# Center of Gravity Position Optimization for Fuel Savings


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## Abstract

Fuel has been considered the biggest airline operating cost [1]. Therefore, it has become imperative that some measures have to be taken to counteract this trend.

This thesis discusses the present and future fuel conservation techniques, evaluating their impact in the fuel consumption, both individually and combined.

Nevertheless, the main focus of this study lies on the optimization of the center of gravity position through the implementation of a computational model which consists in the use of multiquadrics radial basis functions.

The model created showed to be efficient and reliable, considering the associated error minimalism.

Through the analysis of the data obtained with the proposed model, it was also possible to verify that the alteration of center of gravity position leads to a significant variation of the fuel consumption. An optimization of this parameter leads to a decrease of drag rates and engine thrust resulting in a reduction on fuel consumption and emission of pollutants into the atmosphere.



## Keywords

Fuel consumption, Center of Gravity, Multiquadrics.


## 1. Introduction

The fear of an untypical rise of jet fuel price is always present in airlines managers' minds.

Fuel is already the biggest airline operating cost so it is extremely important to take some measures in order to reduce this financial burden.

Some procedures like reduce APU usage and flaps deflection as well as single-engine taxi have been adopted for companies in order to conserve more fuel. Also in the maintenance sector, engine washing and drag reduction programs are already mandatory [2,3].

Nevertheless, is still hard to implement some flight procedures like changing the cruise level with step climbs as the aircraft gets lighter to remain as close as possible to the optimum altitude. Fly at optimum altitude and at Long Range Cruise Mach Number, $M_{LRC}$, will accrue in substantial fuel savings [2].

Choose the right Cost Index is also important. Performing a flight at smaller CI's yield more fuel conservation with no significant time impact [4].

However, it was the influence of the center of gravity location on fuel consumption that this study intended to investigate.

The center of gravity is the theoretical point where the total weight of an aircraft is assumed to be concentrated.

The adequate position of CG will ensure the aircraft maneuverability and stability as well as structure integrity, so it's extremely important to respect the limits associated to this parameter.

The further aft the CG is, the heavier the weight over the main gear will be. Thus CG could only assume positions which are tolerable for it. The same could be seen in the case of a too forward CG but for the nose gear.

The adherence of nose gear in order to facilitate the operations on ground when the rudder is not effective yet has to be guaranteed. To have an appropriate adherence, enough weight has to be applied on the nose gear, hence the position of CG cannot be located too far.

It should also be taken into account that a too forward CG will lead to a hard take-off rotation. On the other hand, the opposite is not desirable, since a too aft position could compromise the tail risking tail strike.

Despite of a forward CG presuppose a more stable aircraft, it cannot be assumed a too forward CG position because it will generate a pitch down moment impossible to be compensated for the Trimmable Horizontal Stabilizer (THS).

When engines are at full throttle a pitch up moment will be compensated by the elevator. However, with an aft CG this pitch up moment will be bigger. So this position cannot be that aft that the pitch moment become impossible to be compensated by the elevator. Other things like the fact of being mandatory that the CG has to be in front of the maneuver point have to be consider.

The CG position can vary through the movement of fuel. This is all controlled by the Fuel Control and Monitoring System (FCMS). It calculates the adequate CG position and compares it with a target value that depends on the aircraft weight. When the right position is displayed, it informs the quantity of fuel that has to be moved and to where [5].

In this study, it was intended to determine an optimum CG position which could minimize the fuel consumption. To that a computational model have been created.

## 2. Methodology

The model implemented is based on multiquadrics radial basis functions.

It is intended to obtain a function similar to the next one:

$$y = f(x) = \lambda_0 + \sum_{k=1}^{m} \lambda_k \, \varphi_k(x) \tag{1}$$

This function aims the approximation of flight data in order to obtain the associated fuel consumption.

$$\dot{m} = f\begin{pmatrix} FL \\ TAS \\ ROC/D \\ Mach \\ Weigth \\ CG \end{pmatrix} \tag{2}$$

The $\varphi_k(x)$ represents the multiquadric function:

$$\varphi_k\,(x) = \sqrt{\sigma^2 + \|x - x_k\|} \tag{3}$$

x represent the problem data, $x_k$ are the centers and $\sigma$ is the shape parameter which is calculated as follows:

$$\sigma = \max(min\|x_i - x_j\|) \tag{4}$$

When the $\varphi_k\,(x)$ are calculated, the $\lambda_k$ could be determined.

It is important to note that the approximation accuracy depends on the selected centers.

In this model the initial centers selected are the inputs data, but they were optimized using the Fuzzy c-means Algorithm, FCMA.

