Modelling and Analysis of Dynamic Models for Air Traffic Flow

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Abstract

This paper describes the achievement of a set of equations in order to represent a discrete model for air traffic dynamics in terminal areas. Such a mathematical model could be used to investigate the occupancy rate of a TMA* at a single instant and has the potential to be used for occupancy rate prediction.

It could also be used by an Air Traffic Controller in order to predict a future over-occupation en thus being able to avoid it. This is why there is an interest to know at each instant an estimate of how many aircraft are in a specific *TMA*

The idea behind it was to make a *Control Theory* approach so that the same classical tools used in other topics such as flight dynamics could be applied to this subject. In order for that to happen there were needed some Network notions along with navigation formulae.

Data retrieved from the website *www.localizatodo.com* together with *Octave* Software allowed the validation of the model. The equations performed as they should, making them applicable to a real scenario, since the utilized data are in fact coming themselves from a real scenario and they are not invented.

Keywords: Control Theory, TMA, Air Traffic Controller, Prediction, Warning, Transponder

1 Introduction

The challenge suggested was to find a set of equations in order to represent a discrete model for air traffic dynamics in terminal areas. Such a mathematical model could be used to investigate the occupation of a TMA at a single instant and has the potential to be used on the prediction of a future occupation.

A part of the answer to this motto is made by this dissertation by making a *Control Theory* approach so that the same classical tools used in other topics such as flight dynamics could be applied to this subject. In order for that to happen there were needed some Network notions along with navigation formulae.

When an aircraft leaves one place we know before it happens that he will follow a path and later enter a certain TMA, unless something goes wrong.Aircraft within a radius of several nautical miles, depending on each TMA, does belong to that TMA. A transition between areas is considered a control input. The states are the number of aircraft at a certain moment within a certain TMA.

If we know the initial amount of aircraft on the zero instant at a particular site then we can calculate the following instant state.

A peculiarity of this system is that it has mass conservation in a sense that aircraft don't just disappear - instead they move around and the total number of aircraft on the system remains the same. This is a very important assumption on which lays an important pillar of this work.

All data required for testing purposes was retrieved from "Localiza Todo" which is a website with the hyperlink *www.localizatodo.com*. Together with several scripts allowed the validation of the model. On this website the transponder signals captured by several antennas are transformed intoinformation from which were retrieved the useful one.

The third model is slightly different from the first two. There is a need to remove abrupt changes inherent to the model in a way that an aircraft would gradually become present on the arriving TMA from the moment he lifted off the departing TMA, instead of just showing up on the arriving TMA and thus giving no anticipation opportunity to the air traffic controllers.

1.1 Description of TMA

Controlled airspace is the airspace within which air traffic control service is provided andsome or all aircraft may be subject to air traffic control. Types of controlled airspace are the *High Level*

*TMA stands for Terminal Manoeuvring Area.

Airspace and the *Low Level Airspace*. Within the second one we can find amongst others the *TMA** or *TCA**.

TMA are established at high volume traffic airports to provide an IFR† control service to arriving, departing and enroute aircraft. [1](pp. 188,192-193)

These definitions may differ from country to country but usually it follows a similar pattern. The general idea is what we need for the specific line of work on this thesis, being each TMA a defined area separated from other TMA for a certain amount of nautical miles.

2 Air Traffic Dynamics Modelling in TMAs

Before modelling a non contiguous system we should study the simpler model. One where there is not a transition between the instant k to the instant $k+1$. One in the discrete time.

Let us consider for the moment that we can be here or there but not in between. This will allow a logic inference on this problem. All variables belong to the natural numbers ($\in \mathbb{N}_0^+$) and $i \neq j$:

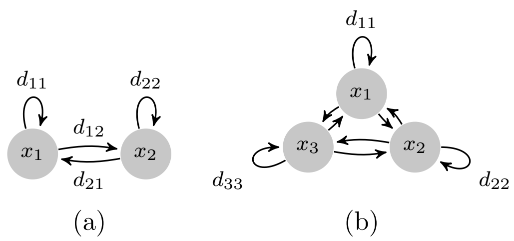

Figure 1: Graphs illustrating all possible interactions at a single discrete instant amongst airports in terms of departures and its growing of complexity as the amount of TMA on the system increases.

This system is different from most systems because of one detail: it has *mass conservation*.For the general purpose $\mathbf{N}(k)$ and $n(k)$ (number of TMA) given any k (time constant) are considered constant:

$$\mathbf{N}(k) = \sum_{i=1}^{n} x_i(k) = \sum_{i=1}^{n} x_i(k+1) \quad (1)$$

$$n(k) = \sum_{i=1}^{n} i = \sum_{i=1}^{n} j \quad (2)$$

What is our *state*? What about our *control*, where is it? The second question seems to have a direct answer that all departures $d_{ij}(k), \forall\{i \neq j\}$ should be the *control*.

In a system with two TMA (figure 1 (a)) the *control* would be d_{12} and d_{21}. Looking at figure 1 (b)it is inferred that when we have n TMA then we have n *states* and $n \times n - n$ *controls*.

