UNIVERSIDADE DA BEIRA INTERIOR

Engenharia

Dynamic Network Modelling and Simulation for Air Traffic Flow Analysis

Tiago Mendes Domingues

Dissertação para a obtenção do Grau de Mestre em
Engenharia Aeronáutica
(2º ciclo de estudos)

Orientador: Prof. Doutor Kouamana Bousson

Covilhã, Outubro de 2011

Dedicated to my family, my girlfriend and my closest friends.

Acknowledgements

First of all I have to thank my supervisor, Prof. Doutor Kouamana Bousson, for he has been the one to show me the north whenever it seemed lost. His wisdom and the calm he transmitted to me allowed to advance step by step never forgetting to aim into the final goal. This is why I admire him and it is the reason why I will always be grateful for all the things he taught me and led me to learn.

Many thanks to my colleagues oriented by my supervisor, specially Paulo Machado and Carlos Velosa. The discussions we had and the availability to help each other and share our experience and knowledge is reflected in several key aspects of this document.

I would like to acknowledge *Fundação para a Ciência e a Tecnologia* (FCT) and *Departamento de Ciências Aeroespaciais* (DCA) for providing me the financial support for this dissertation which came from the MIT-Portugal program on the project “AIRDEV - Business Models for Airport Development and Management”.

My everlasting gratitude falls towards my loving parents Adelino and Maria. They built the foundations of my existence and together with all the rest of my family they nourished my growth and evolution for many years. There I can trust to always have a constant in my life, an anchor to protect me from the wild sea failing on tearing me apart.

To my girlfriend Tânia for her support and for all the strength she gave me. Her words kept me motivated and her actions provided the equilibrium required by this ordeal. There is a big part of her in this work.

I can’t forget my friends either from Covilhã and Amares for their friendship, comradeship and even sacrifice. They won’t blame me for not naming them because they know that most likely someone might be left aside undeservedly.

I would like to thank my co-workers from Masterjet Lisbon who enabled the continuity of my work side by side with the further development of this dissertation. It can be hard sometimes to come from a day at work and still have the spirit to think on yet another set of issues. But with the comprehension and support they gave me it really made things way easier.

Last but not least I recognize that this work could not have been accomplished without the anonymous work of the Open Source community. If there was no LaTeX nor Octave this document could not exist. Also the interface made by the enthusiastic people at *www.localizatodo.com* allowed me to have data for the simulations.

Resumo

Esta dissertação descreve o processo pelo qual se encontraram um conjunto de equações representativas de um modelo discreto de dinâmica de tráfego aéreo em zonas terminais. Tal modelo matemático poderá ser utilizado para investigar a ocupação de uma *TMA** num instante singular, tendo o potencial de ser utilizado para prever a ocupação futura.

Poderia também ser usável por um Controlador de Tráfego Aéreo em prever uma futura sobre-ocupação e portanto ser capaz de a prevenir. É por isso que existe um interesse em saber a cada instante uma estimativa de quantas aeronaves estão numa *TMA* específica.

A idéia por detrás deste assunto foi a de fazer uma abordagem de *Teoria de Controlo* de forma a que as ferramentas clássicas utilizadas noutros tópicos, tais como dinâmica de voo, pudessem ser aplicadas a este problema. Para que isso fosse possível eram necessárias noções de *Network* a par de fórmulas de navegação.

Dados obtidos do sítio web *www.localizatodo.com* em conjunto com o Software *Octave*, permitiram a validação do modelo. As equações comportaram-se como deviam, fazendo este modelo aplicável a um cenário real, já que os dados utilizados de facto provêm eles mesmos de de um cenário real e não de um inventado.

Este trabalho poderia ser utilizado como fundação para a implementação de um sistema de alerta aos Controladores de Tráfego Aéreo do estado de ocupação. Não só isso como também poderia ser um módulo num Controlador Automático de Espaço Aéreo e mesmo noutras aplicações ainda por considerar. Poderia até ser aplicado no tráfego de autocarros ou poderia ser extrapolado para muitos outros campos.

Palavras-chave: Teoria de Controlo, TMA, Controlador de Tráfego Aéreo, Previsão, Aviso, Transponder

*do Inglês *Terminal Manoeuvring Area*, que será definido mais à frente nesta dissertação.

Resumo Alargado

Esta dissertação descreve o processo pelo qual se encontraram um conjunto de equações representativas de um modelo discreto de dinâmica de tráfego aéreo em zonas terminais. Tal modelo matemático poderá ser utilizado para investigar a ocupação de uma *TMA*[†] num instante singular, tendo o potencial de ser utilizado para prever a ocupação futura.

Existem limites de segurança e legais acerca de quantas aeronaves deverão estar dentro de uma *TMA* de uma só vez. Poderia também ser usável por um Controlador de Tráfego Aéreo em prever uma futura sobre-ocupação e portanto ser capaz de a prevenir. É por isso que existe um interesse em saber a cada instante uma estimativa de quantas aeronaves estão numa *TMA* específica.

A idéia por detrás deste assunto foi a de fazer uma abordagem de *Teoria de Controlo* de forma a que as ferramentas clássicas utilizadas noutros tópicos, tais como dinâmica de voo, pudessem ser aplicadas a este problema. Para que isso fosse possível eram necessárias noções de *Network* a par de fórmulas de navegação.

Se uma aeronave deixa um determinado sítio sabemos *a priori* que vai seguir um caminho e mais tarde vai entrar numa *TMA* específica, a menos que algo corra mal. Essa informação é sabida antes sequer de haver descolagem. Isto permite-nos limitar a nossa janela de monitorização e podemos agora focar-nos na existência física de uma aeronave dentro ou fora de uma determinada TMA.

Qualquer aeronave num raio de várias milhas náuticas, dependendo de cada *TMA*, pertence a essa *TMA*. Uma transição entre áreas é considerado um input de controlo.

Sabendo a quantidade inicial de aeronaves no instante inicial num determinado sítio, podemos calcular o instante seguinte adicionando as chegadas e eliminando as partidas. Este sistema tem conservação de massa no sentido em que as aeronaves não desaparecem. Ao invés elas movem-se de um lado para o outro e o número total de aeronaves num sistema continua o mesmo.

Dependendo da complexidade do sistema, outros estados separados poderão ser monitorizados e podemos separar as aeronaves no chão das do ar; permitir alteração de decisão a meio de um voo para dirigir-se a uma *TMA* diferente, etc.

Dados obtidos do sítio web *www.localizatodo.com* em conjunto com o Software *Octave*, permitiram a validação do modelo. Neste sítio web os sinais de transponder capturados são depois transformados em vários bits de informação dos quais se retiram os úteis: Longitude, Latitude e carimbo de tempo. As equações comportaram-se como deviam, fazendo este modelo aplicável a um cenário real, já que os dados utilizados de facto provêm eles mesmos de de um cenário real e não de um inventado.

Os primeiros dois modelos são ligeiramente diferentes do terceiro. Um deles é relativo às *TMA* como se elas fossem contíguas umas das outras e segundo já considera os caminhos entre elas. São passos no caminho de um objectivo, em vez de serem uma solução final.

A inclusão de uma função sigmóide ao modelo foi o último trabalho a ser efectuado. Tem que ver com a previsão e de quanto pesaria uma aeronave a chegar aos olhos do Controlador de Tráfego Aéreo. Existe uma necessidade de remover mudanças

[†] do Inglês *Terminal Manoeuvring Area*, que será definido mais à frente nesta dissertação.

abruptas inerentes ao modelo, de forma que uma aeronave fosse ficando presente gradualmente na *TMA* de chegada a partir do momento que tenha descolado da *TMA* de partida, ao invés de somente aparecer na *TMA* de chegada e assim sendo não dar oportunidade de antecipação aos Controladores de Tráfego Aéreo.

Num trabalho futuro um estimador, tal como um Filtro de Kalman, deveria ser adicionado a este modelo ou até a um modelo similar melhorado, já que este tem a flexibilidade de incorporar novas características de uma forma expedita. Existe uma explicação em como fazer essa adições.

Este trabalho poderia ser utilizado como fundação para a implementação de um sistema de alerta aos Controladores de Tráfego Aéreo do estado de ocupação. Não só isso como também poderia ser um módulo num Controlador Automático de Espaço Aéreo e mesmo noutras aplicações ainda por considerar. Poderia até ser aplicado no tráfego de autocarros ou poderia ser extrapolado para muitos outros campos.

Abstract

This dissertation describes the achievement of a set of equations in order to represent a discrete model for air traffic dynamics in terminal areas. Such a mathematical model could be used to investigate the occupation of a TMA‡ at a single instant and has the potential to be used on the prediction of a future occupation.

There are safety and legal limits to how many aircraft should be inside a TMA at once. It could also be useful for an Air Traffic Controller to predict a future over-occupation and thus being able to prevent it. That is why there is an interest to know at each instant an estimate of how many aircraft are in a specific TMA.

The idea behind it was to make a *Control Theory* approach so that the same classical tools used in other topics such as flight dynamics could be applied to this subject. In order for that to happen there were needed some Network notions along with navigation formulae.

If an aircraft leaves one place we know *a priori* that he will follow a path and later enter a certain TMA, unless something goes wrong. That information is known even before lift off. It limits our monitoring window and we can now focus on the physical existence of an aircraft in or out a determined TMA.

Every aircraft within a radius of several nautical miles, depending on each TMA, does belong to that TMA. A transition between areas is considered a control input.

Knowing the initial amount of aircraft on the zero instant at a particular site we can calculate the following instant state by adding the arrivals and subtracting the departures. This system has mass conservation in a sense that aircraft don't just disappear - instead they move around and the total number of aircraft on the system remains the same.

Depending on the complexity of the system other separated states can be monitored and we could separate the grounded aircraft from the airborne aircraft; allow mid-flight decision change to go to a different TMA, etc.

Data retrieved from the website *www.localizatodo.com* together with *Octave* Software allowed the validation of the model. On this website the captured transponder signals are transformed into several bits of information from which were retrieved the ones useful: Longitude, Latitude and time stamp. The equations performed as they should, making them applicable to a real scenario, since the utilized data are in fact coming themselves from a real scenario and they are not invented.

The first two models are slightly different from the third. One concerns the TMA as if they were contiguous and another considers the paths among each other. They are steps towards an objective, rather than a final solution.

The inclusion of a sigmoid function to the model was the latest work to be done. It has to do with the prediction and how much would an incoming airship weight on the eyes of the Air Traffic Controller. There is a need to remove abrupt changes inherent to the model in a way that an aircraft would gradually become present on the arriving TMA from the moment he lifted off the departing TMA, instead of just showing

‡TMA stands for Terminal Manoeuvring Area which is defined further along on this dissertation.

up on the arriving TMA and thus giving no anticipation opportunity to the air traffic controllers.

In a future work an estimator, such as a Kalman Filter, should be added to this model or even an improved similar model, since it has the flexibility to incorporate new features in a simple way. There is an explanation on how to perform these additions.

This work could be used as a foundation for the implementation of a warning to the Air Traffic Controllers of the occupation status. Not only that but it could be a module on a fully automatic Airspace Controller or even other applications yet to be considered. It could even be applied on the traffic of buses (coaches) and it can be extrapolated to many other fields.

Keywords: Control Theory, TMA, Air Traffic Controller, Prediction, Warning, Transponder

Contents

List of Figures

List of Tables

Chapter 1

Introduction

This dissertation has its origins on the scholarship granted by "AIRDEV" project from which I was fortunate to receive. The challenge suggested was to find a set of equations in order to represent a discrete model for air traffic dynamics in terminal areas. Such a mathematical model could be used to investigate the occupancy rate of a TMA* at a single instant and has the potential to be used for occupancy rate prediction.

The intent of the suggested theme is to perform an analysis, optimization and control of dynamic networks based on models of air traffic in terminal areas. In fact it is important to determine and analyse the static and transitory phases of such networks with the purpose of optimizing and controlling them accordingly with specified performance requirements. These tasks are justified by the escalating importance of optimization of airspace.

A part of the answer to this motto is made by this dissertation by making a *Control Theory* approach so that the same classical tools used in other topics such as flight dynamics could be applied to this subject. In order for that to happen there were needed some Network notions along with navigation formulae.

When an aircraft leaves one place we know before it happens that he will follow a path and later enter a certain TMA, unless something goes wrong. This kind of information is known even before lift off. It limits our monitoring window and we can now focus on the physical existence of an aircraft in or out a determined TMA. Aircraft within a radius of several nautical miles, depending on each TMA, does belong to that TMA. A transition between areas is considered a control input. The states are the number of aircraft at a certain moment within a certain TMA. On the model that considers runways once an aircraft enters a runway it is considered to leave the TMA and vice-versa.

If we know the initial amount of aircraft on the zero instant at a particular site then we can calculate the following instant state. Simply by adding the arrivals and subtracting the departures we could get an answer but we want a system that is a function of the previous state. This will lead to develop three models and to leave some instructions in order to allow further changes. It could for instance incorporate the possibility of allowing mid-flight decision change to go to a different TMA.

A peculiarity of this system is that it has mass conservation in a sense that aircraft don't just disappear - instead they move around and the total number of aircraft on

*TMA stands for Terminal Manoeuvring Area which is defined further along on this dissertation.

the system remains the same. This is a very important assumption on which lays an important pillar of this work.

All data required for testing purposes was retrieved from “Localiza Todo” which is a website with the hyperlink *www.localizatodo.com*. Together with several scripts allowed the validation of the model. On this website the transponder signals captured by several antennas are transformed into several bits of information from which were retrieved the useful ones. The equations performed as they should, making them applicable to a real scenario, since the utilized data are in fact coming themselves from a real scenario i.e. they were not invented.

The third model is slightly different from the first two. The inclusion of a sigmoid function to the model was the latest work to be done. It has to do with the prediction and how much would an incoming airship weight on the eyes of the Air Traffic Controller. There is a need to remove abrupt changes inherent to the model in a way that an aircraft would gradually become present on the arriving TMA from the moment he lifted off the departing TMA, instead of just showing up on the arriving TMA and thus giving no anticipation opportunity to the air traffic controllers. Somehow it breaks with the mass conservation concept because there is a fraction of a plane arriving to a TMA but since it is more like a layer than a base model it cannot be compared with the other two.

Once completed this has the potential to be a part of the answer to the AIRDEV motto. This work could be used as a foundation for the implementation of an advisory to the Air Traffic Controllers of the occupation status. Not only that but it could be a module on a fully automatic Airspace Controller or even other applications yet to be considered. It could even be applied on the traffic of buses (coaches) and it can be changed in order to suit many other subjects.

