimal state feedback, and (8) reduces to

$$F = R^{-1}B^{\mathrm{T}}K^* C^{-1} \tag{11}$$

and (9) becomes

$$K^* A + A^{\mathrm{T}} K^* + Q - K^* BR^{-1}B^{\mathrm{T}} K^* = 0\,. \tag{12}$$

To obtain the optimal gain matrix F, it will be necessary to apply an iterative method. The following computer algorithm can be used [6].

Let

$$F_{n-1} = R^{-1}B^{\mathrm{T}}K_{n-1}L_{n-1}C^{\mathrm{T}}\left[CL_{n-1}C^{\mathrm{T}}\right]^{-1}, \tag{13}$$

where K_n is solution of

$$\begin{aligned} &K_n\left[A - BF_{n-1}C\right] + \left[A - BF_{n-1}C\right]^{\mathrm{T}} K_n + Q + \\ &+ C^{\mathrm{T}} F_{n-1}^{\mathrm{T}} RF_{n-1}C = 0 \end{aligned} \tag{14}$$

and L_{n-1} is solution of

$$L_{n-1}\left[A - BF_{n-1}C\right]^{\mathrm{T}} + \left[A - BF_{n-1}C\right]L_{n-1} + I = 0\,. \tag{15}$$

Start by guessing the initial value for F_0. Then, by (14) and with $n = 1$, obtain K_1. Substitute K_1 into (13). F_1 is still unknown, but it depends only on L_1. Substituting F_1 into (15) one gets a nonlinear equation in only one unknown, L_1. Equation (15) is then solved for a positive definite L_1. This value of L_1 is substituted into (13) to give F_1. This completes the first iteration. The value of F_1 initiates the next iteration [6].

The foregoing has been a brief review of optimal control theory as it applies to the optimal load frequency regulator problem. However, in order to design the load frequency regulator, a major problem still remains: how to select the matrices Q and R. Next we present a technique for specifying the matrices Q and R in the context of optimal state feedback.

III. Proposed Approach

In this paper, without loss of generality, the weighting matrix R is specified as an identity matrix. Thus, the problem then is to specify the weighting matrix Q.

The most common procedure to specify the matrix Q is search by trial and error. Different matrices Q_i result in different gain matrices F_i, which in turn result in different dynamic performances for the closed-loop system. There are many possibilities to test by trial and error, which makes this procedure burdensome.

In order to simplify and accelerate this procedure of trial end error, or even avoid it altogether, a new algorithm, based on [4], to select the weighting matrix Q is proposed.

When the location of the systems poles is assigned, and a suitable gain matrix F is computed, then it is possible to judge on the weight of each state variable upon the control vector. An initial weighting matrix Q_0 may then be selected. This matrix plays a decisive role in making the system poles move close to the desired pole locations, as done by conventional pole assignment techniques.

<u>Algorithm</u>

Step 1- Specify desired location for system poles.

Step 2- Compute the corresponding gain matrix F to allocate the poles Λ in the desired location.

Step 3- Let $Q_0 = diag\left(\left|F\right|\right)$ (i.e. Initialize Q).

Step 4- Vary each value of Q_0, one by one, and compute the eigenvalues of the closed-loop system:

for i=1 : n
$Q_i = Q_0$
for j=1 : $\dfrac{\max\ value\ \ for\ Q_i(i,i)}{\max\ value\ of\ \ Q_0}$
$Q_i\,(i,i) = Q_i\,(i,i) \times j$
compute Λ_{ij}
end
end

Step 5- Retrieve Q for the desired pole location.

Notes:

The more negative the real parts of the eigenvalues specified in step 1 are, the larger the elements values of Q_0 will be.

Step 4 imposes a maximum value for the elements of the weighting matrix Q, as expressed by $\max\ value\ \ for\ Q_i(i,i)$. This prevents feedback gains from becoming too large.