From the definition in the section 1.1 the number of aircraft in a TMA i include the ones landing $\alpha_{ij}(k), \forall\{i \neq j\}$, others already landed $d_{ii}(k)$ and those departing $d_{ij}(k), \forall\{i \neq j\}$. Roughly speaking every aircraft within a radius of several nautical miles, depending on each TMA, does belong to that TMA i.

$$x_i(k) = \sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k) + \sum_{\substack{j=1 \\ i \neq j}}^{n} d_{ij}(k) + d_{ii}(k) \quad (3)$$

The *state* is $x_i(k)$ — the total number of aircraft in TMA i at instant t_k — and it is known empirically by the reasons behind equation (3).

Our *carry*[2] or *evader* represents the amount of aircraft that were static in a TMA together with those who have already landed:

$$\alpha_{ii}(k) = d_{ii}(k-1) + \sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k-1) \quad (4)$$

The number of aircraft at any given instant t_k can also be described as the *carry* together with all the arriving aircraft.

$$x_i(k) = \sum_{\substack{j=1 \\ i \neq j}}^{n} \alpha_{ji}(k) + \alpha_{ii}(k) \quad (5)$$

If nothing bad happens an aircraft that lifts off will arrive at another TMA after a while. It seems logic that all departures will eventually become arrivals after a time leap.

$$\forall\{i \neq j\} : d_{ij}(k) = \alpha_{ij}(k+1) \quad (6)$$

We have now the tools to begin to aim for a set of equations on a familiar form just like it is usual on so many systems where the *state* ahead (t_{k+1}) is a function of the *state* and *control* of the previous time (t_k).

After some algebraic manipulation we get equation (7).

$$\boxed{\begin{aligned} x_i(k) &= x_i(k-1) \; + \\ &+ \textstyle\sum_{\substack{j=1 \\ i \neq j}}^{n} \left[d_{ji}(k-1) - d_{ij}(k-1) \right] \end{aligned}} \quad (7)$$

*Terminal Manoeuvring Area or Terminal Control Area

†Instrument Flight Rules

3 Air Traffic Dynamics Modelling in TMAs and Paths

This section is an evolution from section 2 thatwill guide this one as if it were a cake recipe.

Paths amongst TMA are added to the simpler model so that it incorporates an important reality detail: when *aircraft one* departs from TMA x_i and *aircraft two* departs a while after it may happen that *aircraft two* arrives before to its destiny than *aircraft one*. And no matter what x_j they are heading these equations have to remain valid.

Also we have to consider the landing and lift-off in order to distinguish the airborne aircraft at the TMA from those standing on the ground. Whenever the location of an aircraft indicates that it resides at an inner position within a 7 NM (seven nautical miles) range from the airport it will be considered as belonging to the runway.

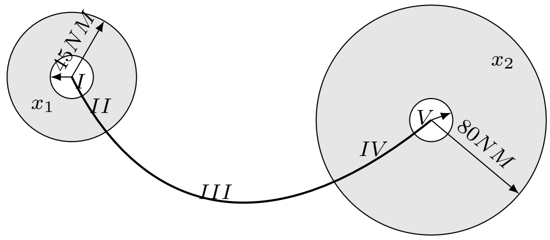

Figure 2: Trajectory of an aircraft from TMA x_1 to TMA x_2 and the generation of the control of the system which are the departures. Table 1 outputs the departures.

Typically between this inner ring and an outer ring of 45 NM (forty five nautical miles) the aircraft will be at the TMA. Sometimes this radius can be stretched due to fitting waiting aircraft within the TMA. All of this and more can be seen in figure 2.

In order to prevent that an aircraft flying above and out of the TMA to be considered, within the data of each aircraft there are two variables i and j meaning "coming from" and "going to", respectively.

The goal which comes from defining i and j is to eventually find these four functions: $\exists o@r_i$, $\exists o@x_i$, $\exists o@r_j$ and $\exists o@x_j$. They evaluate the existence of an aircraft at a particular place just by knowing where does it come from, where does it go and its position in a two dimensional space composed by longitude (L or λ) and latitude (G or γ).

$$\exists o@x_i = \begin{cases} 1 \quad , \quad R^2 \geq (G_o - G_i)^2 + (L_o - L_i)^2 \\ 0 \quad , \quad R^2 < (G_o - G_i)^2 + (L_o - L_i)^2 \end{cases} \tag{8a}$$

$$\exists o@r_i = \begin{cases} 1 \quad , \quad {R_r}^2 \geq (G_o - G_i)^2 + (L_o - L_i)^2 \\ 0 \quad , \quad {R_r}^2 < (G_o - G_i)^2 + (L_o - L_i)^2 \end{cases} \tag{8b}$$

One (1) means it is true and it does exist. Zero (0) means it doesn't. Ahead you should read binaries this way. The meaning of the variables present on equations 8a and 8b are as follows:

(L_i, G_i) and (L_o, G_o): Coordinates of the center of the TMA i and of the position of a specific aircraft o.