Chapter 2

Bibliographic Research

2.1 Control Theory

Control theory originated around 150 years ago when the performance of mechanical governors started to be analysed in a mathematical way. Such governors act in a stable way if all the roots of some associated polynomi- als are contained in the left half of the complex plane. One of the most outstanding results of the early period of control theory was the Routh al- gorithm, which allowed one to check whether a given polynomial had this property. Questions of stability are present in control theory today, and, in addition, to technical applications, new ones of economical and biological nature have been added. Control theory has been strongly linked with mathematics since World War II. It has had considerable influence on the calculus of variations, the theory of differential equations and the theory of stochastic processes.[3]

Without control systems there could be no manufacturing, no vehicles, no computers, no regulated environment—in short, no technology. Control systems are what make machines, in the broadest sense of the term, function as intended. Control systems are most often based on the principle of feedback, whereby the signal to be controlled is compared to a desired reference signal and the discrepancy used to compute corrective control action.[4]

2.2 Navigation Revision

Historical Background

On the early days of Europe's Renaissance the economy depended widely on spice trade. It played an important role on food preservation and preparation since refrigeration as we know it didn't exist back then. But is was also used in medicine, personal grooming, etc.

This market was dependant on the overland transportation from various points in Asia, passing through the Middle East and ending on Europe. The long and hard path did attract travellers despite its dangers. The reason behind it was that even a small bag of pepper could make one individual a rich person.

The Iberian countries, at the very end of this long route, naturally took the lead in exploring alternate routes to obtain these precious goods. In the 1500s Portugal was already a major seafaring country. In 1534 the scholar *Pedro Nunes* was appointed to the newly created Chair of Mathematics at the University of Coimbra. He supposedly had a conversation with a sea captain that stated the annoying fact that whenever he set out, say, eastward, and fixed the rudder to go straight eventually he would find himself turning towards the equator.

As a result of this conversation, in 1537, *Pedro Nunes* published two treatises analysing the geometry of such courses. Rhumb (similar to the Portuguese word "rumo") can be defined as an angle about a meridian.

In 1569 a map was invented that would become to the following years as the standard. It was the *Mercator Map* idealised and constructed by *Gerhardus Mercator*. In it a course would be a straight line drawn on the map since the latitude is distorted accordingly even though the longitude remains unchanged.

Figure 2.1 shows a Mercator's Map where it can be seen the increasingly larger gap between parallels and a disproportion of land masses as we go further away from the equator. Greenland has approximately the same size as Africa on the map when it should be $\frac{1}{5}$ of Africa instead.

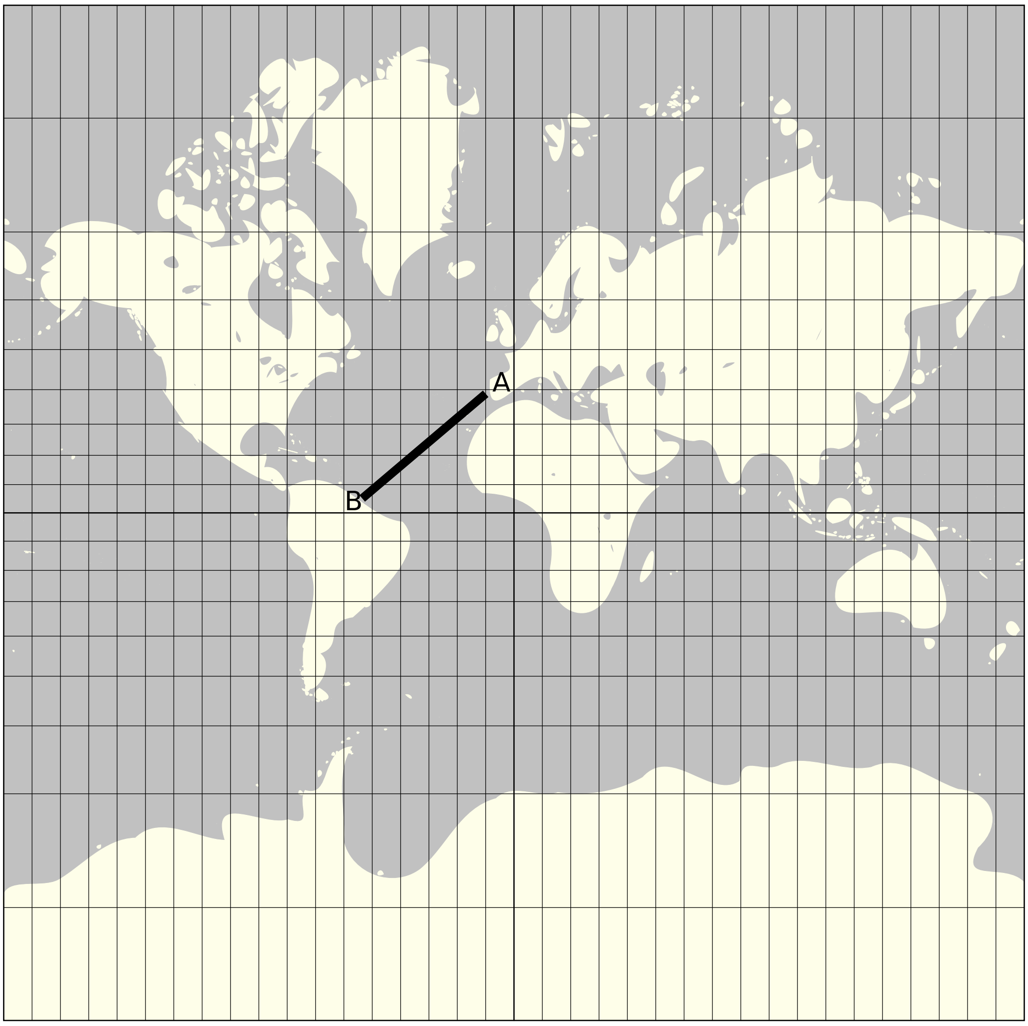

Figure 2.1: World map obtained by a Mercator projection. On this map a loxodromy is a straight line, i.e., a rhumb can be drawn as a segment of a line.

The line between points **A** and **B** is a loxodromy (Figure 2.1).

This information on this subject came from [5].

2.2.1 Navigation Formulae

Given two points **A** and **B** on the face of the earth we may want to know the distance between them. There are many ways of doing that and the several distances we can achieve represent a different kind of situation. We might want to know the distance between two points on the face of the earth as if we would drill a hole through the earth itself. That is not what we want here.

Instead we want the shortest distance on the surface of the earth. In order to have that we could use several orthodromies and that would make a good approximation of the real deal. An orthodromy is a segment of a great circle but it is not the shortest distance between two points **A** and **B**.

And yet the computational requirements to perform the actual real deal are not that much so we are using loxodromy:

$$L_A^* = \frac{10800}{\pi} \ln \left[\tan \left(\frac{L_A^\circ}{2} + 45^\circ \right) \right] \tag{2.1a}$$

$$L_B^* = \frac{10800}{\pi} \ln \left[\tan \left(\frac{L_B^\circ}{2} + 45^\circ \right) \right] \tag{2.1b}$$

$$\tan(V) = \frac{(\Delta G_{AB})_{min}}{\left(\Delta L_{AB}^*\right)_{min}} = \frac{|G_A - G_B|}{|L_A^* - L_B^*|} \tag{2.2}$$

$$\cos(V) = \cos\left[\tan^{-1}(\tan(V))\right] \tag{2.3}$$

$$(\Delta L_{AB})_{min} = |L_A - L_B| \tag{2.4}$$

The loxodromic distance is represented by equation (2.5):

$$(AB)_{NM}^{loxo} = \frac{(\Delta L_{AB})_{min}}{\cos(V)} \tag{2.5}$$

Of course **A** and **B** are the two points on the face of the earth. L or λ is for longitude (xx axis) and G or γ is for latitude (yy axis).

All of the information above came from [6].

2.3 Network Revision

A Graph represents only the structure (or topology) of a system, adding no more information such as distance, direction, etc. It is composed of vertexes connected by edges. Loops may also connect a vertex to itself. There is an example in figure 2.2.

Depending on the author, a *node* could be called a *vertex* or even a *point*. Also an *arc*, a *line* or even an *edge* can be seen as synonyms.

A Complete Graph has a joint between every two vertices by exactly one edge.

Diagraph or Directed Graph is used when there is a preferred direction for an edge. You can see it in figure 2.2.

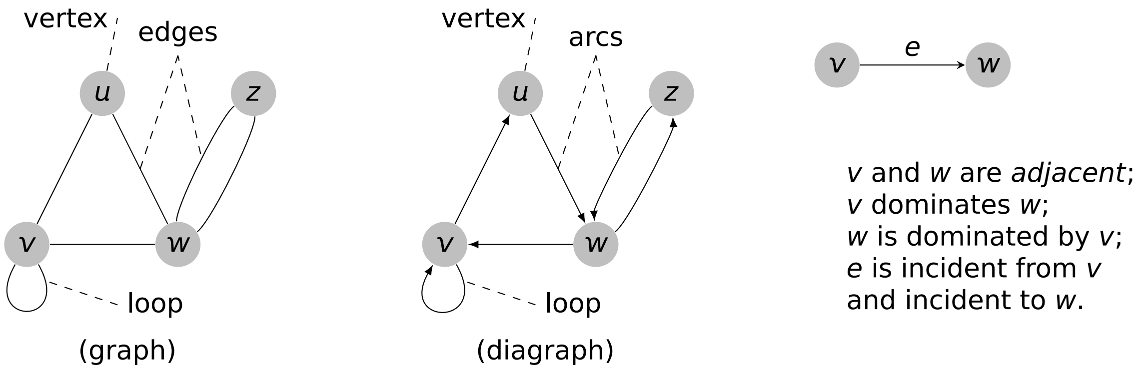

Figure 2.2: Nomenclature on Graphs

Network can be defined as a graph (or a diagraph) that has numerical values beside the corresponding edges. Those numbers are called *weights*. The distance between vertexes is an example of weight given to an edge on a network.

Also a vertex can be given a capacity, the same way as an edge can. The flow on a network can be seen as a running river, traffic along a road, electricity etc. All of them can be studied similarly as a weighted diagraph, thus a network.

Isomorphism of two different graphs is when they have a different shape yet hold the same information. Despite its difference graph (a) is the same as graph (b) on figure 2.3.

Topologically those two are the same even though they look differently. Only the connections amongst nodes matter and not their position in space and appearance.

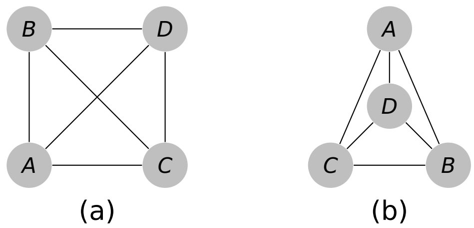

Figure 2.3: Isomorphisms that show two different shapes of a four vertex total graph.

Capacity is the positive value which represents the maximum amount of flow that could pass through an *arc e*. At figure 2.4 $\boldsymbol{f}(\boldsymbol{e})$ is the flow and $c(e)$ represents the capacity.

Once the flow equals the capacity then the arc is said *saturated*.

This section has origin on the book: [7].

2.4 Neural Networks Contribution - Sigmoid

The basic operation of an artificial neuron involves summing its weighted input signal and applying an output, or activation, function. For the input units the identity function is used:

$$f(x) = x, \qquad \forall x \tag{2.6}$$

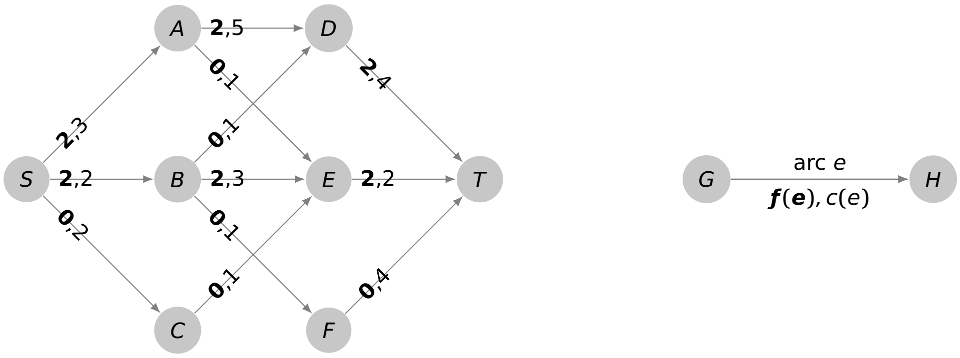

Figure 2.4: Flow from a source S to a sink T and the example of an arc e.

This is the most simple scenario but imagine that we intend to have a *threshold*. We can use a step function to convert the net input, which is a continuously valued variable, to an output unit that is either a binary (1 or 0) or bipolar (1 or -1) signal. The binary step function is also known as the threshold function or Heaviside function (with threshold θ):

$$f(x) = \begin{cases} 1 \quad , \quad x \geq \theta \\ \\ 0 \quad , \quad x < \theta \end{cases} \tag{2.7}$$

Sigmoid functions (S-shaped curves) are useful activation functions. The logistic function and the hyperbolic tangent functions are the most common, because the simple relationship between the value of the function at a point and the value of the derivative at that point reduces the computational burden during training.

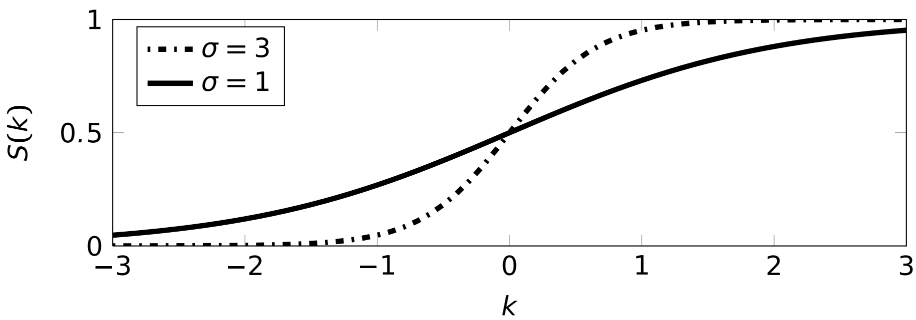

Figure 2.5: Binary sigmoid. Steepness parameters $\sigma = 1$ and $\sigma = 3$.

The logistic function, a sigmoid function with range from 0 to 1, is often used as the activation function for neural nets in which the desired output values either are in the interval between 0 and 1. To emphasize the range of the function, we will call it the *binary sigmoid* (see equation (2.8)); it is also called the *logistic sigmoid*. This function is illustrated in Figure 2.5 for two values of the steepness parameter σ.

$$f(x) = \frac{1}{1+e^{-\sigma x}} \tag{2.8}$$

$$f'(x) = \sigma f(x)\left[1 - f(x)\right] \tag{2.9}$$

The logistic sigmoid function can be scaled to have any range of values that is appropriate for a given problem.