As an example, the value of Q_i is computed ten times with an increment in j of five, for max *value for* $Q_i(i,i) = 100$.

To select the value for $Q_i\,(i,i)$ (step 5), it is convenient to draw the locus of the eigenvalues.

IV. Test Results

For an interconnected system with a large number of generating units, modeling the power control system of each individual generator is complex. Although, simpler models of lower dimension give more understandable results. In general, we may identify groups of generators which are closely coupled internally but relatively weakly coupled between groups. The model and its usefulness are enhanced when the boundaries of the coherent groups of generators coincide with those of an operating company or utility. In this case, under a single operating jurisdiction, the generators are frequently controlled as single unit and in such case the groups are called control areas.

To illustrate the proposed algorithm a four different interconnected areas are considered: one area with hydro power plants–area 1, two with fossil power plants, one with nonreheat steam system configuration–area 2 and the other one with tandem compound, single reheat steam system configuration–area 3 and the last area is considered to have infinite kinetic energy–area 4, as shown in Fig. 1.

This case study can be compared to an interconnection of the power systems of individual utilities. Each company operates independently within its own jurisdiction, but there are contractual agreements about intercompany exchanges of power through the tie-lines and other agreements about operating procedures to maintain system frequency. The adjustment of the command increment, ΔP_{r_i}, is done automatically and according to the contractual agreements.

We have combined the referred *four-area system* and the model was obtained. This model is shown in Fig. 2, using a block diagram representation, and the system is defined by the symbols and parameters shown in Appendix, where $K_p = 1/D_i$ and $T_p = 2H_i/D_i$.

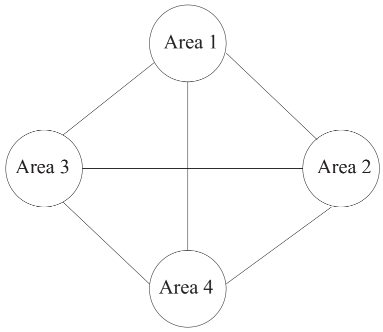

Fig.1 – Case study – four interconnected control areas

As a result a linear model of the system, in state space form, is obtained:

$$\dot{x} = Ax + Bu + \Gamma \Delta P_D \,. \tag{16}$$

The state vector is defined as

$$\begin{aligned} x = [\Delta f_1 \quad \Delta f_2 \quad \Delta f_3 \quad \mathrm{M} \quad \Delta p_{\mathrm{int},1} \quad \Delta p_{\mathrm{int},2} \quad \Delta p_{\mathrm{int},3} \quad \mathrm{M} \\ \Delta p_{M_1} \quad \Delta p_{M_2} \quad \Delta p_{x1} \quad \Delta p_{x2} \quad \Delta p_{M_3} \quad \mathrm{M} \quad . \\ \Delta x_{E_1} \quad \Delta x_{E_2} \quad \Delta x_{E_3} \quad \mathrm{M} \quad \Delta p_{C_1} \quad \Delta p_{C_2} \quad \Delta p_{C_3} \quad] \end{aligned}$$

The control vector and incremental load demand change vector are defined, respectively, as

$$u = \begin{bmatrix} \Delta p_{r1} \\ \Delta p_{r2} \\ \Delta p_{r3} \end{bmatrix}, \ \Delta P_D = \begin{bmatrix} \Delta p_{D_1} \\ \Delta p_{D_2} \\ \Delta p_{D_3} \end{bmatrix}.$$

Substituting the definition of the states and the control into to the seventeen differential equations that define the four-area system we can obtain, respectively, the control matrix B, the disturbance matrix Γ and the state matrix A, that it results in a controllable system but not observable.