R and R_r: Radius of the TMA i and of the runway at TMA i.

For the TMA x_3 that you can see ahead on figure 4 the equation (8) is not valid. Instead the existence of an aircraft on TMA x_3 means only that it is outside the dashed line.

By examination of figure 2 we can get table 1. The transition of *existence* in a previous state to the *non existence* on the next state will give rise to some kind of departure. Upper-case Roman numerals in the figure correspond to those in the table.

Graph in figure 3 represents that interaction. The triangles are the runways which are represented by the inner circle at figure 2 with 7 NM of radius. Circles are the TMA and squares account for the paths. Additionally every state should be seen as if there was a loop making it possible for an aircraft to remain at the same state indefinitely.

In table 2 we can see the relationship between the first model and this model. Once we identify this parallelism a simple substitution on equation (7) and we get equation (9).

$$\begin{cases} p_{ij}(k) = p_{ij}(k-1) + \Big(d_{ii1ij1}(k-1) - d_{ij1jj1}(k-1)\Big) \\ r_i(k) = r_i(k-1) + \Big(d_{ii1ii2}(k-1) - d_{ii2ii1}(k-1)\Big) \\ x_i(k) = x_i(k-1) + d_{ii2ii1}(k-1) - d_{ii1ii2}(k-1) + \\ \qquad + \sum_{\substack{j=1 \\ i \neq j}}^{n} \Big[d_{ji1ii1}(k-1) - d_{ii1ij1}(k-1)\Big] \end{cases} \tag{9}$$

3.1 Simulation

In order to validate a hypothesis for a life improvement idea one should test it on a real scenario where the surrounding conditions are the same as if they were the ones of the final implementation environment.

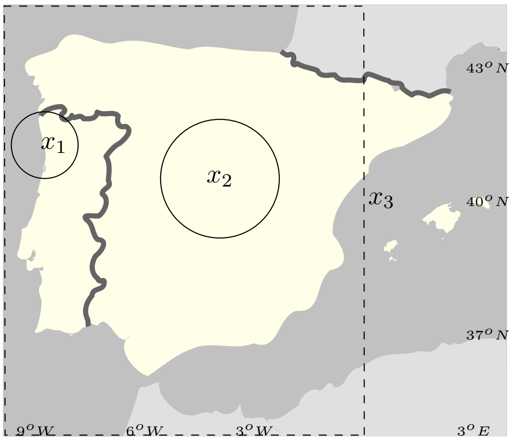

Figure 4: Iberian Peninsula showing TMA x_1, x_2 and a fictional TMA x_3, outside the big square. Source:Adapted from [3]

That ideal *modus operandi* can't always be achieved. When this happens there must be a way to replicate as accurately as possible the data that would be captured during a real-life test of our model.

It has not even been tried out and yet most likely the attempt to use the actual data from any Control Tower recorders would lead to a bureaucracy inferno that could last far beyond the schedule for this present work to be completed.

The work-around was possible due to a website of enthusiastic people who gathered information emitted from the aircraft transponder and that can be received and translated by a radio device. That website is *www.localizatodo.com* (LT).

A menu on this website allows the visitor to export a paraphernalia of data regarding one or several "flights" to a *.kml* file. This file is originally intended to be read by Google Earth™ and Google Maps™ but any text editor can be used to see the content within.

Making use of a bash† script the useful information could be distilled and went to a format that is now readable by Octave™.

If you remember picture 2 from section 3 there are two TMA one bigger than the other. That picture is inspired on the area chosen to perform the test on these models. In a sense, it is an abstraction of figure 4.

Even though there are many more airports in this area than those two it was not important to perform a painstaking job. Since the model could be tested with only three TMA the airports of Madrid and Oporto were chosen. Left aside were all flights passing through this area that were not arriving nor departing from either one of these two TMA; all flights departing from one of these two TMA and arriving to other TMA within this area such as *Faro* or *Santiago*.

It leaves us with a set of routes made by ten (10) different aircraft from 3 hours p.m. (15 hours) up until 9 p.m. (21 hours). A set of three (3) additional aircraft per TMA were added on the early simulations and since they are not disturbing they were not removed.

You might have already noticed in figure 4 that the two main TMA (*Oporto* and *Barajas*) are all-right but TMA 3 is somehow special. Instead of being a circular TMA it is a square and instead of bounding the area that lies within it delimits the area outside that square. It means that the rest of the world is TMA 3 and that it doesn't need a runway. It is as if the aircraft that enter TMA 3 will keep on flying *ad aeternum*. They become stock to be used whenever we might need it.