All of this information came from [8].

Chapter 3

Discrete Model for Air Traffic Dynamics in Terminal Areas

3.1 Description of TMA

Controlled airspace is the airspace within which air traffic control service is provided and within which some or all aircraft may be subject to air traffic control. Types of controlled airspace are the *High Level Airspace* and the *Low Level Airspace*. Within the second one we can find amongst others the *TMA*[*] or *TCA*[*].

TMA are established at high volume traffic airports to provide an IFR[†] control service to arriving, departing and enroute aircraft. [1](pp. 188,192-193)

These definitions may differ from country to country but usually it follows a similar pattern. The general idea is what we need for the specific line of work on this thesis, being each TMA a defined area separated from other TMA for a certain amount of nautical miles. At this stage the difference between Classes (A,B,...,G,...) (take a peek at figure 3.1) is irrelevant.

Nevertheless here goes the explanation:

A, B, C, or D In all of these airspaces there is somewhat a restriction to VFR[‡] flight. All aircraft operating in Class A, B, C or D airspace must be equipped with a transponder and automatic pressure altitude reporting equipment.

E Class E airspace is designated where an operational need exists for controlled airspace but does not meet the requirements for Class A, B, C, or D. Low level airways, control area extensions, transition areas, or control zones established without an operating control tower may be classified as Class E airspace.

G Class G is all uncontrolled domestic airspace. Low-level air routes are contained within Class G airspace.

Source: [1](pp. 193-196)

If you could take a picture of a controlled airspace what you would see would be a series of TMA close or distant from each other in a network built by moving aircraft amongst themselves. A seemingly chaotic version of figure 3.2.

---

[*] Terminal Manoeuvring Area or Terminal Control Area
[†] Instrument Flight Rules
[‡] Visual Flight Rules

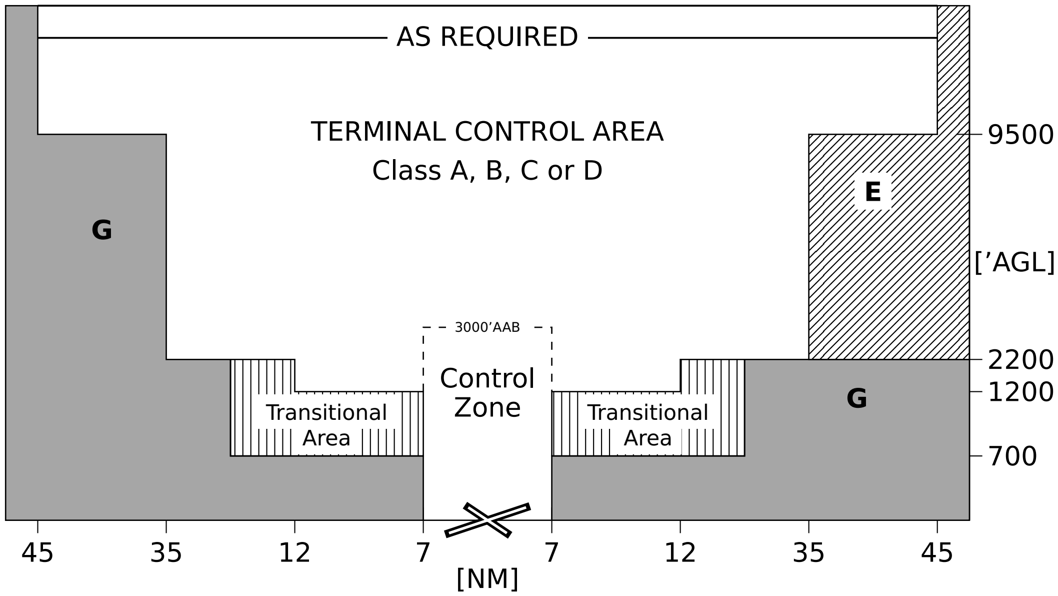

Figure 3.1: Inverse cake shaped schema of a typical TMA or TCA. Source:[1](page 193)

3.2 Air Traffic Dynamics Modelling in TMAs

Before modelling a non contiguous system we should study the simpler model. One where there is not a transition between the instant k to the instant $k+1$. One in the discrete time.

Let us consider for the moment that we can be here or there but not in between. This will allow a logic inference on this problem. All variables belong to the natural numbers ($\in \mathbb{N}_0^+$) and $i \neq j$:

t_{k+1}	—	time instant immediately after t_k ($t_{k+1} = t_k + \Delta t$);
$x_i(k)$	—	number of aircraft at the TMA i at the instant t_k;
$\alpha_{ij}(k)$	—	number of aircraft coming from i and arriving to j at the instant t_k;
$d_{ij}(k)$	—	amount of aircraft leaving i and going to j at the instant t_k;
$d_{ii}(k)$	—	aircraft static on x_i at the instant t_k;
$\alpha_{ii}(k)$	—	aircraft static on x_i at t_k, considering the immediate early landings;
$\mathbf{N}(k)$	—	total number of aircraft in the system at the instant t_k;
$n(k)$	—	total number of TMA in the system at the instant t_k;

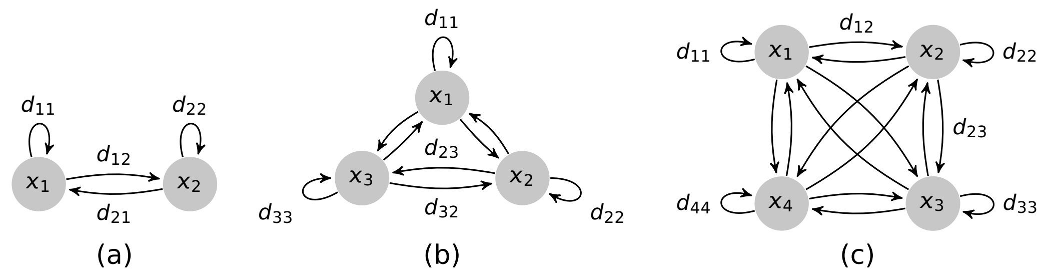

Figure 3.2: Graphs illustrating all possible interactions at a single discrete instant amongst airports in terms of departures and its growing of complexity as the amount of TMA on the system increases.

This system is different from most systems because of one detail: it has *mass conservation* (just like Antoine Lavoisier would have it!). When modelling a differential equation for a typical *predator/prey* situation if you have 3 wolves and 2 sheep you got 5 individuals total and when a wolf eats a sheep you end up with 4 individuals. Instead of this type of system in ours a plane does not get "eaten" by the airport and wherever they depart or arrive they always exist.

A different thing is to say that a new plane has entered the system or that a new TMA has been defined. That can be added to our system but for the general purpose $\mathbf{N}(k)$ and $n(k)$ given any k are considered constant:

$$\mathbf{N}(k) = \sum_{i=1}^{n} x_i(k) = \sum_{i=1}^{n} x_i(k+1) \tag{3.1}$$

$$n(k) = \sum_{i=1}^{n} i = \sum_{i=1}^{n} j \tag{3.2}$$

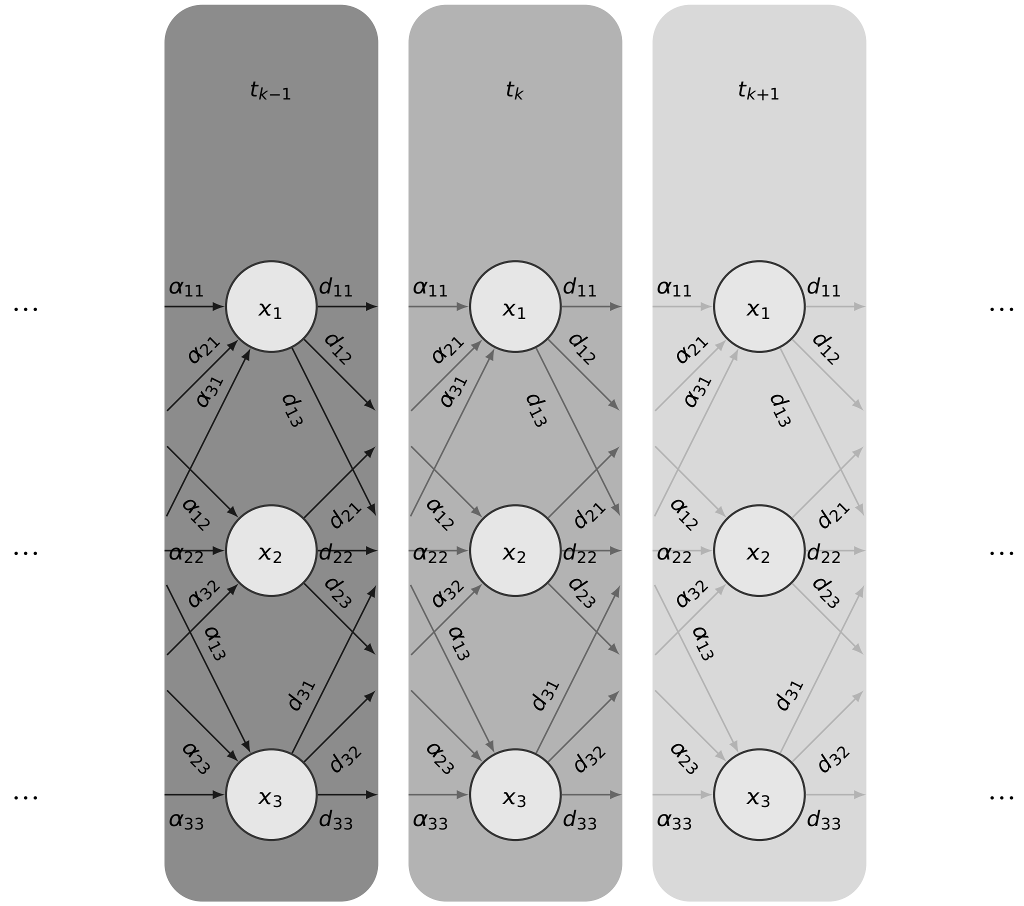

Figure 3.3: Representative graph about the interaction between three airports.

From here on equations will pop up from my understanding of figure 3.3. The first two questions arise for our system. What is our *state*? What about our *control*, where is it? The second question seems to have a direct answer that all departures $d_{ij}(k), \forall\{i \neq j\}$ should be the *control*. It might be a little bit early to state this but it seems that our *evader* is the same as our *control*. All this has to do with section 2.1 because we are trying to find x_k (state) and u_k (control).

In a system with two TMA (figure 3.2 (a)) the *control* would be d_{12} and d_{21}. Looking at figures 3.2 (b) and (c) it is inferred than when we have n TMA then we have n *states* and $n \times n - n$ *controls*.

From the definition in the section 3.1 the number of aircraft in a TMA i include the ones landing $\alpha_{ij}(k), \forall\{i \neq j\}$, others already landed $d_{ii}(k)$ and those departing $d_{ij}(k), \forall\{i \neq j\}$. Roughly speaking every aircraft within a radius of several nautical miles, depending on each TMA, does belong to that TMA i.

$$x_i(k) = \sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k) + \sum_{\substack{j=1 \\ i \neq j}}^{n} d_{ij}(k) + d_{ii}(k) \tag{3.3}$$

The *state* is $x_i(k)$ — the total number of aircraft in TMA i at instant t_k — and it is known empirically by the reasons behind equation (3.3).

Another question arises. What is the *pursuer* for our *evader*? I'll call it *carry* making a parallel to digital electronics. Further information on this subject can be found at [9]. Our *carry* or *evader* represents the amount of aircraft that were static in a TMA together with those who have already landed at the same TMA:

$$\alpha_{ii}(k) = d_{ii}(k-1) + \sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k-1) \tag{3.4}$$

Just by observing figures 3.3 and 3.4 we can notice that the number of aircraft at any given instant t_k can also be described as the *carry* together with all the arriving aircraft.

$$x_i(k) = \sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k) + \alpha_{ii}(k) \tag{3.5}$$

If nothing bad happens an aircraft that lifts off will arrive at another TMA after a while. It seems logic that all departures will eventually become arrivals after a time leap.

$$\forall\{i \neq j\} : d_{ij}(k) = \alpha_{ij}(k+1) \tag{3.6}$$

We have now the tools to begin to aim for a set of equations on a familiar form just like it is usual on so many systems where the *state* ahead (t_{k+1}) is a function of the *state* and *control* of the previous time (t_k).

$$x_i(k+1) = f\Big(x_i(k), d_{ij}(k)\Big), \forall i \neq j$$

So equations (3.3) and (3.5) won't do the job because we need to know the next state of the system depending on the previous.