$$B' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{0.5T_{G_1}} & 0 & 0 & 0 & 0 & \frac{1}{T_{G_1}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{G_2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{G_3}} & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma' = \begin{bmatrix} \frac{-K_{P_1}}{T_{P_1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-K_{P_2}}{T_{P_2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-K_{P_3}}{T_{P_3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$
A=\begin{bmatrix}
\frac{-1}{T_{P_1}} & 0 & 0 & \frac{-K_{P_1}}{T_{P_1}} & 0 & 0 & \frac{K_{P_1}}{T_{P_1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-1}{T_{P_2}} & 0 & 0 & \frac{-K_{P_2}}{T_{P_2}} & 0 & 0 & \frac{K_{P_2}}{T_{P_2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{-1}{T_{P_3}} & 0 & 0 & \frac{-K_{P_3}}{T_{P_3}} & 0 & 0 & 0 & 0 & \frac{K_{P_3}}{T_{P_3}} & 0 & 0 & 0 & 0 & 0 & 0 \\
T_{12}+T_{13}+T_{1i} & -T_{12} & -T_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-T_{21} & T_{21}+T_{23}+T_{2i} & -T_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-T_{31} & -T_{32} & T_{31}+T_{32}+T_{3i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{0.5R_1T_{G_1}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{0.5T_W} & 0 & 0 & 0 & 0 & \frac{T_{G_1}+T_W}{0.5T_{G_1}T_W} & 0 & 0 & \frac{-1}{0.5T_{G_1}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_t} & 0 & 0 & 0 & 0 & \frac{1}{T_t} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_1} & 0 & 0 & 0 & 0 & \frac{1}{T_1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_2} & \frac{-1}{T_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_1}{T_3}-\frac{K_1}{T_1}-\frac{K_2}{T_2} & \frac{K_2}{T_3}+\frac{K_3}{T_3}-\frac{K_2}{T_2} & \frac{-1}{T_3} & 0 & 0 & \frac{K_1}{T_1} & 0 & 0 & 0 \\
\frac{-1}{R_1T_{G_1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{G_1}} & 0 & 0 & \frac{1}{T_{G_1}} & 0 & 0 \\
0 & \frac{-1}{R_2T_{G_2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{G_2}} & 0 & 0 & \frac{1}{T_{G_2}} & 0 \\
0 & 0 & \frac{-1}{R_3T_{G_3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{G_3}} & 0 & 0 & \frac{1}{T_{G_3}} \\
0 & 0 & 0 & -K_{i_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -K_{i_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -K_{i_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