Taking advantage of equation 9 and of the data formerly acquired from LT we get, amongst others, the graphic of figure 5. Figure 5 allows us to realize the relative scale between each ele-

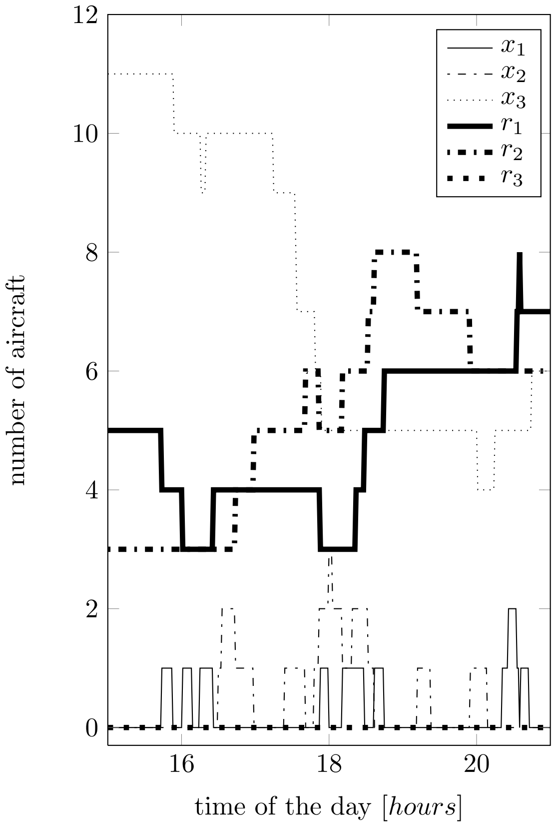

Figure 5: Number of aircraft on each TMA and each TMA's runway at a determined time of the day.

†Bourne-Again SHell is a command language interpreter. For info: http://www.gnu.org

ment. The way they are represented in figure 7 scale might not be perceptible.

Figure 6 holds the information of all paths $p_{ij}(k)$. An aircraft is either on a path $p_{ij}(k)$ (figure 6); inside a TMA $x_i(k)$ (figure 5 and figure 7); or already at the runway $r_i(k)$ (figure 5). In all figures either on section 3.1 and on section 4.3 the *yy* axis represents the *number of aircraft* at that instant.

The last subplot of figure 7 is explained through equation (10). As announced in section 2 this system has mass conservation in its conception. As we can see on figure 7 $N(k)$ remains the same whatever is the location of an aircraft: TMA; runway or path. This pattern will not be observed for the third model at figure 8 as we will be able to see on the next section - section 4.3.

$$N(k) = \sum_{i=1}^{n} x_i(k) + \sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}(k) + \sum_{i=1}^{n} r_i(k) \tag{10}$$

What we have proved here is that this model can identify the places where the aircraft are and when they are there based on the information of there departures. The "Control Theory" approach to this problem opens many doors and enables to see it from this new perspective where we have a *state* and a *control* which are suited for manipulation.

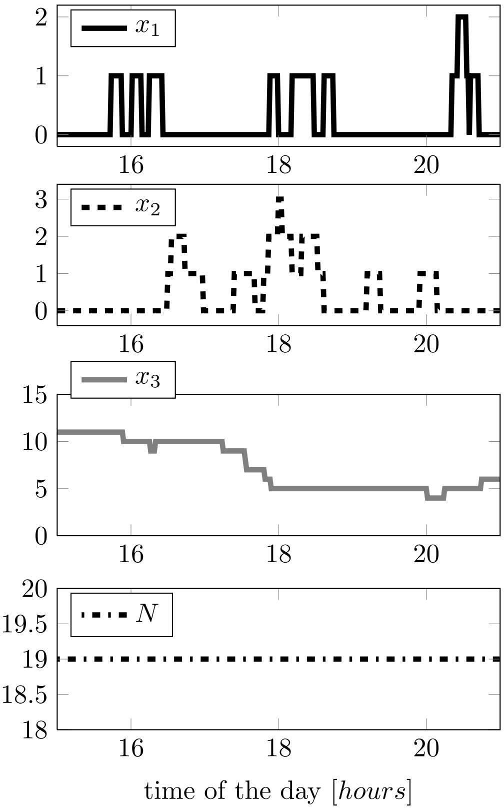

Figure 7: Individually the number of aircraft on each TMA and each TMA's runway at a determined time of the day.

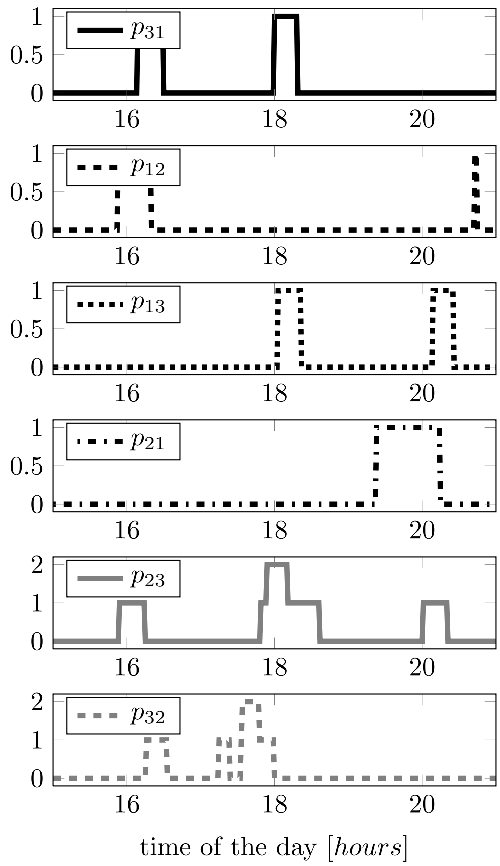

Figure 6: Number of aircraft departed from a TMA i currently on a path that will eventually lead to a TMA j at a certain period of the afternoon.