A better more simple equation would take use of all equations above and provide a useful one. The algebraic manipulation of those equations will eventually give us the answer. By equalling the equations (3.3) and (3.5):

$$\sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k) + \alpha_{ii}(k) = \sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k) + d_{ii}(k) + \sum_{\substack{j=1 \\ i \neq j}}^{n} dij(k)$$

$$\Rightarrow \alpha_{ii}(k) = d_{ii}(k) + \sum_{\substack{j=1 \\ i \neq j}}^{n} d_{ij}(k+1) \tag{3.7a}$$

$$\Rightarrow d_{ii}(k) = \alpha_{ii}(k) - \sum_{\substack{j=1 \\ i \neq j}}^{n} d_{ij}(k+1) \tag{3.7b}$$

Starting with equation (3.5) and replacing its elements using equations (3.6) and (3.4):

$$\begin{aligned} x_i(k) &= \sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k) + \alpha_{ii}(k) = \\ &= \overbrace{\sum_{\substack{j=1 \\ i \neq j}}^{n} d_{ij}(k-1)}^{(3.6)} + \overbrace{d_{ii}(k-1) + \sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k-1)}^{(3.4)} = \end{aligned}$$

$$x_i(k) = d_{ii}(k-1) + \sum_{\substack{j=1 \\ i \neq j}}^{n} \left[d_{ji}(k-1) + \underbrace{d_{ji}(k-2)}_{(3.6)} \right] \tag{3.8}$$

The next step is to make disappear $d_{ii}(k-1)$. To make it happen first we should express equations (3.5) and (3.7a) in terms of $k-1$.

$$\begin{cases} x_i(k-1) = \alpha_{ii}(k-1) + \sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k-1) \\ \alpha_{ii}(k-1) = d_{ii}(k-1) + \sum_{\substack{j=1 \\ i \neq j}}^{n} d_{ij}(k-1) \end{cases}$$

$$\Rightarrow x_i(k-1) = d_{ii}(k-1) + \sum_{\substack{j=1 \\ i \neq j}}^{n} \left[d_{ij}(k-1) + d_{ji}(k-2) \right]$$

$$\Rightarrow d_{ii}(k-1) = x_i(k-1) - \sum_{\substack{j=1 \\ i \neq j}}^{n} \left[d_{ij}(k-1) + d_{ji}(k-2) \right] \tag{3.9}$$

Now that we have $d_{ii}(k-1)$ we can finally replace (3.9) on equation (3.8).

$$\boxed{x_i(k) = x_i(k-1) + \sum_{\substack{j=1 \\ i \neq j}}^{n} \left[d_{ji}(k-1) - d_{ij}(k-1) \right]} \tag{3.10}$$

Here is an example of how a traffic could occur. The numbers came from the example on figure 3.4 and they confirm equation (3.10):

$$
\begin{aligned}
x_2(k+1) &= x_2(k) + \Big(d_{12}(k) - d_{21}(k)\Big) + \Big(d_{32}(k) - d_{23}(k)\Big) \\
&= 7 + \Big(0 - 0\Big) + \Big(0 - 2\Big) \\
&= 5
\end{aligned}
$$

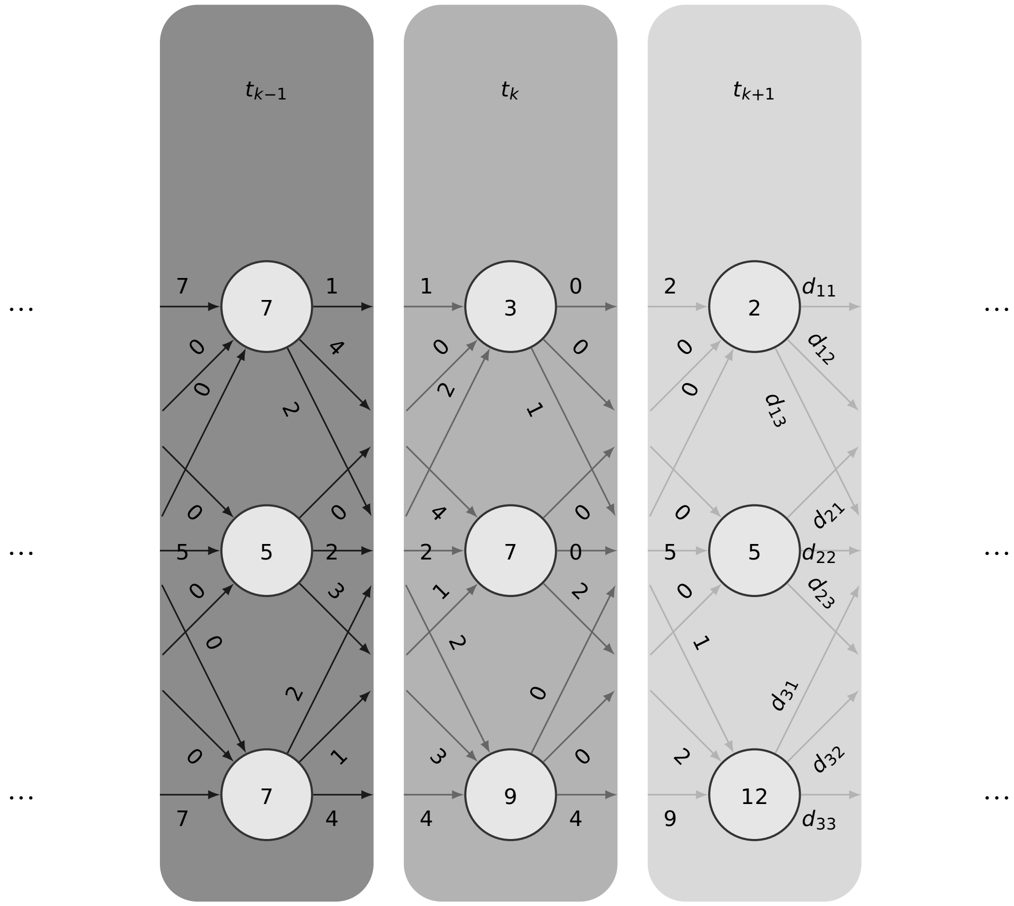

Figure 3.4: An example for figure 3.3.

3.3 Air Traffic Dynamics Modelling in TMAs and Paths

This section is nothing but an evolution from section 3.2. The former section will guide this one as if it were a cake recipe.

Paths amongst TMA are added to the simpler model so that it incorporates an important reality detail: when *aircraft one* departs from TMA x_i and *aircraft two* departs a while after it may happen that *aircraft two* arrives before to its destiny than *aircraft one*. And no matter what x_j they are heading these equations have to remain valid.

Also we have to consider the landing and lift-off in order to distinguish the airborne aircraft at the TMA from those standing on the ground. Since we aren't controlling the altitude this is made according to section 3.1 of this dissertation. Whenever the location of an aircraft indicates that it resides at an inner position within a 7 NM (seven nautical miles) range from the airport it will be considered as belonging to the runway.

Typically between this inner ring and an outer ring of 45 NM (forty five nautical

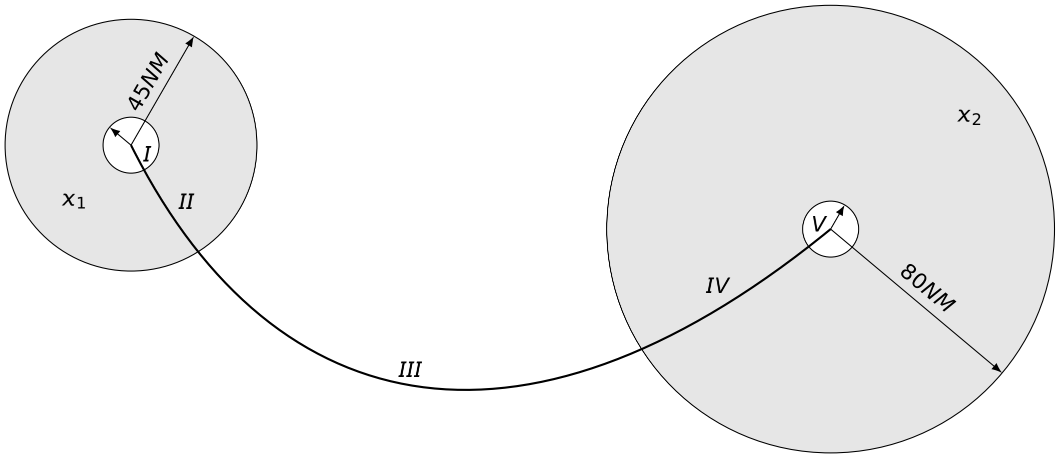

Figure 3.5: Trajectory of an aircraft from TMA x_1 to TMA x_2 and the generation of the control of the system which are the departures. Table 3.1 outputs the departures.

miles) the aircraft will be at the TMA. Sometimes this radius can be stretched due to fitting waiting aircraft within the TMA. All of this and more can be seen in figure 3.5

In order to prevent that an aircraft flying above and out of the TMA to be considered, within the data of each aircraft there are two variables i and j meaning "coming from" and "going to", respectively.

The goal which comes from defining i and j is to eventually find these four functions: $\exists o@r_i$, $\exists o@x_i$, $\exists o@r_j$ and $\exists o@x_j$. They evaluate the existence of an aircraft at a particular place just by knowing where does it come from, where does it go and its position in a two dimensional space composed by longitude (L or λ) and latitude (G or γ).

$$\exists o@x_i = \begin{cases} 1 \quad , \quad R^2 \geq (G_o - G_i)^2 + (L_o - L_i)^2 \\ \\ 0 \quad , \quad R^2 < (G_o - G_i)^2 + (L_o - L_i)^2 \end{cases} \tag{3.11a}$$

$$\exists o@r_i = \begin{cases} 1 \quad , \quad {R_r}^2 \geq (G_o - G_i)^2 + (L_o - L_i)^2 \\ \\ 0 \quad , \quad {R_r}^2 < (G_o - G_i)^2 + (L_o - L_i)^2 \end{cases} \tag{3.11b}$$

One (1) means it is true and it does exist. Zero (0) means it doesn't. The meaning of the variables present on equations 3.11a and 3.11b are as follows:

(L_i, G_i): Coordinates of the center of the TMA i.

(L_o, G_o): Coordinates of the position of a specific aircraft o.

R: Radius of the TMA i (usually 45 $[NM]$).

R_r: Radius of the runway at TMA i (10 $[NM]$ seems to me a reasonable value).

For the TMA x_3 that you can see ahead on figure 4.1 the equation (3.11) is not valid. Instead the existence of an aircraft on TMA x_3 means only that it is outside the dashed line.

By examination of figure 3.5 we can get table 3.1. The transition of *existence* in a previous state to the *non existence* on the next state will give rise to some kind of departure. Upper-case Roman numerals in the figure correspond to those in the table.

Table 3.1: Trajectory of an aircraft from TMA x_1 to TMA x_2 and the generation of the control of the system which are the departures. Figure 3.5 illustrates the departures.

(Figure 3.5)	V	I	I	II	II	III	III	IV	IV	V	V	I
i	3	1	1	1	1	1	1	1	1	1	1	2
j	1	2	2	2	2	2	2	2	2	2	2	1
$\exists o@r_i$	0	1	1	0	0	0	0	0	0	0	0	1
$\exists o@x_i$	0	1	1	1	1	0	0	0	0	0	0	1
$\exists o@r_j$	1	0	0	0	0	0	0	0	0	1	1	0
$\exists o@x_j$	1	0	0	0	0	0	0	1	1	1	1	0
Increment	—		d_{ii2ii1}		d_{ii1ij1}		d_{ij1jj1}		d_{jj1jj2}		—	

Graph in figure 3.6 represents that interaction. The triangles are the runways which are represented by the inner circle at figure 3.5 with 7 NM of radius. Circles are the TMA and squares account for the paths. Additionally every state should be seen as if there was a loop making it possible for an aircraft to remain at the same state indefinitely.

A Graph like the one on figure 3.3 is shown on appendix A along with the deduction of the equations of the current model. But the one on figure 3.6 is enough, for the growing of complexity requires some simplifications.

Notice that paths only receive aircraft from one source and only send it to one destination. Either that or they send aircraft for themselves to the future instant through the omitted loop.

A problem with nomenclature will arise about the writing of departure indexes. The chosen solution uses the index *1 (one)* for the circles and squares and the index *2 (two)* for the triangles. In figure 3.6 $d_{11\underline{2}11\underline{1}}$ and $d_{21\underline{1}11\underline{1}}$ have those indexes underlined.

Also an index i at x_i and at r_i will be represented by the index ii in the departure. As opposed to ij at p_{ij} which remains ij in the departure.

The departures can be seen as matrices in order to i, j and time k (figure 3.7). The equivalence between t and k can be understood at appendix B.

That said we can start the following of our recipe by identifying the two key aspects of our new system and then compare it with the model from the previous section (3.2). In table 3.2 we can see the relationship between the first model and this model. Once we identify this parallelism a simple substitution on equation (3.10) and we get equation (3.12). The full deduction can bee seen on appendix A.

Table 3.2: Comparison between equations in sections 3.2 and 3.3

Section 3.2	$x_i(k)$	$\sum_{j=1, i\neq j}^{n} \alpha_{ji}(k)$	$\sum_{j=1, i\neq j}^{n} d_{ij}(k)$	α_{ii}
Section 3.3	$p_{ij}(k)$	$\sum_{j=1, i\neq j}^{n} \alpha_{ii1ij1}(k)$	$\sum_{j=1, i\neq j}^{n} d_{ij1jj1}(k)$	$\alpha_{ij1ij1}(k)$
	$x_i(k)$	$\alpha_{ii2ii1}(k) + \sum_{j=1, i\neq j}^{n} \alpha_{ji1ii1}(k)$	$d_{ii1ii2}(k) + \sum_{j=1, i\neq j}^{n} d_{ii1ij1}(k)$	$\alpha_{ii1ii2}(k)$
	$r_i(k)$	$\sum_{j=1, i\neq j}^{n} \alpha_{ii1ii2}(k)$	$\sum_{j=1, i\neq j}^{n} d_{ii2ii1}(k)$	$\alpha_{ii2ii2}(k)$

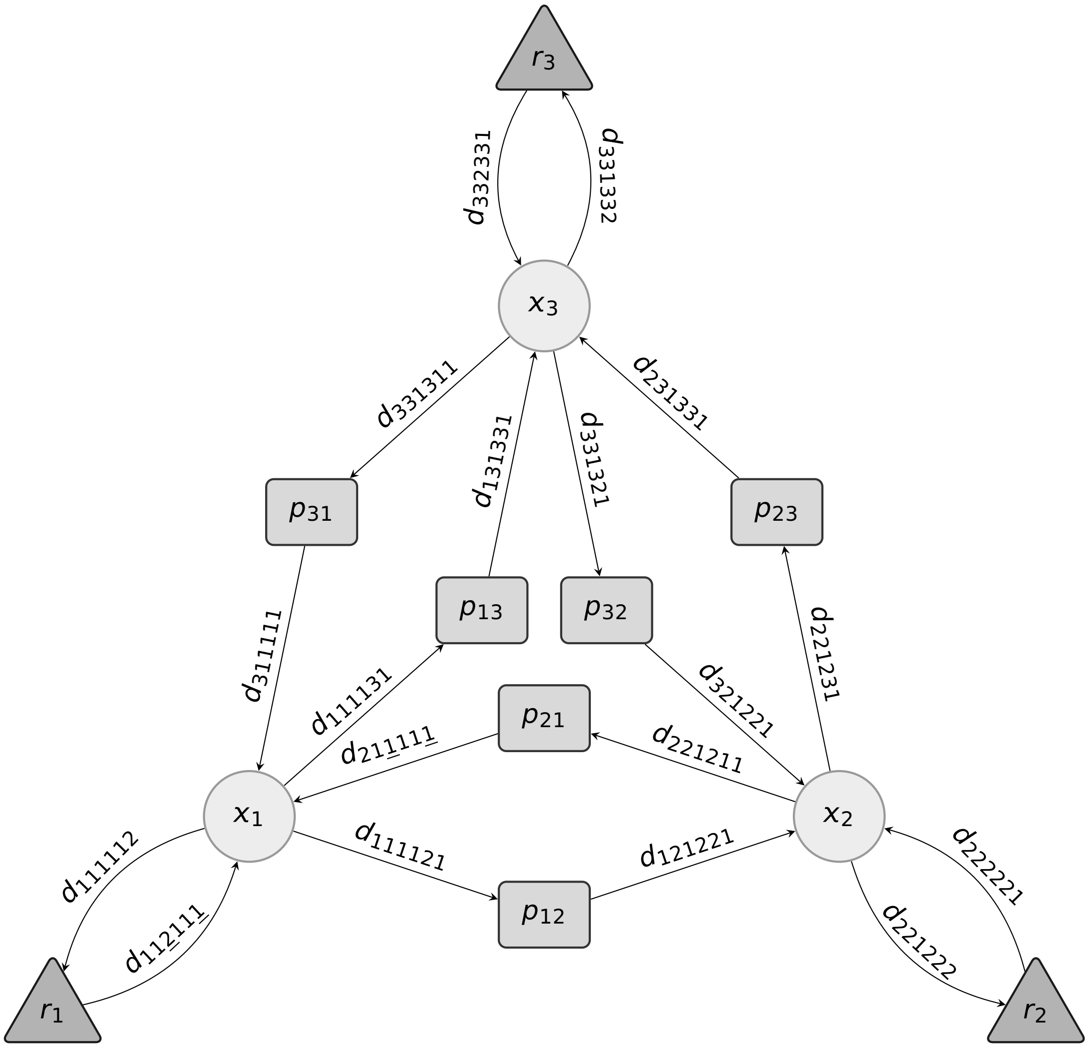

Figure 3.6: Graph representing our case study. The interaction is pictured between airports 1 and 2 considering the paths between them and the runways.