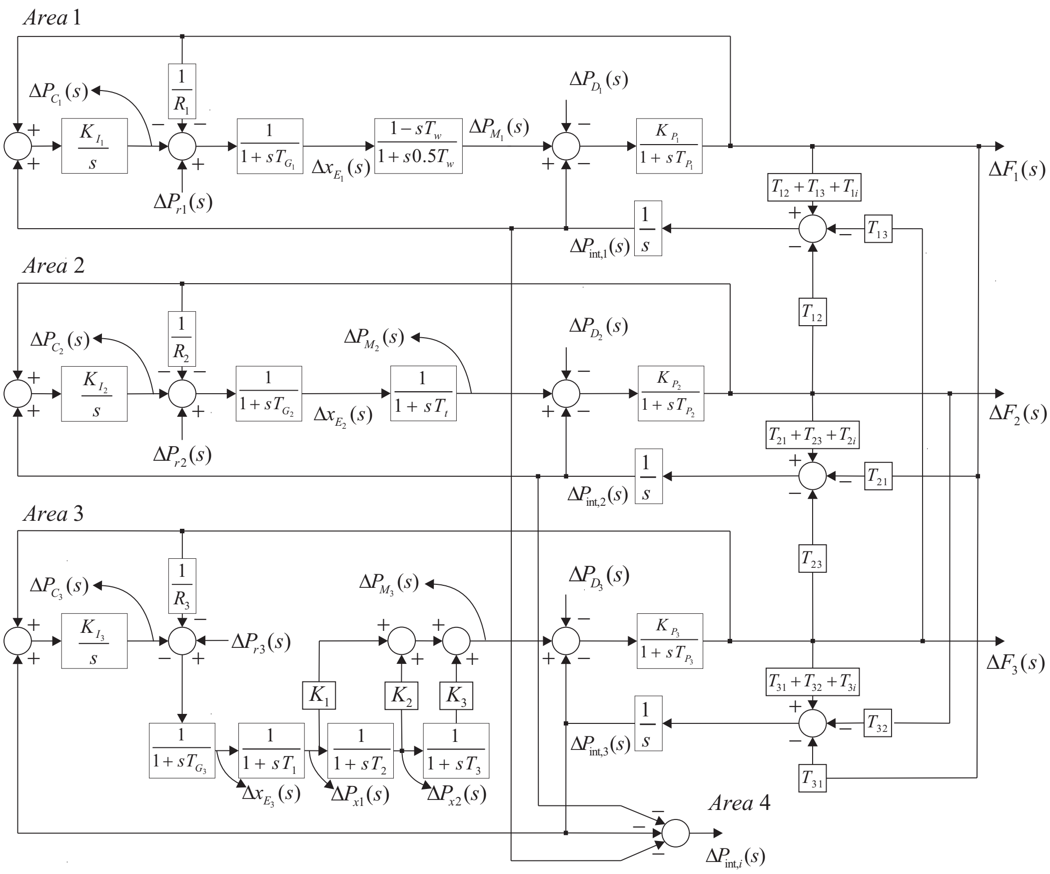

Fig.2 – Block diagram of four-area perturbation model.

Applying the proposed algorithm, for the parameters given in appendix, we can specify the weighting matrix Q,

$$Q = diag([\,85\;\;85\;\;85\quad M\;8\;\;8\;\;8\quad M\;68\;\;68\;\;68\;\;68\;\;68\quad M \\ 8.5\;\;8.5\;\;8.5\quad M\;8\;\;8\;\;8\,]),$$

corresponding to state variables.

The output feedback gains matrix, F, respectively, for the state variable according to the control area i, is defined as:

$$F = \left[F_{SP}\; M F_{INT}\; M F_T\; M F_R\; M F_{CI}\right].$$

This gain matrix F, for convenience and with purposes of more understandable presentation of results, is subdivide into five submatrixes corresponding, respectively, to the output feedback gains of the state variable according to the control area i (F_{SP} – feedback gains for the incremental frequency deviation, F_{INT} – feedback gains for the net incremental real power exported from area i, F_T – feedback gains for the governor, F_{CI} – feedback gains for the integral control) and are:

$$F_{SP} = \begin{bmatrix} 7.73 & 0 & 0 \\ 0 & 10.75 & 0 \\ 0 & 0 & 10.04 \end{bmatrix}, \quad F_{INT} = \begin{bmatrix} -15.04 & 0 & 0 \\ 0 & -2.41 & 0 \\ 0 & 0 & -3.21 \end{bmatrix},$$

$$F_T = \begin{bmatrix} 3.07 & 0 & 0 & 0 & 0 \\ 0 & 13.12 & 0 & 0 & 0 \\ 0 & 0 & 5.29 & 10.38 & 2.40 \end{bmatrix},$$

$$F_R = \begin{bmatrix} 22.33 & 0 & 0 \\ 0 & 2.69 & 0 \\ 0 & 0 & 3.45 \end{bmatrix}, \quad F_{CI} = \begin{bmatrix} -1.83 & 0 & 0 \\ 0 & -2.46 & 0 \\ 0 & 0 & -1.88 \end{bmatrix}.$$

The corresponding eigenvalues for the system without and with optimal output control are shown in Table I.