4 Sigmoid Model

The idea is to have a model where there are no sudden changes to the eyes of the air traffic controller. An aircraft would gradually appear on the arriving TMA instead of showing up abruptly.

4.1 The Sigmoid Function

Taking advantage on the properties of the sigmoid function [4] we can make a few changes in order to have a more well behaved formula that can suit our purposes, i.e. equation (11). A function that could go back and forth according to our desires.

$$S_{ijo}^{(2)} = \left[1 + e^{-\sigma \times \left(k - \frac{T_t}{2} - k_a\right)}\right]^{-1} \tag{11}$$

After some trial and error the equation 11 seems perfect for the job. In case there is some distrust on behalf of the reader here goes some proof. We start by defining an interval where the formula will work and with a total time of T_t:

$$\sigma = \frac{7}{T_t} \qquad , T_t = k_z - k_a \tag{12}$$

A steepness (σ) having nice characteristics is the one in equation (12). Those characteristics are such that for however different k_a and k_z the steepness remains the same.

In the beginning and in the end of this sigmoid we want our counter-domain to be zero and one, respectively.

On this article 1 means true and 0 means false but it can also mean 1 for already there and 0 for completely apart from somewhere.

4.2 Final Sigmoid Model

Now it starts to become interesting. The whole magic will happen between II and IV of table 1 and figure 2. We will work the paths so that they show up progressively on the graphics of each TMA. The whole section III of figure 2 and the transitions from and to it will be scrutinized.

Looking at table 1 section III is when all $\exists o@\ldots$ are zero. Let us invent a new matrix M_{ijo} encasing them together:

$$M_{ijo} = \exists o@\left[r_i, x_i, r_j, x_j\right] \tag{13}$$

The place to implement our sigmoid function $S^{(2)}_{ijo}$ from equation (11) is where M_{ijo} is zero and where M_{ijo} is something else it doesn't matter to us. Since it will later on multiply with the paths it is convenient to be zero so it will suppress the paths that have nothing to do with it.

$$S_{ijo} = \begin{cases} S^{(1)}_{ijo} = 0 & , \quad M_{ijo} \neq \vec{0} \\ \\ S^{(2)}_{ijo} & , \quad M_{ijo} = \vec{0} \end{cases} \tag{14}$$

Now we will call the loxodromic distance between two points **A** and **B** as $D_{A,B}$ to make things more simple. This $D_{A,B}$ represents the loxodromic distance from [5].

Let a be a point on the transition between II and III; l be a point somewhere in the middle of III; and z the point in the transition between III and IV (relative to figure 2). We want an estimate on the mean velocity of the object (which happens to be an aircraft) in l so that we have a prediction of the time stamp on the instant z to be calculated at each time step.

$$\hat{V}_l = \frac{D_{l,1}}{t_{l,1}} \tag{15}$$

I already told you what the l stands for on the subscript of $D_{l,1}$. The 1 represents the first moment when we are aware of the aircraft. Since we have our velocity estimate $\hat{V}_l$ let us try and find $k_z(k)$ - the time where the aircraft reaches TMA j.

$$k_z(k) = \frac{D_{l,z}}{\hat{V}_l} \tag{16}$$

In order to know $k_z(k)$ we first have to find $D_{l,z}$. The distance from a to z is composed of the distance from a to l plus the distance from l to z:

$$D_{a,z} = D_{a,l} + D_{l,z} \tag{17}$$

And the distance from a to the center of TMA j is the distance from a to z plus the radius of TMA j:

$$D_{a,j} = D_{a,z} + R_j \tag{18}$$

Playing around with these two equations we have:

$$D_{a,j} - R_j = D_{a,l} + D_{l,z} \Leftrightarrow$$

$$\Leftrightarrow D_{a,z} = D_{a,l} + D_{l,z} \tag{19}$$

With this last piece of the puzzle qe can now roughly predict $k_z(k)$.

Please notice that this model doesn't take into account the acceleration or deceleration of the aircraft. It also doesn't care for the 2D trajectory (from up above) it follows; much less its 3D trajectory. It makes this model susceptible to error.

The success of it is only incident on showing the possibility or not possibility of implementing a prediction to the model in section 3. A model regarding flight dynamics and having several Kalman Filters might become a project whose scale is much bigger than this dissertation.

Nevertheless the following equation is fine to be implemented:

$$\boxed{\chi_j(k) = x_j(k) + \left[\sum_{i=1}^{n} p_{ij}(k)\left(\sum_{o=1}^{O} S_{ijo}(k)\right)\right]} \tag{20}$$

Here O is the total number of aircraft an o the step aircraft. It uses the values of $x_j(k)$ and $p_{ij}(k)$ from the previous model and shapes them to our needs.