$$\begin{cases} p_{ij}(k) = p_{ij}(k-1) + \Big(d_{ii1ij1}(k-1) - d_{ij1jj1}(k-1)\Big) \\ r_i(k) = r_i(k-1) + \Big(d_{ii1ii2}(k-1) - d_{ii2ii1}(k-1)\Big) \\ x_i(k) = x_i(k-1) + d_{ii2ii1}(k-1) - d_{ii1ii2}(k-1) + \\ \qquad + \sum_{\substack{j=1 \\ i\neq j}}^{n} \Big[d_{ji1ii1}(k-1) - d_{ii1ij1}(k-1)\Big] \end{cases} \tag{3.12}$$

Needless to say that all of these equations work for any n. That is why they are in order to i and j so they can apply to any number of TMA. To include other features in the system (like allowing the transition between paths without passing through a TMA) one has to make a graph like the one on figure 3.6 or like figure 3.3 and then make an equivalence table similar to 3.2. From that table the equations can be easily deduced.

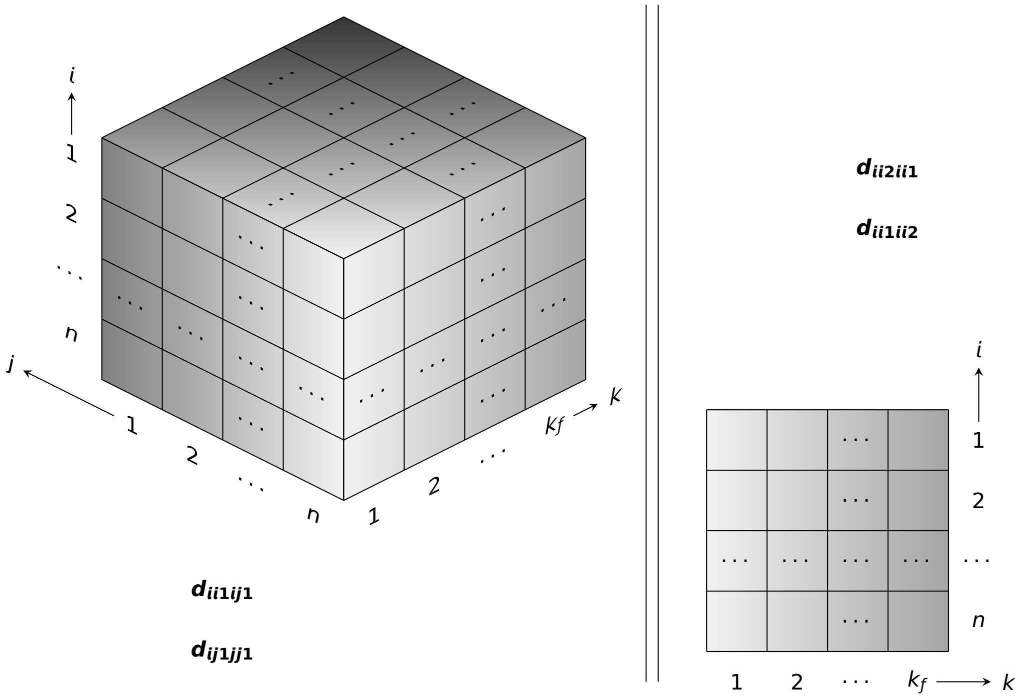

Figure 3.7: 3D and 2D matrices for the control, i.e., the departures.

3.4 Sigmoid Model

The idea is to have a model where there are no sudden changes to the eyes of the air traffic controller. An aircraft would gradually appear on the arriving TMA instead of showing up abruptly. This would allow a more intuitive prediction on the amount of aircraft arriving and the level of stress about to be implemented on the TMA we are at.

Even though the paths already provide a notion there is not a sense of progressiveness to the system. A workaround is proposed on the following pages but this issue is vast enough for a dissertation to be written only with such a model in mind.

3.4.1 The Sigmoid Function

Taking advantage on the properties of the sigmoid function 2.8 we can make a few changes in order to have a more well behaved formula that can suit our purposes, i.e. equation (3.13). A function that could go back and forth according to our desires.

$$S_{ijo}^{(2)} = \left[1 + e^{-\sigma \times \left(k - \frac{T_t}{2} - k_a\right)}\right]^{-1} \tag{3.13}$$

After some trial and error the equation 3.13 seems perfect for the job. In case there is some distrust on behalf of the reader here goes some proof. We start by defining an interval where the formula will work and with a total time of T_t:

$$T_t = k_z - k_a \tag{3.14}$$

$$\sigma = \frac{7}{T_t} \tag{3.15}$$

A steepness (σ) having nice characteristics is the one in equation (3.15). Those characteristics are such that for however different k_a and k_z the steepness remains the same.

In the beginning and in the end of this sigmoid we want our counter-domain to be zero and one, respectively. This solution provides some values near enough on it:

When $k = k_a$

$$-\sigma \times \left(k - \frac{T_t}{2} - k_a\right) = -\frac{7}{k_z - k_a} \times \left(k_a - \frac{k_z - k_a}{2} - k_a\right) = \frac{7}{2}$$

This is where the function (3.13) is near zero:

$$S_{ijo}^{(2)}(k_a) = \left[1 + e^{\frac{7}{2}}\right]^{-1} \approx 0.0293$$

When $k = k_z$

$$-\sigma \times \left(k - \frac{T_t}{2} - k_a\right) = -\frac{7}{k_z - k_a} \times \left(k_z - \frac{k_z - k_a}{2} - k_a\right) = -\frac{7}{k_z - k_a} \times \left(\frac{k_z - k_a}{2}\right) = -\frac{7}{2}$$

And this is where the function (3.13) is near one:

$$S_{ijo}^{(2)}(k_z) = \left[1 + e^{-\frac{7}{2}}\right]^{-1} \approx 0.9707$$

On this dissertation 1 means true and 0 means false but it can also mean 1 for already there and 0 for completely apart from somewhere.

3.4.2 Final Sigmoid Model

Now it starts to become interesting. The whole magic will happen between II and IV of table 3.1 and figure 3.5. We will work the paths so that they show up progressively on the graphics of each TMA. The whole section III of figure 3.5 and the transitions from and to it will be scrutinized.

Looking at table 3.1 section III is when all $\exists o@\ldots$ are zero. Let us invent a new matrix M_{ijo} encasing them together:

$$M_{ijo} = \exists o@\left[r_i, x_i, r_j, x_j\right] \tag{3.16}$$

The place to implement our sigmoid function $S_{ijo}^{(2)}$ from equation (3.13) is where M_{ijo} is zero and where M_{ijo} is something else it doesn't matter to us. Since it will later on multiply with the paths it is convenient to be zero so it will suppress the paths that have nothing to do with it.

$$S_{ijo} = \begin{cases} S_{ijo}^{(1)} = 0 & , \quad M_{ijo} \neq \vec{0} \\ \\ S_{ijo}^{(2)} & , \quad M_{ijo} = \vec{0} \end{cases} \tag{3.17}$$

Now we will call the loxodromic distance between two points **A** and **B** as $D_{A,B}$ to make things more simple. This $D_{A,B}$ represents the $(AB)^{loxo}_{NM}$ from equation (2.5).

Let a be a point on the transition between II and III; l be a point somewhere in the middle of III; and z the point in the transition between III and IV (relative to figure 3.5). We want an estimate on the mean velocity of the object (which happens to be an aircraft) in l so that we have a prediction of the time stamp on the instant z to be calculated at each time step.

$$\hat{V}_l = \frac{D_{l,1}}{t_{l,1}} \tag{3.18}$$

I already told you what the l stands for on the subscript of $D_{l,1}$. The 1 represents the first moment when we are aware of the aircraft. Since we have our velocity estimate $\hat{V}_l$ let us try and find $k_z(k)$ - the time where the aircraft reaches TMA j.

$$k_z(k) = \frac{D_{l,z}}{\hat{V}_l} \tag{3.19}$$

In order to know $k_z(k)$ we first have to find $D_{l,z}$. The distance from a to z is composed of the distance from a to l plus the distance from l to z:

$$D_{a,z} = D_{a,l} + D_{l,z} \tag{3.20}$$

And the distance from a to the center of TMA j is the distance from a to z plus the radius of TMA j:

$$D_{a,j} = D_{a,z} + R_j \tag{3.21}$$

Playing around with these two equations we have:

$$D_{a,j} - R_j = D_{a,l} + D_{l,z} \Leftrightarrow$$

$$\Leftrightarrow D_{a,z} = D_{a,l} + D_{l,z} \tag{3.22}$$

With this last piece of the puzzle qe can now roughly predict $k_z(k)$.

Please notice that this model doesn't take into account the acceleration or deceleration of the aircraft. It also doesn't care for the 2D trajectory (from up above) it follows; much less its 3D trajectory. It makes this model susceptible to error.

The success of it is only incident on showing the possibility or not possibility of implementing a prediction to the model in section 3.3. A model regarding flight dynamics and having several Kalman Filters might become a project whose scale is much bigger than this dissertation.

Nevertheless the following equation is fine to be implemented:

$$\boxed{\chi_j(k) = x_j(k) + \left[\sum_{i=1}^{n} p_{ij}(k)\left(\sum_{o=1}^{O} S_{ijo}(k)\right)\right]} \tag{3.23}$$

Here O is the total number of aircraft an o the step aircraft. It uses the values of $x_j(k)$ and $p_{ij}(k)$ from the previous model and shapes them to our needs.

Chapter 4

Simulation and Validation

Since it is merely a means to an end the model of section 3.2 was not simulated. But the important ones (3.3 and 3.4) were.

4.1 Air Traffic Dynamics Modelling in TMAs and Paths

In order to validate a hypothesis for a life improvement idea one should test it on a real scenario where the surrounding conditions are the same as if they were the ones of the final implementation environment.

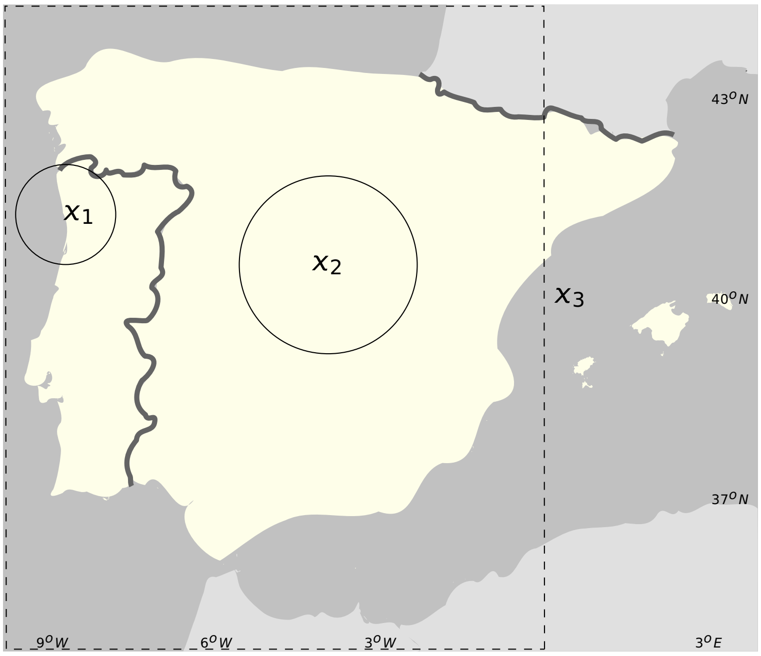

Figure 4.1: Iberian Peninsula showing TMA x_1, x_2 and a fictional TMA x_3, outside the big square. Source:Adapted from [2]

That ideal *modus operandi* can't always be achieved. When this happens there must be a way to replicate as accurately as possible the data that would be captured during a real-life test of our model.

It has not even been tried out and yet most likely the attempt to use the actual data from any Control Tower recorders would lead to a bureaucracy inferno that could last far beyond the schedule for this present work to be completed.