TABLE I.
SYSTEM EIGENVALUES FOR BOTH CONVENTIONAL AND OPTIMAL OUTPUT CONTROL

System eigenvalues with conventional control	System eigenvalues with optimal output control applied
-24.75	-212.00
-10.80	-16.31
-5.28	-4.29
-6.19	-2.50
-3.30	-0.28
-2.72	$-17.79 \pm j8.68$
-1.68	$-12.96 \pm j6.35$
$-0.35 \pm j4.25$	$-2.61 \pm j1.95$
$-0.25 \pm j3.22$	$-0.84 \pm j2.32$
$-0.30 \pm j1.56$	$-0.5 \pm j0.06$
$-0.87 \pm j1.34$	$-0.23 \pm j\,0.09$
$-0.20 \pm j0.38$	

A 10% step-load increase ($\Delta P_D = 0.1\,pu\,MW$) is applied to area 1 and subsequent deviations of Δf_1, Δf_2, Δf_3 and $\Delta P_{\text{int},1}$, $\Delta P_{\text{int},2}$, $\Delta P_{\text{int},3}$, $\Delta P_{\text{int},\infty}$ are studied. Fig. 3 and Fig. 4 illustrate the time evolution of the system frequency deviations, for each area, with classic control and with optimal output control, respectively. Fig. 5 and Fig. 6 illustrate the time evolution of the tie-line power deviation, for each area, with classic control and with optimal output control, respectively.

From these results, along with many others obtained but not discussed herein, some conclusions can be made: (1) the performance of the system with decentralized optimal output state feedback controller, Fig. 4 and Fig. 6, exhibits better damping than with the conventional control (Fig. 3 and Fig. 5) – optimal output controllers are especially effective in damping fast the oscillatory tie-line load and frequency deviations; (2) considering the system eignevalues, was expected the increase of the stability margin of optimal solution as Fig. 4 and Fig. 6 shows.

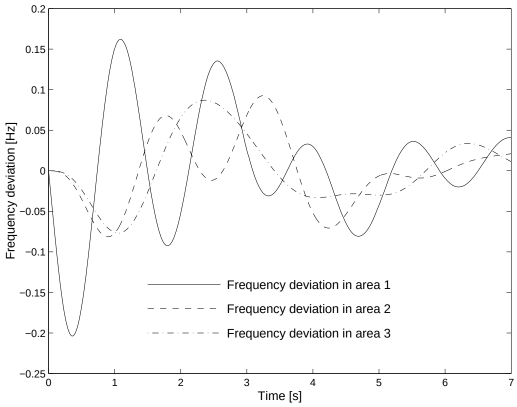

Fig.3 – Step response of system frequency deviations with classic control

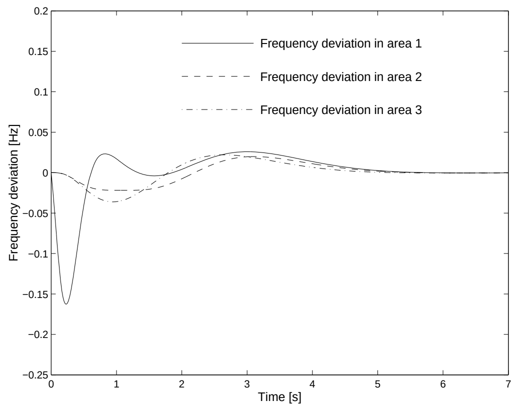

Fig.4 – Step response of system frequency deviations with optimal output state feedback controller

The longer term effects, as shown in Fig. 3 and Fig. 4, shows that, since the outside world is infinite (one area considered to have infinite kinetic energy), frequency is essentially back to normal and therefore the powers of each area are back to normal, by increasing the mechanical power on area 1 – the basic principle of pool operation (scheduled interchanges of tie-line power are maintained and each area absorbs its own load changes) are satisfied. Thus, the interconnection is effective on delivering power to the control area 1, specially with optimal control where the step-load increase in area 1 is satisfied simultaneous and in equal parts, in the first second, by the other areas and, in the next seconds, by the area considered to have infinite kinetic energy.