4.3 Simulation

The sigmoid model is taking the paths $p_{ij}(k)$ and adding them progressively with the arriving TMA $x_j(k)$. There is a need to remove abrupt changes inherent to the model in a way that an aircraft would gradually become present on the arriving TMA from the moment he lifted off the departing TMA, instead of just showing up on the arriving TMA and thus giving no anticipation opportunity to the air traffic controllers.

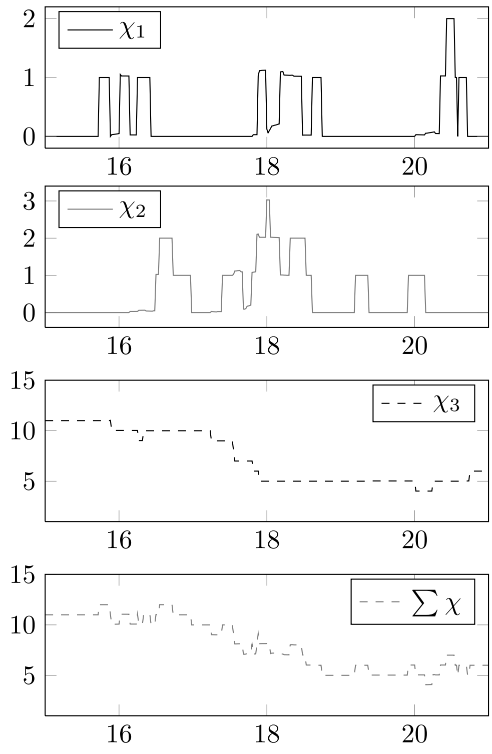

Figure 8: Number of aircraft on each TMA at a determined time of the day regarding the incoming aircraft as well.

A feature inherent to such a model if that the number of aircraft will no longer be constant. Think about it this way — we are considering that a fraction of one aircraft is arriving to a TMA. It will start as, let us say, about 0.1; then it will increase to 0.4; ...; 0.8; until it reaches 1. It is only when it is inside TMA $x_j(k)$ we consider it a full aircraft.

Equation (21) allows us to see this disparity in the last plot of figure 8.

$$\sum \chi(x) = \sum_{i=1}^{n} \chi_i(k) \tag{21}$$

4.4 One Aircraft Only Sigmoid

It could be interesting to see one aircraft by itself in order to comprehend the effects of this model individually. The way that the *Octave* simulation program was written didn't make the task easy. Another solution was plotted instead of the obvious first choice.

In order to see individually what could happen if there was just one aircraft to study it was performed a simulation with nine (9) aircraft. Each one of these aircraft have the same set of data and are identical to each other, i.e., they are copies of the first one.

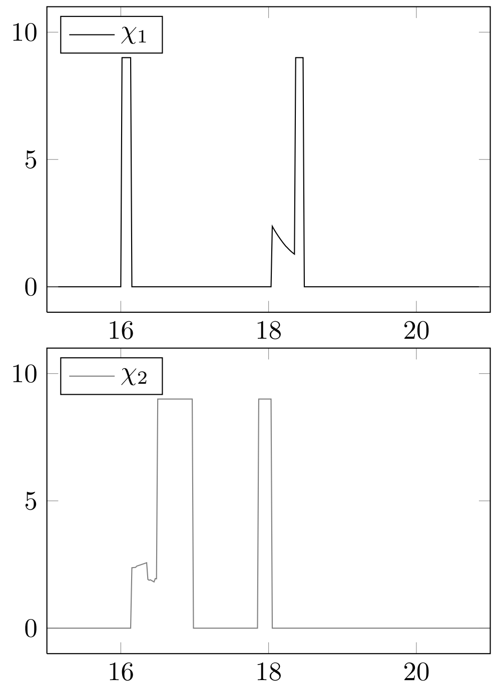

Figure 9: Nine identical aircraft travelling between two TMA. It regards the incoming aircraft as well.

As expected the graphics at figure 11 portrait an aircraft departing from TMA x_1 a while after 04:00 p.m. (16:00) and arriving to TMA x_2 around 04:30 p.m. (16:30). The voyage to return took more or less the same time and the aircraft arrived a few minutes before 06:30 p.m. (18:30).

Figure 10 corroborates the information provided by figure 11. The arrivals to the runways are not portrayed on either one of these graphics.