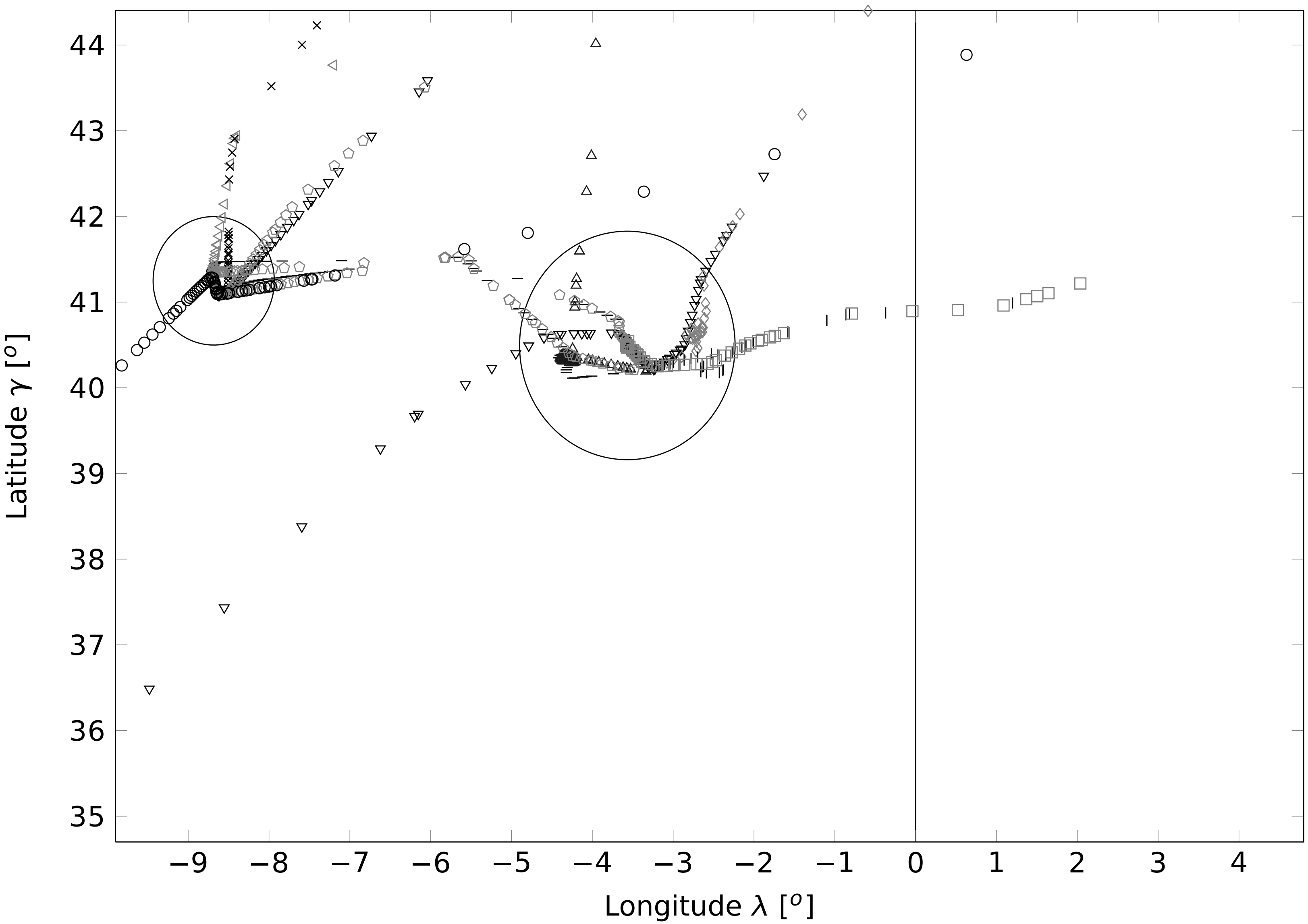

Figure 4.2: All aircraft routes at all times in a single snapshot. It resembles a long exposure photograph.

The work-around was possible due to a website of enthusiastic people who gathered information emitted from the aircraft transponder and that can be received and translated by a radio device. That website is *www.localizatodo.com* (LT).

A menu on this website allows the visitor to export a paraphernalia of data regarding one or several "flights" to a *.kml* file. This file is originally intended to be read by Google Earth™ and Google Maps™ but any text editor can be used to see the content within.

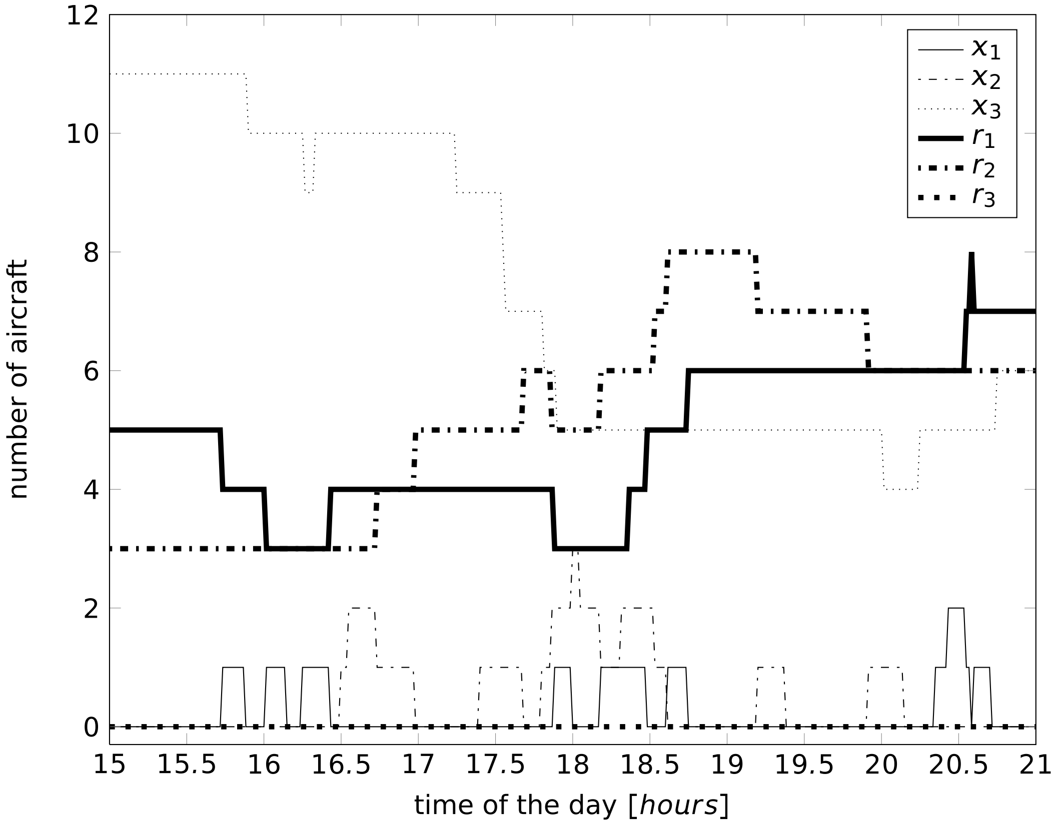

Figure 4.3: Number of aircraft on each TMA and each TMA's runway at a determined time of the day.

Making use of a bash* script the useful information could be distilled and went to a format that is now readable by Octave™.

If you remember picture 3.5 from section 3.3 there are two TMA one bigger than the other. That picture is inspired on the area chosen to perform the test on these models. In a sense, it is an abstraction of figure 4.1.

Even though there are many more airports in this area than those two it was not important to perform a painstaking job. Since the model could be tested with only three TMA the airports of Madrid and Oporto were chosen. Left aside were all flights passing through this area that were not arriving nor departing from either one of these two TMA; all flights departing from one of these two TMA and arriving to other TMA within this area such as *Faro* or *Santiago*.

It leaves us with a set of routes made by ten (10) different aircraft from 3 hours p.m. (15 hours) up until 9 p.m. (21 hours). A set of three (3) additional aircraft per TMA were added on the early simulations and since they are not disturbing they were

*Bourne-Again SHell is a command language interpreter. For info: http://www.gnu.org not removed.

Each item on figure 4.2 represents a time instant of a particular aircraft in a single position, which in his turn is represented by a particular mark different from every other mark on any other aircraft. If for instance we select all the circles it shows us the trajectory followed by the aircraft who's symbol is a circle. The Big circle is *Madrid — Barajas* and that TMA spreads throughout 80 nautical miles. *Oporto — Sá Carneiro* is the other smaller TMA with a radius of 45 nautical miles.

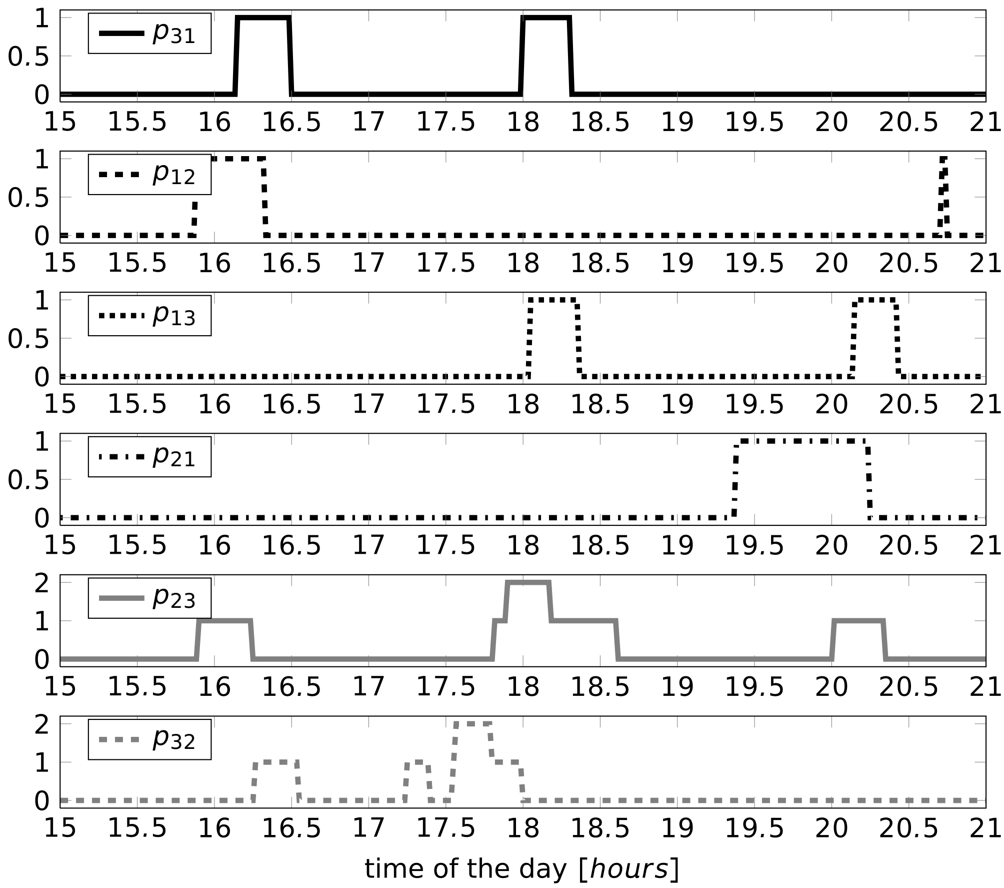

Figure 4.4: Number of aircraft departed from a TMA i currently on a path that will eventually lead to a TMA j at a certain period of the afternoon.

The negative signal on the values at the abscissa axis of figure 4.2 is the same as a *W* to denote that direction as *West*. The vertical line marks the zero of longitude (λ).

You might have already noticed in figure 4.1 that the two main TMA (*Oporto* and *Barajas*) are all-right but TMA 3 is somehow special. Instead of being a circular TMA it is a square and instead of bounding the area that lies within it delimits the area outside that square. It means that the rest of the world is TMA 3 and that it doesn't need a runway. It is as if the aircraft that enter TMA 3 will keep on flying *ad aeternum*. It might as well happen like that or any other way because it doesn't really matter to us and we don't need to know what happened to them. They become stock to be used whenever we might need it.

Taking advantage of equation 3.12 and of the data formerly acquired from LT we get, amongst others, the graphic of figure 4.3. Figure 4.3 allows us to realize the relative scale between each element. The way they are represented in figure 4.5

scale might not be perceptible.

Figure 4.4 holds the information of all paths $p_{ij}(k)$. An aircraft is either on a path $p_{ij}(k)$ (figure 4.4); inside a TMA $x_i(k)$ (figure 4.3 and figure 4.5); or already at the runway $r_i(k)$ (figure 4.3). In all figures either on section 4.1 and on section 4.2 the *yy* axis represents the *number of aircraft* at that instant.

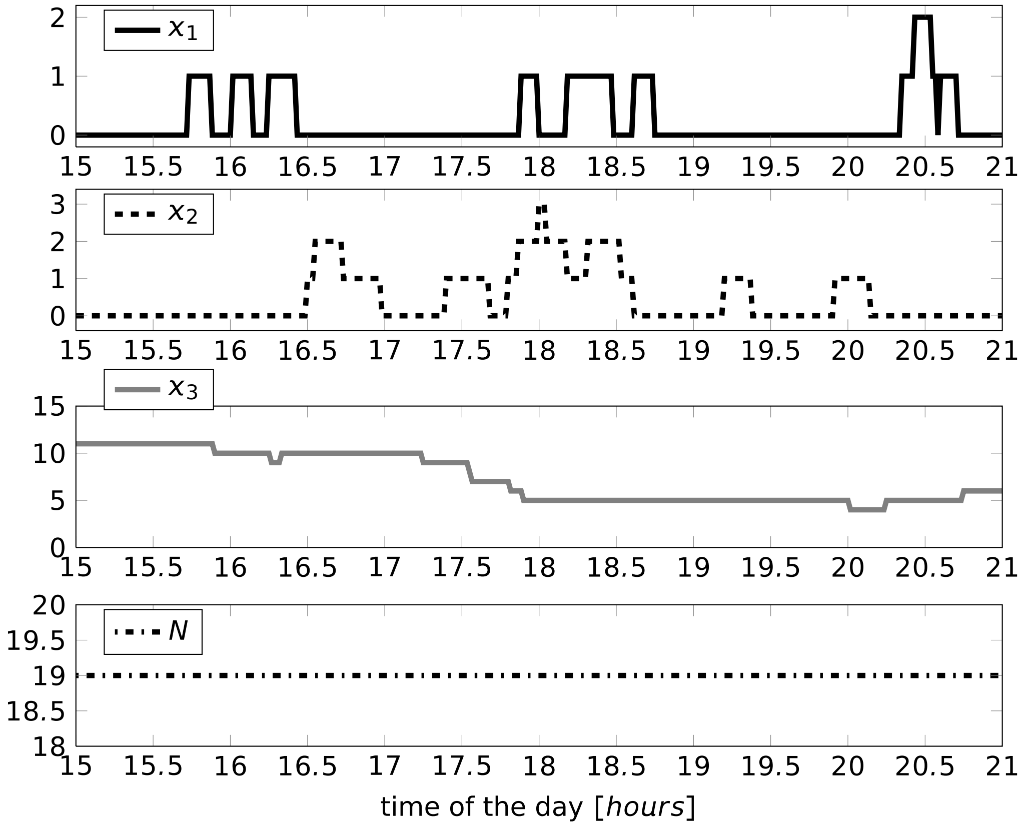

Figure 4.5: Individually the number of aircraft on each TMA and each TMA's runway at a determined time of the day.

The last subplot of figure 4.5 is explained through equation (4.1). As announced in section 3.2 this system has mass conservation in its conception. As we can see on figure 4.5 $N(k)$ remains the same whatever is the location of an aircraft: TMA; runway or path. This pattern will not be observed for the third model at figure 4.6 as we will be able to see on the next section - section 4.2.

$$N(k) = \sum_{i=1}^{n} x_i(k) + \sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}(k) + \sum_{i=1}^{n} r_i(k) \tag{4.1}$$

What we have proved here is that this model can identify the places where the aircraft are and when they are there based on the information of there departures. The “Control Theory” approach to this problem opens many doors and enables to see it from this new perspective where we have a *state* and a *control* which are suited for manipulation.