Also, by examining the optimal gains, it is seen that the gains, corresponding to the incremental frequency deviation variable and to the net incremental real power exported from area *i*, impose more influence by this states, with less significant contribution of the other states.

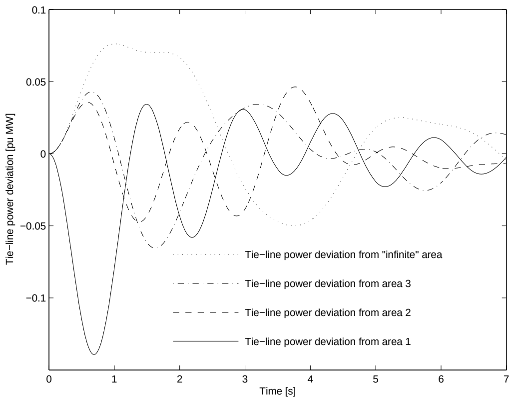

Fig.5 – Step response of system tie-line power deviation with classic control

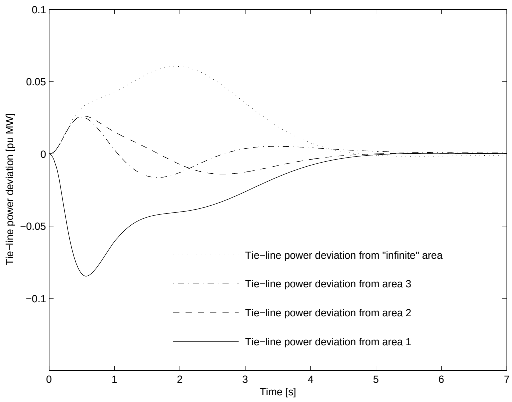

Fig.6 – Step response of system tie-line power deviation with optimal output state feedback controller

V. CONCLUSION

The frequency control problem of a large power system (a multi-area electric energy system) was solved through the application of a new technique that allows the specification of the weighting matrices *Q* and *R*. As a conclusion, the optimal output state feedback, together with the proposed algorithm for specifying *Q* and *R*, yields results that are robust, and according to the desired performance.

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APPENDIX

System symbols and parameters

Δf_i incremental frequency deviation in area *i*

ΔP_{M_i} incremental generated real power (mechanical power) deviation in area *i*

ΔP_{x1} and ΔP_{x2} incremental fraction generated power in tandem compound, single reheat turbine

ΔP_{D_i} incremental load demand change in area *i*

ΔP_{r_i} incremental command signal to governor speed changer in area *i*

$\Delta P_{\text{int},i}$ net incremental real power exported from area *i*

$\Delta P_{\text{int},\infty}$ net incremental real power exported from area considered to have infinite kinetic energy

H_i inertia constant ($H_1 = 0.06$, $H_2 = 0.1$ and $H_3 = 0.16$ in seconds)

T_{G_i} governor time constant ($T_{G_1} = 0.08$, $T_{G_2} = 0.1$ and $T_{G_3} = 0.2$ in seconds)

R_i drop constant ($R_1 = 5$, $R_2 = 2.5$ and $R_3 = 2.6$ in $Hz/pu\ MW$)

T_{ij} transmission constant (considered equal for all interconnections, $T_{ij} = 0.5$ in $pu\ MW/Hz$)

T_w hydro turbine time constant ($T_w = 0.1$ in seconds)

T_T nonreheat turbine time constant ($T_T = 0.3$ in seconds)

T_1, T_2 and T_3 tandem compound, single reheatturbine time constants ($T_1 = 0.2$, $T_2 = 6$ and $T_3 = 0.4$ in seconds)

K_1, K_2 and K_3 tandem compound, single reheat cylinder fractions ($K_1 = 0.3$, $K_2 = 0.4$ and $K_3 = 0.3$)

D_i system damping ($D_1 = D_2 = D_3 = 0.01$ in $pu\ MW/Hz$)