The "cherry on top of the cake" would happen if both the steps of the graphic on picture 10 were perfect S shaped curves starting on zero until they reached the value of nine. Since it doesn't happen there is certainly something

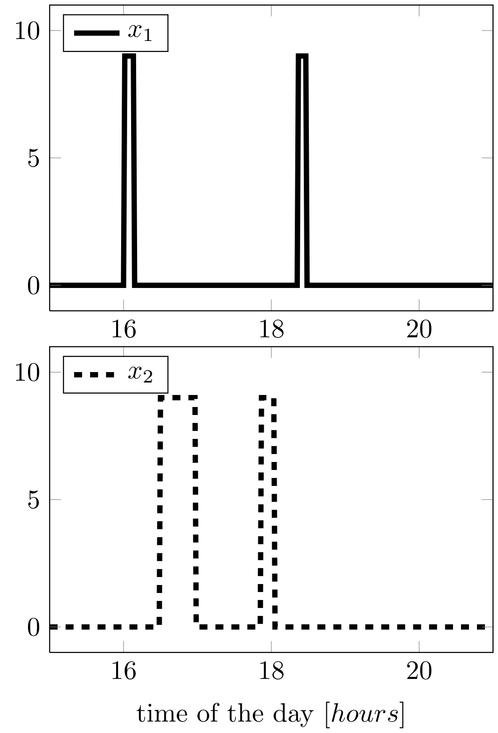

Figure 10: Nine identical aircraft travelling between two TMA. It denotes when they are in which TMA.

wrong with the witchcraft that guesses $k_z(k)$ — something is wrong with subsection 4.2.

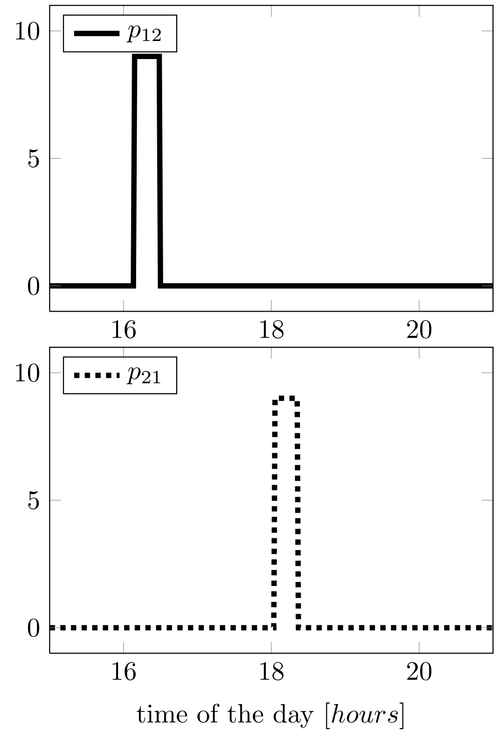

Figure 11: Paths of nine identical aircraft amongst two TMA.

The same sympthoms of figure 9 can be observed in figure 8. Anything different from this resemblance would point either towards some kind of code issue or instead to some better news. Neither one happened so it means that this model is not perfect. All the shortcomings and achievements of this model are discussed in the conclusion chapter at 5.

5 Conclusions and Future Work

This work started with a discrete model for Air Traffic Dynamics in which there was not a sense of in between and all TMA were contiguous. It was a mean to an end and it provided the mathematical basis to work on.

A second model was idealized and provided enough depth to accommodate paths between different airports and even a separation between the TMA and the runway. This increased model breaks up with the last one in a sense that it can already be used in a real scenario and it is lo longer just a theoretical algebraic manipulation.

At this stage the flexibility to incorporate new features in a simple way started to take shape and it is definitely a plus of this work. Changes for the model to match the requirements of a certainly more complex real life application should be easily implemented in order to create an improved similar model.

To identify the control and the state of each model was also a great achievement for it opens doors for some Control Theory approaches. Optimisation and discrete-time to continuous-time transformation will now become easier to perform.

Sigmoid model was an attempt to achieve the ultimate goal which is to implement a sense of prediction to this model. It could be seen as a layer working above the second model in order to transform the data provided into a more humanized perspective.

By picking up the numbers from the in-traffic aircraft and adding them progressively into the number of aircraft on the destination TMA we could provide a more intuitive and less abrupt advisory for the air traffic controllers.

To identify the control and the state of each model was also a great achievement for it opens doors for some Control Theory approaches. Optimisation and discrete-time to continuous-time transformation will now become easier to perform.

In a future work an estimator, such as a Kalman Filter, should be added to this model. It may come a time when we don't know the data from all airports. And in fact we don't need to know all data from all airports all over the world. It is not even logical. A Kalman Filter could help in providing an estimation on the incoming traffic from airports where there is somehow a restriction to release data.

The third model (section 4) has a few shortcomings that must be highlighted on this conclusion. The beautiful S-shape is not present as it was intended to be. Some few characteristics of the model might explain this scenario. First of all there is not enough data to perform a flight dynamics analysis.

Also the script program itself could aid on providing an explanation for what happened.A huge disregard on the early stages of this project was the disposal of data such as the instantaneous velocity of each aircraft in every point. Even the altitude could have come in handy.

Another Kalman Filter would have to be implemented here in order to uniform the data into sets distant from each other of a specific time step. This is prediction that should be made on the basic inputs even before the large models should be remotely considered. Octave script was designed to copy the same data to the following time step up until a new set of data would emerge from the transponder. The frequency on which this information arrived is different from aircraft to aircraft and it doesn't start at the same time making a mess that had to be dealt with somehow.