This algorithm consists in clusters creation and a posterior determination of the point that could represent efficiently the all cluster. This was made through the Euclidian distance calculation between all the points of the partition.

Mathematically it begins with the minimization of the following equation: [6]

$$J_m(U,V) = \sum_i \sum_k u_{ik}^m\, d^2\,(x_k, V_i) \tag{5}$$

With:

$$u_{ik} = \frac{1}{\sum_{i=1}^{c}(d(x_k,V_i)/d(x_k,V_j))^{2/(m-1)}} \tag{6}$$

$$V_i = \frac{\sum_{k=1}^{n}(u_{ik})^m\ x_k}{\sum_{k=1}^{n}(u_{ik})^m} \tag{7}$$

And where:

$$d^2(x_k,V_i) = \|x_k - V_i\|^2 \tag{8}$$

With:

$$d^2(x_k, V_i) > 0 \tag{9}$$

Note that:

$U = [u_{ik}] \in \mathbb{R}^{cn}$ is a fuzzy c-partition of $n$ unlabelled data set $X = \{x_1, \dots, x_n\} \in \mathbb{R}^{pn}$;

$m$ is the fuzziness index;

V is a set of c fuzzy cluster centers $V = (V_1, \dots, V_c) \in \mathbb{R}^{pc}$;

Finishing all that is finally possible to obtain the function that will give the best CG position correspondent to a minimum fuel consumption.

## 3. Results & Discussion

In a primary approach this model was tested for the aircraft A and considered reliable in the climb, cruise and descent data approximation with an associated error of $3.84 * 10^{-6}$ using all the centers available. However, it was seen that using only half of the possible centers the approximation was already satisfactory like proven in the graphic below:

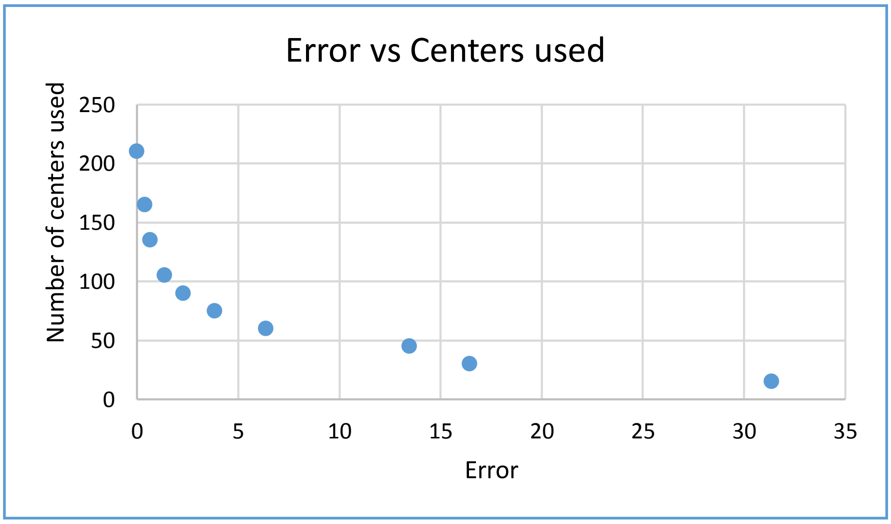



**Figure 1 - Error depending on the number of centers used**

Secondly and for study effects only cruise phase was considered owing to its long duration. The results obtained for the aircraft A were:

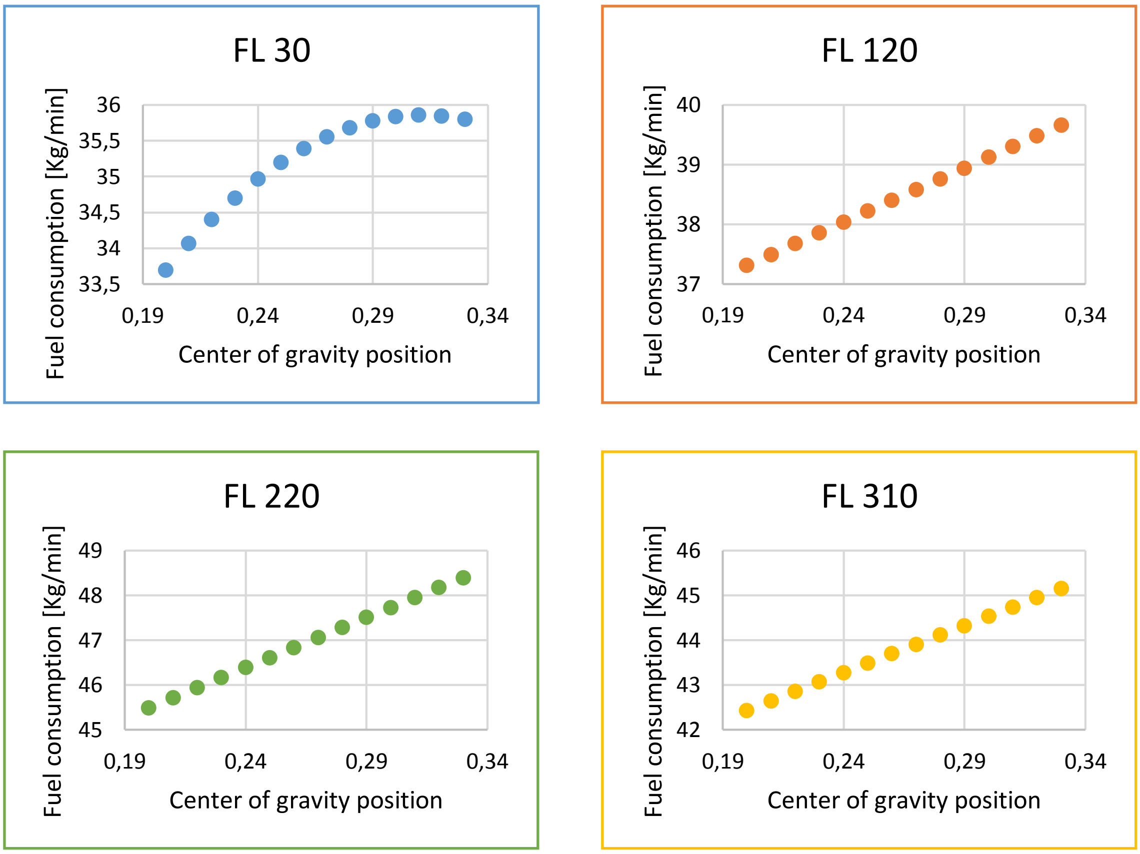
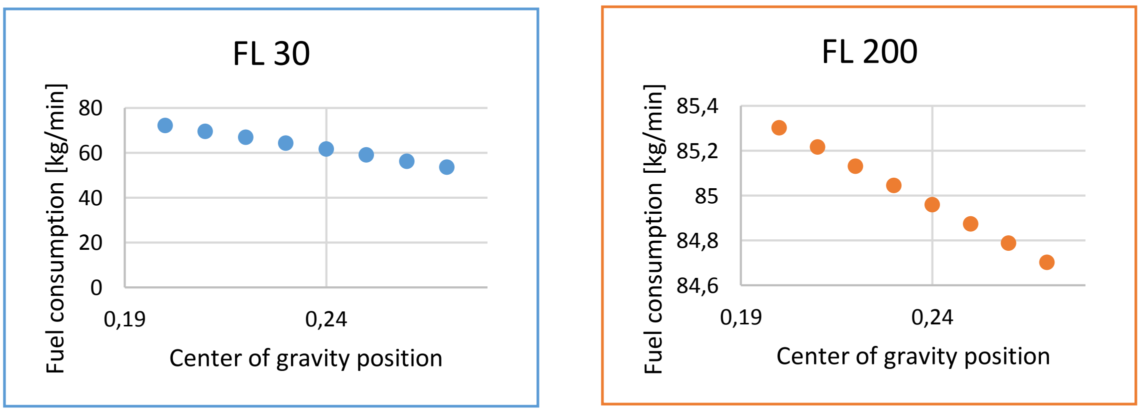



**Figure 2 - Variation of Fuel consumption according to the center of gravity position for Aircraft A**

This graphics are a sample of the results obtained. It is possible to see that for all flight levels the optimum center of gravity positon is the forward one. In a cruise FL 310 it is attainable to save 3,07 Kg/min adopting a 20% Mac position compared to the 27%Mac. Considering a 120min cruise about 368Kg of fuel can be saved.

The same tests were made to the aircraft B. The results were:



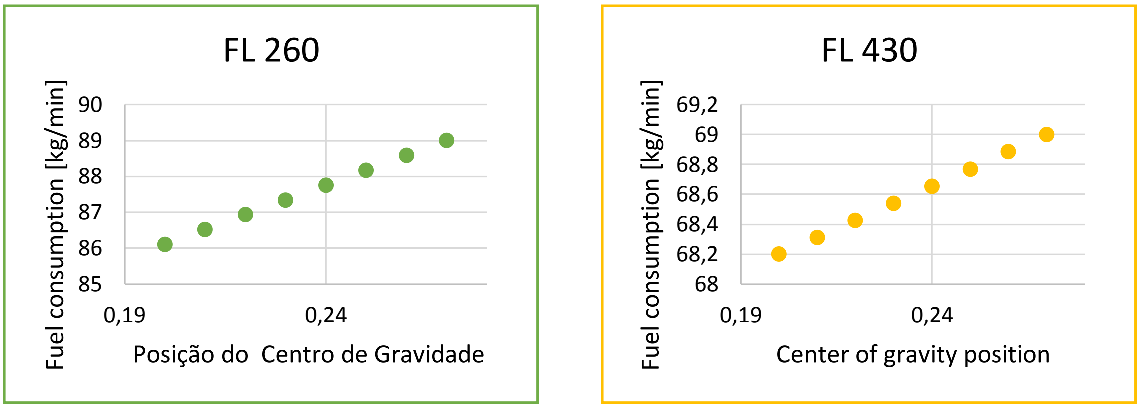



**Figure 3 - Variation of Fuel Consumption according to the Center of Gravity Position for Aircraft B**

In this case, it is possible to observe that until FL200 included, the center of gravity position which minimizes the fuel consumption is the after one. However, for FL superiors to the FL 200 the best CG position to reach a minimum fuel consumption in cruise is 20%Mac. So, it can be concluded that for short range flights the optimum CG position that should be selected is 27%Mac like shown in blue on Figure 4. However, in medium and long range flights the CG position to save fuel should be 20%Mac (green profile). In a cruise at FL430 it is possible to save 96Kg of fuel adopting this profile when comparing to a flight using a CG of 27%MAC. Nevertheless, if the after CG was the selected one, the climb and descent phases should be prolonged in order to avoid this penalties in cruise flight like shown in red on the graphic below:

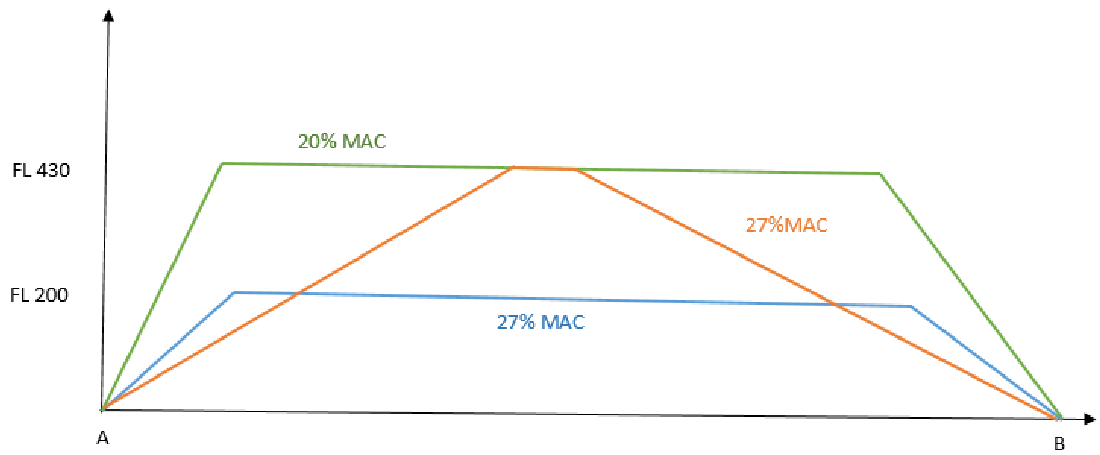



**Figure 4 - Flight profiles for different CG position adoption**

## 4. Conclusions

With this study it was possible to see that CG position varies depending on aircraft type. It was also perceptible that the fuel saves are also different.

By adopting the correct CG position it is possible to save a lot a fuel as well as diminish emissions of pollutants into the atmosphere.

The multiquadric model has shown to be a simple and reliable tool for data approximation so it can easily be introduced in airlines industry in the future.

## 5. References


[1] Viegas, J., Leal, N., "Fuel Saving: Uma ferramenta fundamental para o aumento da eficiência energética e de competitividade do sector da aviação comercial", Palestra ministrada na Universidade da Beira Interior, 2012.

[2] Airbus, *"Getting to grips with Fuel Economy"*, Flight Operations Support & Line Assistance, France, October 2004.

[3] ATR, *"Fuel saving: Contributing to a sustainable air transport development"*, 2008.

[4] Roberson, B., "Fuel Conservation Strategies: Cost Index Explained", *Boeing Commercial Aeromagazine*, 2007.

[5] Airbus, *"Getting to grips with Weight and Balance"*, Flight Operations Support & Line Assistance, France.

[6] Zahid, N., Abouelala, O., Limouri, M., Essaid, A., "Fuzzy clustering based on k-nearest neighbours rule", *Fuzzy Sets and Systems*, 120 (2001) 239-247.