4.2 Sigmoid Model

The sigmoid model is taking the paths $p_{ij}(k)$ and adding them progressively with the arriving TMA $x_j(k)$. There is a need to remove abrupt changes inherent to the model in a way that an aircraft would gradually become present on the arriving TMA from the moment he lifted off the departing TMA, instead of just showing up on the arriving TMA and thus giving no anticipation opportunity to the air traffic controllers.

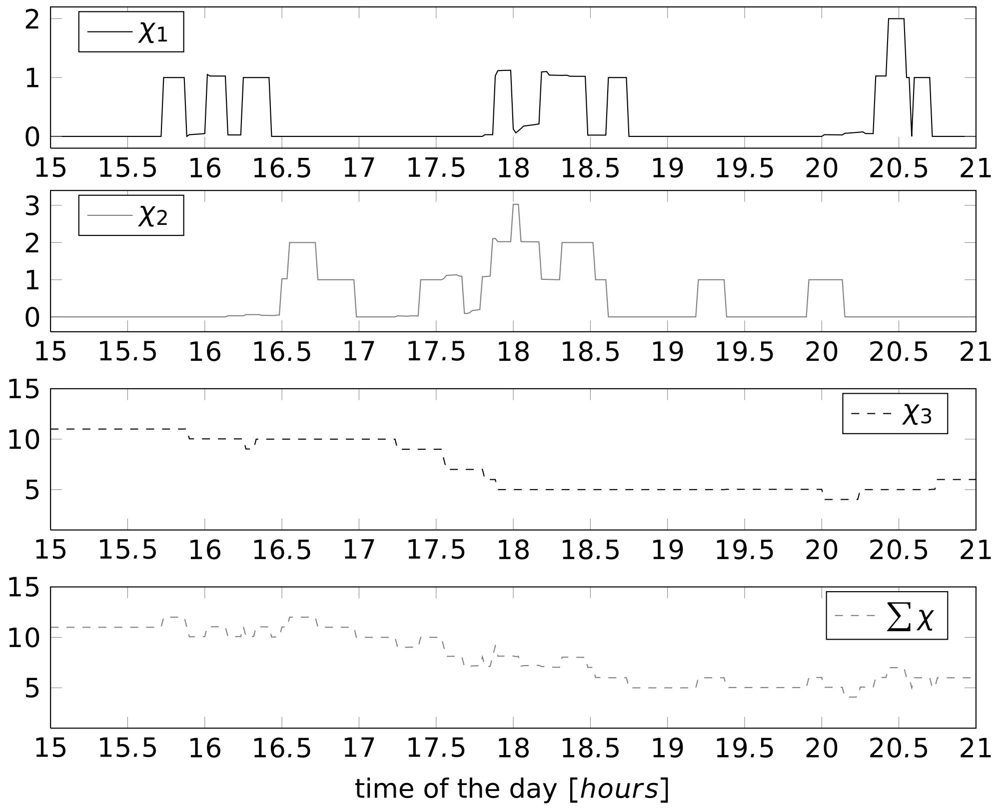

Figure 4.6: Number of aircraft on each TMA at a determined time of the day regarding the incoming aircraft as well.

A feature inherent to such a model if that the number of aircraft will no longer be constant. Think about it this way — we are considering that a fraction of one aircraft is arriving to a TMA. It will start as, let us say, about 0.1; then it will increase to 0.4; . . . ; 0.8; until it reaches 1. It is only when it is inside TMA $x_j(k)$ we consider it a full aircraft.

The way to correct this would be to pick up the remaining of $p_{ij}(k)$ and add it to our fraction of an aircraft resulting in one full aircraft. The fraction of the aircraft comes from $p_{ij}(k)$ so there is where the remaining of it should be gathered from.

Since this kind of job is pointless to our purpose we will leave it as it is. Equation (4.2) allows us to see this disparity in the last plot of figure 4.6.

$$\sum\chi(x) = \sum_{i=1}^{n}\chi_i(k) \tag{4.2}$$

4.2.1 One Aircraft Only Sigmoid

It could be interesting to see one aircraft by itself in order to comprehend the effects of this model individually. The way that the *Octave* simulation program was written didn't make the task easy. Another solution was plotted instead of the obvious first choice.

In order to see individually what could happen if there was just one aircraft to study it was performed a simulation with nine (9) aircraft. Each one of these aircraft have the same set of data and are identical to each other, i.e., they are copies of the first one.

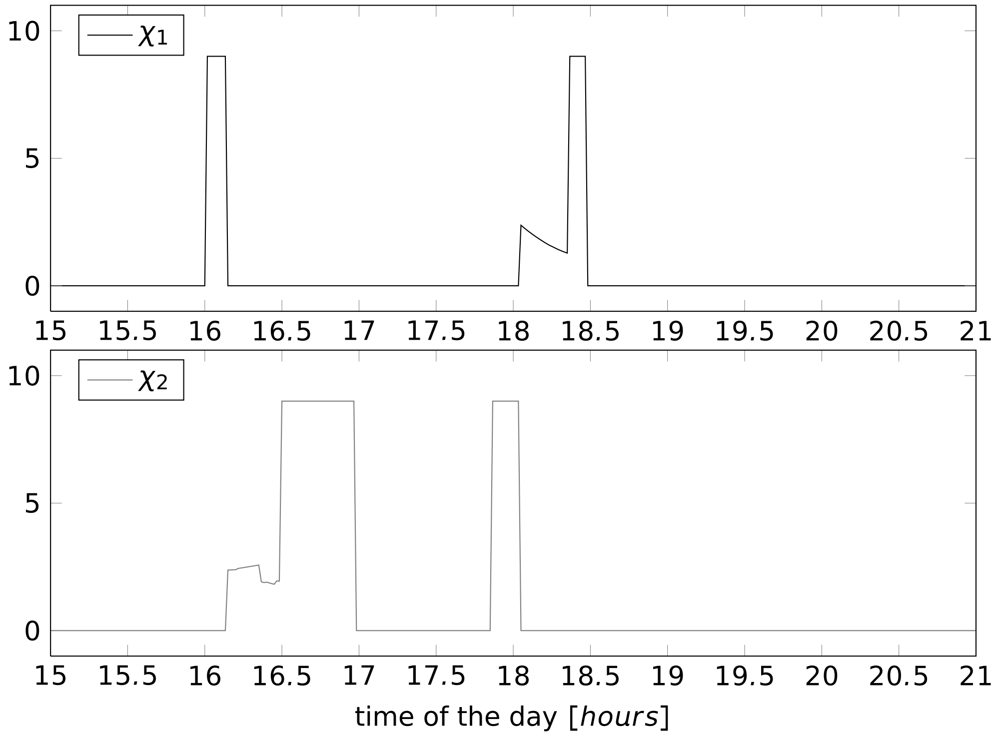

Figure 4.7: Nine identical aircraft travelling between two TMA. It regards the incoming aircraft as well.

As expected the graphics at figure 4.9 portrait an aircraft departing from TMA x_1 a while after 04:00 p.m. (16:00) and arriving to TMA x_2 around 04:30 p.m. (16:30). The voyage to return took more or less the same time and the aircraft arrived a few minutes before 06:30 p.m. (18:30).

Figure 4.8 corroborates and confirms the information provided by figure 4.9. The arrivals to the runways are not portrayed on either one of these graphics.

The "cherry on top of the cake" would happen if both the steps of the graphic on picture 4.8 were perfect S shaped curves starting on zero until they reached the value of nine. Since it doesn't happen there is certainly something wrong with the witchcraft that guesses $k_z(k)$ — something is wrong with subsection 3.4.2.

The same sympthoms of figure 4.7 can be observed in figure 4.6. Anything different from this resemblance would point either towards some kind of code issue or instead to some better news. Neither one happened so it means that this model is not perfect. All the shortcomings and achievements of this model are discussed in the conclusion chapter at 5.

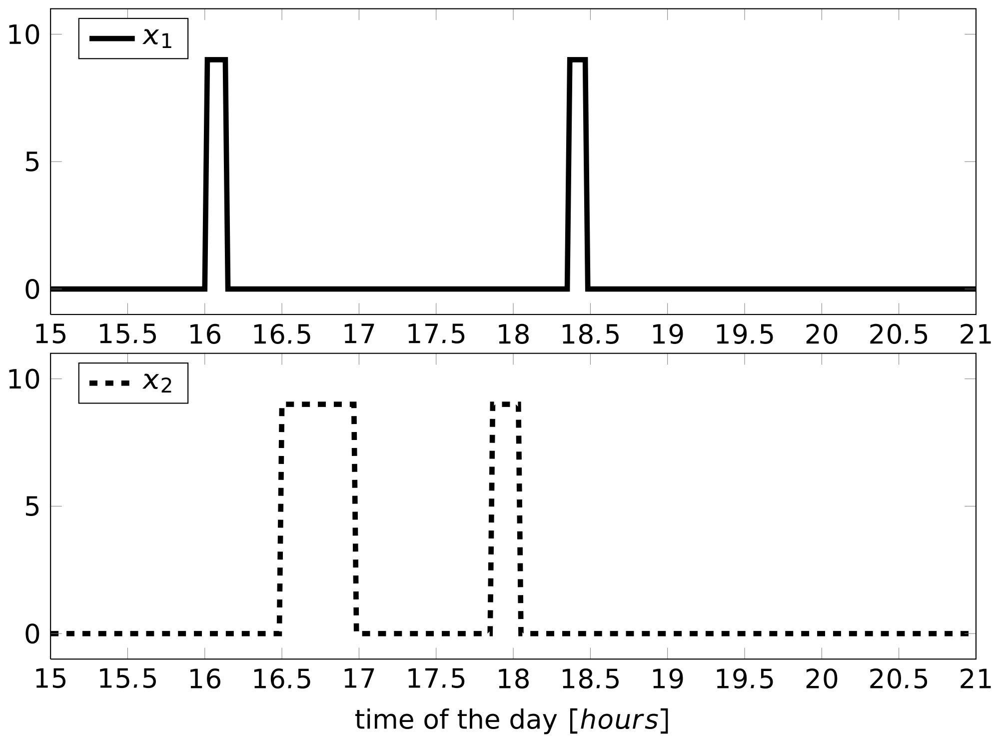

Figure 4.8: Nine identical aircraft travelling between two TMA. It denotes when they are in which TMA.

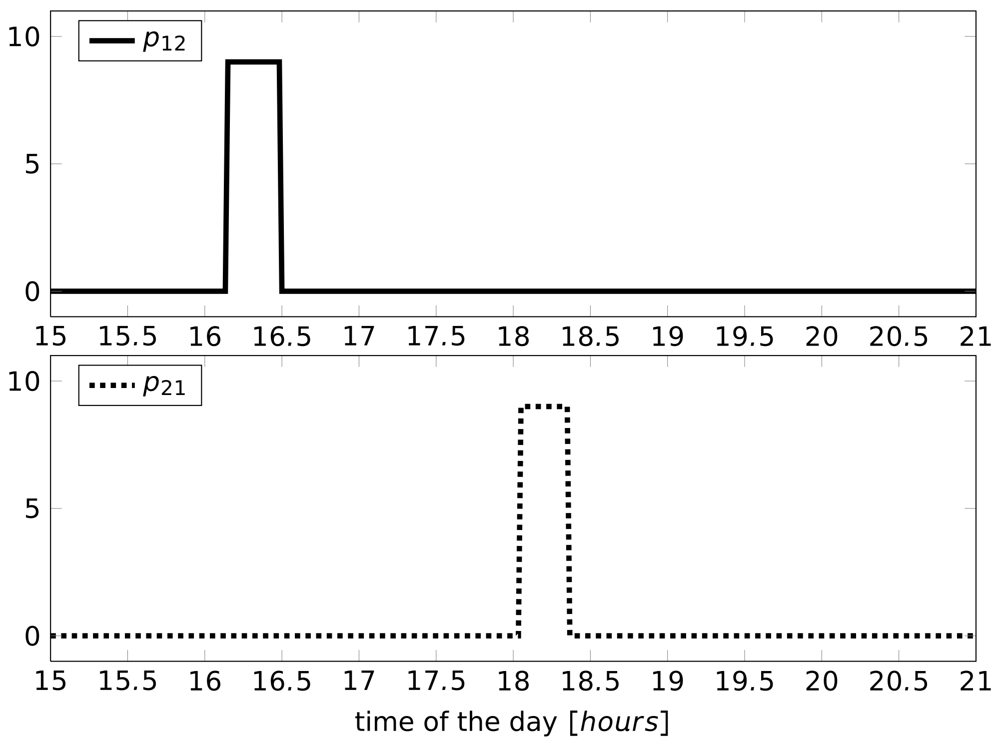

Figure 4.9: Paths of nine identical aircraft amongst two TMA.

Chapter 5

Conclusions and Future Work

This work started with a discrete model for Air Traffic Dynamics in which there was not a sense of in between and all TMA were contiguous. It was a mean to an end and it provided the mathematical basis to work on.

A second model was idealized and provided enough depth to accommodate paths between different airports and even a separation between the TMA and the runway. This increased model breaks up with the last one in a sense that it can already be used in a real scenario and it is lo longer just a theoretical algebraic manipulation.

At this stage the flexibility to incorporate new features in a simple way started to take shape and it is definitely a plus of this work. Changes for the model to match the requirements of a certainly more complex real life application should be easily implemented in order to create an improved similar model.

To identify the control and the state of each model was also a great achievement for it opens doors for some Control Theory approaches. Optimisation and discrete-time to continuous-time transformation will now become easier to perform.

Sigmoid model was an attempt to achieve the ultimate goal which is to implement a sense of prediction to this model. It could be seen as a layer working above the second model in order to transform the data provided into a more humanized perspective.

By picking up the numbers from the in-traffic aircraft and adding them progressively into the number of aircraft on the destination TMA we could provide a more intuitive and less abrupt advisory for the air traffic controllers.

Arc numbering of the Dynamic Network was not an easy one because all those interlinked variables in the end had to have names that made sense. And since it was to be implemented on a computer these names had to be computer friendly as well.

The third model (sections 3.4 and 4.2) has a few shortcomings that must be highlighted on this conclusion. The beautiful S-shape is not present as it was intended to be. Some few characteristics of the model might explain this scenario. First of all there is not enough data to perform a flight dynamics analysis. And yet it is a far-fetched hypothesis to even consider having a system where you can find in one place the information needed for a flight dynamics analysis of several aircraft at the same time and in real-time. Also the script program itself could aid on providing an explanation for what happened.