Not all is bad because it has proven that once there is a proper $k_z(k)$ prediction model then the *sigmoid model* can be used. I will be able to show air traffic controllers a realistic scenario whilst being intuitive enough for them to understand if they are or will become in trouble in the near future.

This work could be used as a foundation for the implementation of an advisory to the Air Traffic Controllers of the occupation status. Not only that but it could be a module on a fully automatic Airspace Controller or even other applications yet to be considered. It could even be applied on the traffic of buses (coaches) and it can be extrapolated to many other fields.

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Table 1: Trajectory of an aircraft from TMA x_1 to TMA x_2 and the generation of the control of the system which are the departures. Figure 2 illustrates the departures.

(Figure 2)	V	I	I	II	II	III	III	IV	IV	V	V	I
i	3	1	1	1	1	1	1	1	1	1	1	2
j	1	2	2	2	2	2	2	2	2	2	2	1
$\exists o@r_i$	0	1	1	0	0	0	0	0	0	0	0	1
$\exists o@x_i$	0	1	1	1	1	0	0	0	0	0	0	1
$\exists o@r_j$	1	0	0	0	0	0	0	0	0	1	1	0
$\exists o@x_j$	1	0	0	0	0	0	0	1	1	1	1	0
Increment	—		d_{ii2ii1}		d_{ii1ij1}		d_{ij1jj1}		d_{jj1jj2}		—	

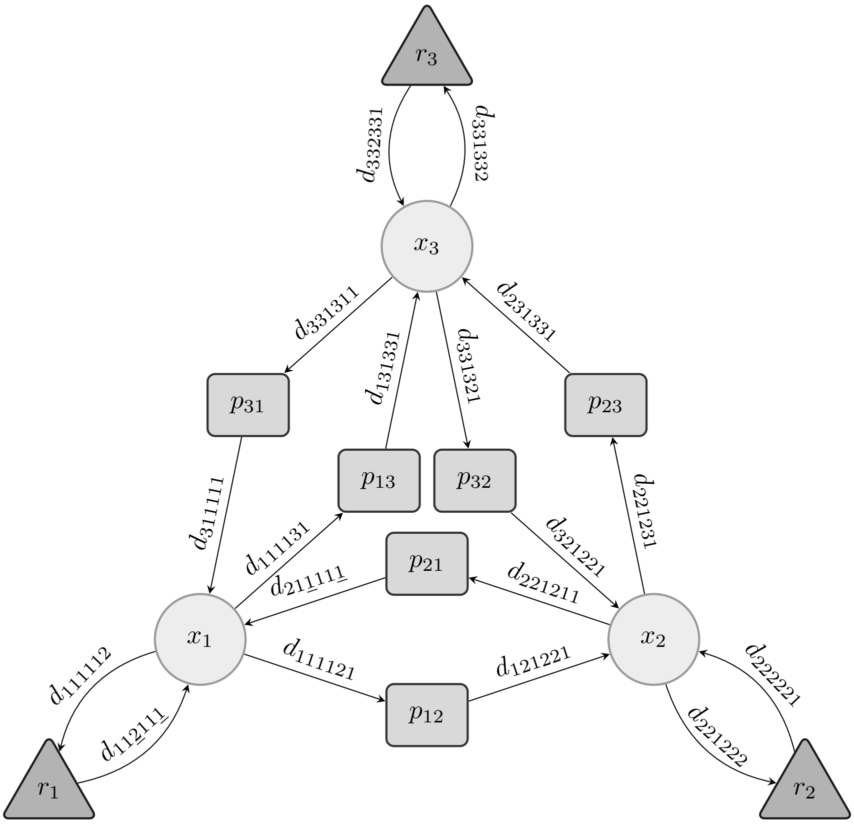

Figure 3: Graph representing our case study. The interaction is pictured between airports 1 and 2 considering the paths between them and the runways.

Table 2: Comparison between equations in sections 2 and 3

Section 2	$x_i(k)$	$\sum_{j=1, i\neq j}^{n} \alpha_{ji}(k)$	$\sum_{j=1, i\neq j}^{n} d_{ij}(k)$	α_{ii}
Section 3	$p_{ij}(k)$	$\sum_{j=1, i\neq j}^{n} \alpha_{ii1ij1}(k)$	$\sum_{j=1, i\neq j}^{n} d_{ij1jj1}(k)$	$\alpha_{ij1ij1}(k)$
	$x_i(k)$	$\alpha_{ii2ii1}(k) + \sum_{j=1, i\neq j}^{n} \alpha_{ji1ii1}(k)$	$d_{ii1ii2}(k) + \sum_{j=1, i\neq j}^{n} d_{ii1ij1}(k)$	$\alpha_{ii1ii2}(k)$
	$r_i(k)$	$\sum_{j=1, i\neq j}^{n} \alpha_{ii1ii2}(k)$	$\sum_{j=1, i\neq j}^{n} d_{ii2ii1}(k)$	$\alpha_{ii2ii2}(k)$