An historical set of data could contribute to invent several covariance matrices. Those matrices applied in a Kalman Filter could improve the accuracy of the forecast on the value of $k_z(k)$. A huge disregard on the early stages of this project was the disposal of data such as the instantaneous velocity of each aircraft in every point. Even the altitude could have come in handy.

In a future work an estimator, such as a Kalman Filter, should be added to this model. It may come a time when we don't know the data from all airports. And in fact we don't need to know all data from all airports all over the world. It is not even logical. A Kalman Filter could help in providing an estimation on the incoming traffic from airports where there is somehow a restriction to release data.

Another Kalman Filter would have to be implemented on data acquisition in order to uniform the data into sets distant from each other of a specific time step. This is prediction that should be made on the basic inputs even before the large models should be remotely considered. As you can imagine it has not been done. Octave script was designed to copy the same data to the following time step up until a new set of data would emerge from the transponder. The frequency on which this information arrived is different from aircraft to aircraft and it doesn't start at the same time making a mess that had to be dealt with somehow.

The early choice that allowed the first two models to work later on the sigmoid model became a headache. How can we predict a velocity when the last set of data says the object didn't move? The idea of getting the first set of data to solve this problem ended up on a rude approximation of our desired $k_z(k)$ and subsequent shortcoming on the S-shaped graphic.

Not all is bad because it has proven that once there is a proper $k_z(k)$ prediction model then the *sigmoid model* can be used. It will be able to show air traffic controllers a realistic scenario whilst being intuitive enough for them to understand if they are or will become in trouble in the near future.

This work could be used as a foundation for the implementation of an advisory to the Air Traffic Controllers of the occupation status. Not only that but it could be a module on a fully automatic Airspace Controller or even other applications yet to be considered. It could even be applied on the traffic of buses (coaches) and it can be extrapolated to many other fields.

In conclusion we see that there is still much work ahead on perfecting these models. A few Kalman Filters could be of use. Also the model will certainly increase in complexity once we hear the requests from those to whom this kind of system would help on a daily basis. Several distinct people with different studies are needed to implement such a system because this is certainly not a one man job. A good team could make something happen out of this.

Bibliography

[1] M. of Transport, *Transport Canada Aeronautical Information Manual*. tp 14371e ed., 2010.

[2] Wikipédia, "Phare du cap mondego — wikipédia, l'encyclopédie libre," 2011. [En ligne; Page disponible le 28-juin-2011], `http://fr.wikipedia.org/w/index.php?title=Phare_du_cap_Mondego&oldid=64082425`.

[3] J. Zabczyk, *Mathematical Control Theory - An Introduction*. Birkhäuser, 1995.

[4] J. Doyle *et al.*, *Feedback Control Theory*. Macmillan Publishing Co., 1990.

[5] J. Alexander, "Loxodromes: A Rhumb Way to Go," *Mathematics Magazine*, vol. 77, p. 349, Dec. 2004.

[6] G. Siouris, *Aerospace Avionics Systems: A Modern Synthesis*. Academic Press, 1993.

[7] A. Dolan and J. Aldous, *Networks and algorithms: an introductory approach*. Milton Keynes, UK: J. Wiley & Sons, 1993.

[8] L. Fausett, *Fundamentals of neural networks : architectures, algorithms, and applications*. Prentice-Hall Englewood Cliffs, NJ, 1994.

[9] A. Padilla, *Sistemas Digitais*. Mc Graw-Hill, 2000.

Appendix A

Deduction of equations (3.12)

To perform this deduction figure A.1 will be used as figure 3.3 was used in section 3.2.

Also table 3.2 together with equations 3.3 to 3.10 are like a receipt for the following equations.

$$\begin{cases} p_{ij}(k) = \alpha_{ii1ij1}(k) + d_{ij1jj1}(k) + d_{ij1ij1}(k) \\ x_i(k) = \left(\alpha_{ii2ii1}(k) + \sum_{\substack{j=1 \\ i\neq j}}^{n} \alpha_{ji1ii1}(k)\right) + \left(d_{ii1ii2}(k) + \sum_{\substack{j=1 \\ i\neq j}}^{n} d_{iiij}(k) + d_{iiii}(k)\right) + d_{ii1ii1}(k) \\ r_i(k) = \alpha_{ii1ii2}(k) + d_{ii2ii1}(k) + d_{ii2ii2}(k) \end{cases} \tag{A.1}$$

Our *carry* will become:

$$\begin{cases} \alpha_{ij1ij1}(k+1) = \alpha_{ii1ij1}(k) + dij1ij1(k) \\ \alpha_{ii1ii1}(k+1) = d_{ii1ii1}(k) + \left(\alpha_{ii2ii1}(k) + \sum_{\substack{j=1 \\ i\neq j}}^{n} \alpha_{ji1ii1}(k)\right) \\ \alpha_{ii2ii2}(k+1) = d_{ii2ii2}(k) + \alpha_{ii1ii2}(k) \end{cases} \tag{A.2}$$

Also from observing the figure we get:

$$\begin{cases} p_{ij}(k) = \alpha_{ij1ij1}(k) + \alpha_{ii1ij1}(k) \\ x_i(k) = \alpha_{ii1ii1}(k) + \left(\alpha_{ii2ii2}(k) + \sum_{\substack{j=1 \\ i\neq j}}^{n} \alpha_{ji1ii1}(k)\right) \\ r_i(k) = \alpha_{ii1ii2}(k) + \alpha_{ii2ii2}(k) \end{cases} \tag{A.3}$$

The departures from figure 3.7 can be translated to arrivals.

$$\begin{cases} \forall\,\{i \neq j\} : d_{ii1ij1}(k) = \alpha_{ii1ij1}(k+1) \\ \forall\,\{i \neq j\} : d_{ij1jj1}(k) = \alpha_{ij1jj1}(k+1) \\ \forall\,\{i \neq j\} : d_{ii2ii1}(k) = \alpha_{ii2ii1}(k+1) \\ \forall\,\{i \neq j\} : d_{ii1ii2}(k) = \alpha_{ii1ii2}(k+1) \end{cases} \tag{A.4}$$

From now on it is only mathematical manipulation. Using equation (A.1) together

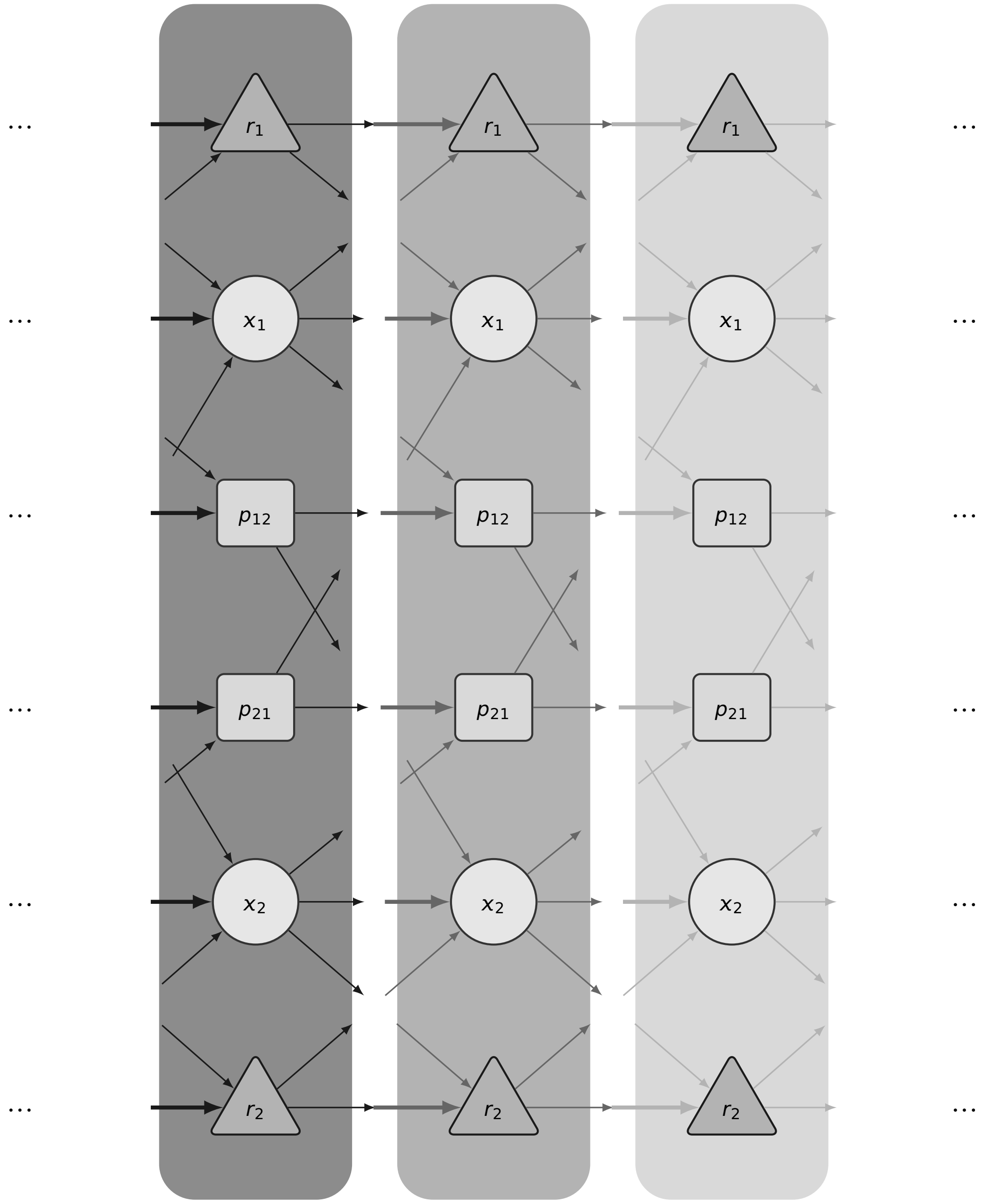

Figure A.1: Representative graph about the interaction between two airports considering the paths (p_{ij}) between them and also the control zone (r_i).

with (A.3):

$$\Rightarrow \begin{cases} \alpha_{ij1ij1}(k) = d_{ij1ij1}(k) + d_{ij1jj1}(k) \\ \alpha_{ii1ii1}(k) = \left(d_{ii1ii2}(k) + \sum_{\substack{j=1 \\ i \neq j}}^{n} d_{ii1ij1}(k) \right) + d_{ii1ii1}(k) \\ \alpha_{ii2ii2}(k) = d_{ii2ii1}(k) + d_{ii2ii2}(k) \end{cases} \tag{A.5}$$

Using equations (A.3) combined with (A.4) and (A.2):

$$\begin{cases} p_{ij}(k) = d_{ii1ij1}(k-2) + d_{ij1ij1}(k-1) + d_{ii1ij1}(k-1) \\ x_i(k) = d_{ii1ii1}(k-1) + \left(d_{ii2ii1}(k-1) + \sum_{\substack{j=1 \\ i\neq j}}^{n} d_{ji1ii1}(k-1)\right) + \left(d_{ii2ii1}(k-2) + \sum_{\substack{j=1 \\ i\neq j}}^{n} d_{ji1ii1}(k-2)\right) \\ r_i(k) = d_{ii1ii2}(k-1) + d_{ii2ii2}(k-1) + d_{ii1ii2}(k-2) \end{cases} \tag{A.6}$$

Picking equations (A.3) and (A.5):

$$\begin{cases} d_{ij1ij1}(k-1) = p_{ij}(k-1) - d_{ij1jj1}(k-1) - d_{ii1ij1}(k-2) \\ d_{ii1ii1}(k-1) = d_{ii2ii1}(k-2) + d_{ii1ii2}(k-1) + \sum_{\substack{j=1 \\ i\neq j}}^{n} \left[d_{ji1ii1}(k-2) + d_{ii1ij1}(k-1) \right] - x_i(k-1) \\ d_{ii2ii2}(k-1) = r_i(k-1) - d_{ii1ii2}(k-2) - d_{ii2ii1}(k-1) \end{cases} \tag{A.7}$$

And last but not least:

$$\boxed{\begin{cases} p_{ij}(k) = p_{ij}(k-1) + \left(d_{ii1ij1}(k-1) - d_{ij1jj1}(k-1)\right) \\ r_i(k) = r_i(k-1) + \left(d_{ii1ii2}(k-1) - d_{ii2ii1}(k-1)\right) \\ x_i(k) = x_i(k-1) + d_{ii2ii1}(k-1) - d_{ii1ii2}(k-1) + \\ \qquad + \sum_{\substack{j=1 \\ i\neq j}}^{n} \left[d_{ji1ii1}(k-1) - d_{ii1ij1}(k-1) \right] \end{cases}} \tag{A.8}$$

Appendix B

Relationship between time and time constant

Even though it is a very simple concept it is still worth to explain due to the large amount of times that k, t or t_k come up in this dissertation.

The time t belongs to a one dimensional rational space but for our convenience the variable k always takes the value of a natural number.

$$t \in \mathbb{R} \tag{B.1a}$$

$$k \in \mathbb{N} \tag{B.1b}$$

The value 0 (zero) was intentionally left out from k for it adds no advantages and would bring the disadvantage that, on a computer program, the position 0 is non existent on a vector or on a matrix.

So now we understand that the initial t is t_i but also t_1 since $k_i = 1$.

There should be a plain relationship between every t and every k in such a way that for every k corresponds a t_k.

That relationship is linear (which follow equation (B.2)) so knowing the initial point and the final point (table B.1) we can get the line segment equation for their equivalence.

$$y = m \times x + b \tag{B.2}$$

Table B.1: Initial, step and final points for t_k and k

	initial	step	final
k	1	1	k_f
t	t_i	d_t	t_f

From the nature of k we can take for granted that k_f will always be:

$$k_f = \frac{t_f - t_i}{d_t} + 1 \tag{B.3}$$

Equation B.3 happens because k starts at 1 (one) and has the same number of elements as t.

m and b from equation (B.2) can be calculated from poins one and two with:

$$\begin{cases} m = \dfrac{y_1 - y_2}{x_1 - x_2} \\ b = \dfrac{x_1 \times y_2 - x_2 \times y_1}{x_1 - x_2} \end{cases} \tag{B.4}$$

Substituting x by k and y by t after some mathematical manipulation it becomes:

$$t = f(k) = d_t \times k + t_i - d_t \tag{B.5a}$$

$$k = f(t) = \frac{t - t_i}{d_t} + 1 \tag{B.5b}$$

As long as we respect (B.1) equations (B.5) are valid.